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Domain Walls in Strongly Coupled Theories

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Abstract

Domain walls in strongly coupled gauge theories are discussed. A general mechanism is suggested automatically leading to massless gauge bosons localized on the wall. In one of the models considered, outside the wall the theory is in the non-Abelian confining phase, while inside the wall it is in the Abelian Coulomb phase. Confining property of the non-Abelian theories is a key ingredient of the mechanism which may be of practical use in the context of the dynamic compactification scenarios.

In supersymmetric ($N = 1$) Yang-Mills theories the energy density of the wall can be *exactly* calculated in the strong coupling regime. This calculation presents a further example of non-trivial physical quantities that can be found exactly by exploiting specific properties of supersymmetry. A key observation is the fact that the wall in this theory is a BPS-saturated state.

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1. Introduction Domain walls are inherent to field theories with spontaneously broken discrete symmetries. This phenomenon – occurrence of the domain walls – is quite common in solid state physics. In high energy physics, in many models, such symmetries are present too, for instance, a discrete symmetry associated with CP. Domain walls then naturally appear in course of evolution of our Universe and must be taken into account in cosmological considerations. Discussion of the issue started over twenty years ago [1]. Recently domain walls in supersymmetric (SUSY) theories (along with other topological or non-topological defects) were proposed as a possible mechanism for dynamical compactification which, simultaneously, ensure spontaneous SUSY breaking [2]. Within this approach, the matter our world is built from is nothing but the zero modes localized on the wall.

In this Letter we consider new classes of the domain walls which appear in the strongly coupled gauge theories. The first model is an example of a domain wall which traps massless gauge particles on the wall. Localizing the gauge bosons in the core of the defects is important for building realistic phenomenology based on the dynamical compactification scenarios [2]. Spinless bosons and spin-1/2 fermions, whose interactions are properly arranged, admit localized zero modes on the wall, a well-known fact. To the best of our knowledge, no fully satisfactory mechanism was suggested for localizing the massless vector gauge fields so far although attempts in this direction were reported in the literature (e.g. [3]).

The second example of an unusual domain wall is specific to SUSY gauge theories. This particular wall is not suitable for dynamical compactification. Nevertheless, we find this object extremely interesting since it has a remarkable property. Although the wall appears in the strong-coupling theory, its energy density per unit area ε is *exactly* calculable. In a sense, the situation reminds that with the monopole mass in the Seiberg-Witten solution [4], although we deal with $N = 1$ supersymmetry, not $N = 2$. Explicit calculation of ε, to be carried out below, helps clarify one old question in supersymmetric Yang-Mills theory, which is in the focus of the ongoing polemics. It is known that the supersymmetric gluodynamics possesses $Z_{2T(G)}$ symmetry, a remnant of the anomalous $U(1)$ [5]. (Here $T(G)$ is the Dynkin index for the adjoint representation of the gauge group G, normalized in such a way that for $SU(N)$ the index $T(SU(N)) = N$.) A controversy continues as to whether this $Z_{2T(G)}$ is spontaneously broken in a standard way, implying the conventional domain walls, or there is an additional superselection rule, implying that the vacuum angle θ varies not from 0 to 2π, as usually believed, but, rather from 0 to $2\pi T(G)$. Arguments *pro* and *contra* were given; they are summarized in Ref. [6]. Our result, confirming a finite value of ε, favors the first option.

2. Massless gauge fields on the wall A mechanism we suggest does not depend on whether or not the theory at hand is supersymmetric. To elucidate the idea we will consider a simple (non-supersymmetric) example. Assume we have $SU(2)$ Yang-Mills theory whose matter sector contains:

(i) one left- and one right-handed fermion doublet field $(\psi_L)_\alpha$ and $(\psi_R)_\alpha$ in the fundamental representation of $SU(2)$, one scalar field χ^a in the adjoint representation, and one real scalar field η carrying no color indices. The interaction Lagrangian has the form

$$\mathcal{L} = -\frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu} + \bar{\psi}_L \, \not{D} \psi_L + \bar{\psi}_R \, \not{D} \psi_R - \left(h \eta \bar{\psi}_L \psi_R + \text{h.c.} \right) +$$

$$\frac{1}{2} (D_\mu \chi^a)^2 - \frac{1}{2} \lambda' (\chi^2 + \kappa^2 - v^2 + \eta^2)^2 + \frac{1}{2} (\partial_\mu \eta)^2 - \lambda (\eta^2 - v^2)^2 \, , \tag{1}$$

where $G^a_{\mu\nu}$ is the gluon field strength tensor, v and κ are positive parameters of dimension of mass assumed to be much larger than the scale parameter Λ of the $SU(2)$ gauge theory at hand [1], λ and λ' are (small) dimensionless coupling constants, g is the gauge coupling constant.

The theory is obviously Z_2 invariant under the transformation $\eta \to -\eta$ and $\psi_L \to i\psi_L, \quad \psi_R \to -i\psi_R$. In the true vacuum of the theory this symmetry is spontaneously broken, and the field η develops a vacuum expectation value (VEV),

$$\eta = v \ \text{ or } \ - v \, . \tag{2}$$

Correspondingly, the self-interaction potential for χ is stable, and the gauge $SU(2)$ is not spontaneously broken. The theory is in the confining phase. All observable degrees of freedom are bound states of gluons and/or matter fields, with masses [2] of order of Λ. The mass of the η quantum is of order

$$m = \sqrt{2\lambda} \, v \, . \tag{3}$$

The theory has a stable domain wall interpolating between two different vacua in Eq. (2),

$$\eta_0 = v \tanh (mz) \, . \tag{4}$$

For definiteness the wall is placed in the $\{x, y\}$ plane; the width of the wall in the z direction is of order of m^{-1}.

Let us consider gauge-non-singlet massless modes localized on the wall. First, there are two massless fermion doublet modes localized on the membrane,

$$(\psi_{L,R})_\alpha = (\nu_{L,R})_\alpha \, \mathrm{e}^{-2h \int_0^z \eta_0(z') dz'} \tag{5}$$

where ν depends only on x, y and t, and $\gamma_z \nu = \nu$. Localization of these modes is due to the index theorem [7] and has nothing to do with the gauge dynamics. This is

[1] More exactly, we will assume that $\lambda v^2 \ll \Lambda^2$ but $\sqrt{\lambda} v^2 \gg \Lambda^2$. The latter requirement is introduced for simplicity. If this requirement is imposed we can ignore the shift of the vacuum energy due to the gluon condensate outside the wall.

[2] In the $SU(2)$ gauge theory with one quark flavor there is an "accidental" global $SU(2)$ symmetry due to the fact that anti-doublet in $SU(2)$ is the same as doublet. If this global symmetry is spontaneously broken we might get massless "pions". This is not a serious problem, however. To avoid massless "pions" suffice it to consider gauge $SU(3)$. Another possibility is to assume that $hv \sim \Lambda$, in which case the would-be pions acquire a mass of order Λ.

simply because of the topologically non-trivial boundary conditions on the fermion mass, $m_\psi(-\infty) = -m_\psi(+\infty) = -hv$. The localization scale is governed by h and becomes infinite if $h \to 0$. One of the crucial observations of the present work is that the gauge-charged fermions (or scalars), as well as massless gauge fields can stay localized on the wall even in the limit $h = 0$ due to confining gauge dynamics outside the wall (see below).

To see that this is indeed the case consider the behavior of the χ in the classical wall background. As was mentioned, far away from the wall, when η is close to v, the self-interaction potential for χ is stable and there is no spontaneous breaking of the gauge $SU(2)$. However, inside the wall $\eta \approx 0$, and the self-interaction potential for χ becomes unstable. It is not difficult to check that for the wide rang of parameters χ becomes tachionic in the core and develops a vacuum expectation value $\chi^2 \sim v^2$.

Indeed, consider [3] a linearized equation for small perturbations in $\chi^a = \delta_{3a}\chi_0 \mathrm{e}^{-i\omega t}$ in the kink background (4)

$$\left\{-\partial_z^2 + \lambda' \left[\kappa^2 + v^2(\tanh^2(mz) - 1)\right]\right\}\chi_0 = \omega^2\chi_0\,, \tag{6}$$

This equation, say, for $\kappa = 0$ is a one-dimensional Shrödinger equation with a negative-definite potential which is known to have a normalizable bound-state solution with negative ω^2. Due to continuity this bound-state solution should persist for a finite range of non-vanishing κ's. Thus, χ becomes tachionic, marking an instability of the $\chi = 0$ solution in the core of the defect.

This means that inside the wall the $SU(2)$ gauge symmetry is spontaneously broken down to $U(1)$. Two out of three gluons acquire very large masses of order of v. The third gluon becomes a photon. Two degrees of freedom in the χ^a field are eaten up by the Higgs mechanism, the remaining degree of freedom is neutral. The "quarks" ψ_α have charges $\pm 1/2$ with respect to the surviving photon.

Let us have a closer look at the theory emerging in this way. Outside the wall the theory has a wider gauge invariance, $SU(2)$, and is in the non-Abelian confining phase. The $U(1)$ gauge invariance is maintained everywhere – inside and outside the wall. The light degrees of freedom inside the wall are massless "quarks", interacting through the photon exchange. We disregard the non-interacting degrees of freedom. The theory inside the wall is in the Abelian Coulomb phase. The photon and the light "quarks" can not escape in the outside space because there they become a part of the $SU(2)$ theory with no states lighter than Λ. The three-dimensional observer confined inside the wall needs energies of order Λ to be able to feel that his/her Universe is actually embedded in the four-dimensional world.

Let us parenthetically note that the Abelian Coulomb phase in 2+1 dimensions confines electric charges since the potential grows logarithmically with distance. The three-dimensional electromagnetic coupling constant α will be of order of m, this is also a typical mass of the neutral bound states whose size will be of order $L \sim$

[3] The line of reasoning we follow here is analogous to that of Ref. [8] for superconducting cosmic strings.

$(\mu m)^{-1/2}$. (Recent work [9] discusses a related issue – the behavior of the fermion zero modes trapped in the $(2+1)$-dimensional wall in a delocalized electromagnetic field dispersed in four dimensions.)

Needless to say that a similar mechanism will work for trapping, say, $SU(2)$ gluons inside the wall submerged into, say, $SU(3)$ environment, or in any other problem of this type.

3. Domain wall in supersymmetric gluodynamics Consider the simplest SUSY gauge theory, supersymmetric $SU(2)$ gluodynamics. The Lagrangian of the theory is

$$\mathcal{L} = \frac{1}{2g^2}\mathrm{Tr}\int d^2\theta W^2 = \frac{1}{g^2}\left\{-\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + i\lambda^{a\dagger}{}_{\dot\alpha}\partial_\mu(\bar\sigma^\mu)^{\dot\alpha\alpha}\lambda^a_\alpha\right\}, \tag{7}$$

where λ is the gluino field (in the Weyl representation). Witten's index of this theory is 2, which means that it has two degenerate supersymmetric vacua [5]. The vacua are marked by the order parameter $\lambda\lambda$ (note that the order parameter is λ^2 or $\lambda^{\dagger 2}$, not $\lambda\lambda^\dagger$). One can always adjust the vacuum angle θ in such a way that the corresponding VEV is

$$\mathrm{Tr}\langle\lambda^2\rangle = \Lambda^3 \ \text{ or } \ -\Lambda^3\,, \tag{8}$$

where Λ is the scale parameter of SUSY gluodynamics. The θ dependence of the condensates can be found on general grounds [10, 11],

$$\langle\lambda^2\rangle = \Lambda^3 \mathrm{e}^{i\theta/2}\,,$$

so that at $\theta = 2\pi$ the vacua interchange. In this Letter we will limit ourselves to $\theta = 0$.

The Z_4 symmetry of the model is spontaneously broken by the gluino condensate (8) down to Z_2. If this is a conventional discrete symmetry breaking, there must exist a wall interpolating between the two vacua. The explicit construction of the wall, routinely done in the weak coupling theories, is impossible, however, since λ^2 is a composite operator, and we are in the strong coupling regime. A way out can be indicated.

The key point is as follows. Superalgebra of the model under consideration has a very peculiar central extension which reveals itself only in the wall-like situations. How $N = 1$ SUSY can have a central extension is explained in detail in Ref. [2]. Briefly, there exists a trivially conserved "current" $J^{\mu\nu\alpha} = \epsilon^{\mu\nu\alpha\beta}\partial_\beta\lambda^2$; the corresponding charge is an anti-symmetric tensor assuming non-vanishing values only in the presence of walls. More specifically,

$$\{Q^\dagger_{\dot\alpha}Q^\dagger_{\dot\beta}\} = \frac{N_c}{4\pi^2}\left(\Sigma^i\right)_{\dot\alpha\dot\beta}\int d^3x\,\partial_i(\mathrm{Tr}\lambda\lambda)\,; \tag{9}$$

here $Q^\dagger$ is the supercharge, and $\Sigma^i = \sigma^i\sigma^2$ where σ^i stands for the Pauli matrices. Equation (9) is a quantum anomaly which will be discussed in more detail in subsequent publications. For completeness we give here the result referring to the gauge group $SU(N_c)$, rather than to $SU(2)$.

In the true vacuum, when all excitations are localized, the integral on the right-hand side vanishes identically. On the wall it reduces to

$$qV_2 = \frac{1}{2\pi^2} V_2 \mathrm{Tr}(\lambda\lambda)\,,$$

(for $SU(2)$), i.e. a non-vanishing central charge emerges. In the sector with the given (non-zero) value of the central charge the masses of all states can be shown to satisfy a quantum Bogomol'nyi bound,

$$M \geq qV_2\,. \tag{10}$$

The lower bound is achieved on the BPS-saturated states [12, 13]. The domain wall in supersymmetric gluodynamics has to be such a state. Although we have no rigorous proof of this statement, we see no reason which could prevent the BPS saturation. Moreover, in the weakly coupled models with the non-vanishing central charge one can easily verify that the wall is a BPS-saturated state. See e.g. the so-called minimal wall in Ref. [2]. For the minimal wall the equlity between ε and the central charge is explicitely derived in [2]. (Similar relation was first observed in a two-dimensional model in [12]). For the BPS-saturated states one half of the supersymmetry transformations act trivially, i.e. supersymmetry is preserved.

The only scenario with no BPS saturated states is when supersymmetry is completely broken by the solution. Although we mention this possibility, it is hard to imagine it is realized in SUSY gluodynamics. Additional arguments in favor of the BPS-saturated wall are provided by consideration of the weakly coupled SQCD plus holomorphy arguments, see below.

If the wall in SUSY gluodynamics is the BPS-saturated state (with half of SUSY transformations acting trivially), the energy density of the wall is nothing but the value of the central charge q,

$$\varepsilon = |q| = \frac{1}{2\pi^2} |\mathrm{Tr}\langle\lambda\lambda\rangle|\,. \tag{11}$$

It remains to be added that the gluino condensate in supersymmetric gluodynamics was exactly calculated in Ref. [10] for unitary and orthogonal groups, and in Ref. [14] for all other groups. Moreover, the wall profile of the order parameter $\lambda\lambda$ is related to the Lagrangian density,

$$\partial_z(\lambda\lambda) = G_{\alpha\beta}G_{\alpha\beta} + i\lambda^\dagger_{\dot\alpha}\partial^{\dot\alpha\beta}\lambda_\beta\,.$$

For arbitrary gauge group there are $(1/2)T(G)[T(G)-1]$ different walls, whose energy densities reduce to

$$\varepsilon = \frac{N_c}{8\pi^2}|\mathrm{e}^{2\pi ik/T(G)} - \mathrm{e}^{2\pi i\ell/T(G)}||\langle\lambda\lambda\rangle|\,,$$

where k and ℓ are integers running from 0 to $T(G)-1$. The fact that ε is proportional to the gluino condensate is not surprising by itself, since both quantities are of order of Λ^3. What is non-trivial is the exact proportionality coefficient. Inside the wall there lives a massless composite boson and a massless composite fermion; a half of supersymmetry is preserved.

If one does not want to rely on the previous results on the gluino condensate one can calculate directly ε using the same strategy as was first suggested in Ref. [10] for exact calculation of the gluino condensate. Namely, one additional quark flavor is introduced, with the mass term m. Thus, the original supersymmetric gluodynamics is substituted by SQCD with one flavor. If the scale parameter of SQCD with one flavor is $\tilde{\Lambda}$ and $m \ll \tilde{\Lambda}$, the theory turns out to be in the weakly coupled Higgs phase [15]. Then the construction of the domain wall can be carried out, and ε calculated. The result depends on the bare mass parameter m in a holomorphic way; therefore, the exact m dependence can be found, much in the same way as in Refs. [10, 11]. Then we can tend $m \to \infty$, making the matter fields very heavy. If they are very heavy, they can be integrated out, and we return back to SUSY gluodynamics. The result for ε will still be valid. The last step to be done is to express the parameters of SQCD with one light flavor in terms of parameters relevant to SUSY gluodynamics.

Let us outline some basic elements of the procedure. The structure of the $SU(2)$ model with one flavor is exhaustively described in the review [16]; the reader is referred to this paper for details and definitions. One flavor is comprised of two chiral superfields, $S^{\alpha f}$, where α is the $SU(2)$ index, while f is the subflavor index, $f = 1, 2$. Classically the model has a one-dimensional vacuum valley (flat direction) parametrized by VEV of the composite operator $S^2 = S^{\alpha f} S_{\alpha f}$. Quantum-mechanically, a superpotential is generated along the valley [15]; it has the form

$$\mathcal{W} = \frac{\tilde{\Lambda}^5}{S^2} + \frac{m}{4} S^2 \,, \tag{12}$$

where we have included also the (tree level) mass term. The latter stabilizes the theory eliminating the run-away vacuum. Note that the mass term *explicitly* breaks the original continuous R symmetry of the theory down to Z_2, under which $S^2 \to -S^2$. It is a spontaneous breakdown of this discrete subgroup in the vacuum that gives rise to a domain wall solution. If m is small, the fields residing in S^2 are light, the vacuum expectation value of S^2 is large, the $SU(2)$ symmetry (as well as Z_2) is spontaneously broken, the gluons acquire a large mass (so that they are actually W bosons) and can be integrated over. As a result of this integration, the superpotential in Eq. (12) is generated through instantons. Equation (12) is exact – it has neither perturbative nor non-perturbative corrections [15, 10]. The vacuum expectation values of S^2 are

$$\langle S^2 \rangle = 2\tilde{\Lambda}^{5/2} m^{-1/2} \ \text{ or } \ -2\tilde{\Lambda}^{5/2} m^{-1/2} \,. \tag{13}$$

The low-energy theory is that of one chiral superfield; it resembles the Wess-Zumino model [17]. The only difference is a slightly unusual form of the superpotential, but its particular form is unimportant for our purposes. The domain wall in the Wess-Zumino model was discussed in detail in Ref. [2] (the "minimal wall"). In the case at hand it is possible, in principle to find an explicit solution interpolating between two vacua of Eq. (13), as a function of z, which will be valid almost everywhere. A small interval, of the size of order $\tilde{\Lambda}^{-1}$, near the origin where the value of S^2 is small, is a strong coupling region. The semiclassical description of the wall in terms of one superfield S^2 is invalid here, since in this region excitations corresponding to composite gauge invariant operators generically have masses of order $\tilde{\Lambda}$. The correct description requires many degrees of freedom. Outside the above narrow region the wall is properly described semiclassically by a profile of S^2. The width of the wall in the z direction is of order m^{-1}. Thus, our wall is a two-component construction. Our ignorance of the small central region does not preclude us from calculating the wall energy density ε exactly, provided that the description is continuous. Indeed, the central charge q is related to the integral

$$J^{\mu\nu} = \int d^3x \varepsilon^{0\mu\nu\rho} \partial_\rho \mathcal{W}(S^2) \to \left[\mathcal{W}(S^2)_{z\to+\infty} - \mathcal{W}(S^2)_{z\to-\infty}\right] .$$

At large separations from the wall we are approaching the true vacuum, where the theory is in the weakly coupled Higgs phase, and the values of $\mathcal{W}(S^2)_{z\to\pm\infty}$ are exactly known. In this way we find that

$$q = \text{a known const.} \times \tilde{\Lambda}^{5/2} m^{1/2} .$$

Moreover, the condition that 1/2 of SUSY transformations act trivially (equivalent to the BPS saturation) is explicitely satisfied everywhere in the weak coupling region. By holomorphy we conclude it must be satisfied in the central region as well.

Tending $m \to \infty$ and eliminating $\tilde{\Lambda}$ in favor of Λ, the scale parameter of supersymmetric gluodynamics, we reproduce Eq. (11).

Conclusions The peculiar features of the domain wall discussed above are due to intricacies of the non-Abelian gauge dynamics. The confining property of gluodynamics and the dynamical mass gap generation is the basic element of the mechanism we suggested for trapping massless gauge bosons inside the wall. A task that lies ahead is using this mechanism in the context of the dynamic compactification scenarios outlined in [2].

As it happened more than once in the past, miracles of supersymmetric ($N = 1$) gauge dynamics allowed us to exactly calculate the energy density of the supersymmetric wall in the strong coupling regime. This new example of non-trivial physical quantity that can be found exactly by exploiting specific properties of supersymmetry is interesting by itself. Moreover, it settles the issue of spontaneous breaking of $Z_{2T(G)}$ in supersymmetric gluodynamics *versus* a new superselection rule, which was debated over a decade, in favor of the first option.

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