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ENHANCEMENT OF SUPERSYMMETRY

NEAR 5d BLACK HOLE HORIZON

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ABSTRACT

Geometric Killing spinors which exist on $AdS_{p+2} \times S^{d-p-2}$ sometimes may be identified with supersymmetric Killing spinors. This explains the enhancement of unbroken supersymmetry near the p-brane horizon in d dimensions. The corresponding p-brane interpolates between two maximally supersymmetric vacua, at infinity and at the horizon. New case is studied here: $p = 0$, $d = 5$. The details of supersymmetric version of the very special geometry are presented. We find the area-entropy formula of the supersymmetric 5d black holes via the volume of S^3 which depends on charges and intersection matrix.

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1 Introduction

The enhancement of supersymmetry near the p-brane horizon is an interesting phenomenon. The corresponding p-branes interpolate between two maximally supersymmetric vacua: $\mathcal{M}^d$ at infinity and $AdS_{p+2} \times S^{d-p-2}$ near the horizon [1, 2, 3]. The generic reason for the enhancement of supersymmetry is the following. Any anti-deSitter space as well as any sphere admit Killing spinors which we will call geometric Killing spinors. The relation of this geometric Killing spinor, which has a dimension of a Dirac spinor, to the Killing spinor of unbroken supersymmetry requires further investigation. In theories with N=2 supersymmetry the unbroken supersymmetry of the p-brane has dimension of one half of the Dirac spinor. If near the horizon the dimension of supersymmetric Killing spinor becomes that of the Dirac spinor, we have a enhancement of supersymmetry. In some cases it has already been established that the Killing spinor defined by the zero mode of the gravitino transformation at the near horizon geometry of the p-brane solutions coincides with the geometric spinors. These cases include : d=4, p=0 with $AdS_2 \times S^2$ [1, 2]; d=10, p=3 with $AdS_5 \times S^5$; d=11, p=2 with $AdS_3 \times S^7$ and d=11, p=5 with $AdS_7 \times S^3$ [3]. The near near horizon geometry of these p-branes is known to be maximally supersymmetric. In d=4 the integrability condition for the Bertotti-Robinson geometry near the black hole horizon $AdS_2 \times S^2$ was proved in [4, 2] using the fact that this geometry is conformally flat and that the graviphoton field strength is covariantly constant.

Remarkably, the supersymmetry near the 5d black hole horizon has not yet been studied. Moreover, the unbroken supersymmetry of the 5d black holes was established in [5] only for black holes of pure N=2, d=5 supergravity without vector multiplets. In particular the Strominger-Vafa 5d black holes [6] and the rotating generalization of them [7] have not yet been embedded into a particular 5d supersymmetric theory and the unbroken supersymmetry of either solution has not been checked directly. An analogous situation holds with the more general 5d static and rotating black holes found in [8]. There are various indications, however, that these solutions have unbroken supersymmetry.

The purpose of this paper is to find out whether the 5-dimensional black holes near the horizon show the enhancement of the supersymmetry. Specifically we will try to identify the supersymmetric Killing spinors admitted by black holes near the horizon with the geometric Killing spinors of $AdS_2 \times S^3$. We will also derive the area formula for generic solutions in N=2 theory interacting with arbitrary number of vector multiplets as the volume of the S^3 and express it as the function of charges and the intersection matrix.

For the d-dimensional manifold which is a product space $AdS_{p+2} \times S^{d-p-2}$ the geometric Killing spinors are given by the product of Killing spinors on AdS_{p+2} and S^{d-p-2}. On AdS_{p+2} and on S^{d-p-2} the Killing spinor equations are

$$\hat{\nabla}_a \eta(x) \equiv (\nabla_a + c_1 \tilde{\gamma}_a)\eta(x) = 0 \ , \qquad a = 0, 1, \ldots, p+1 \ , \tag{1}$$

$$\hat{\nabla}_\alpha \varsigma(y) \equiv (\nabla_\alpha + c_2 \gamma_\alpha)\varsigma(y) = 0 \ , \qquad \alpha = p+2, \ldots, d-1. \tag{2}$$

Here γ_α is the γ-matrix of the $d-p-2$-dimensional Euclidean space and the commutator $[\tilde{\gamma}_a, \tilde{\gamma}_b]$ equals $-[\gamma_a, \gamma_b]$ where γ_a is the γ-matrix of the $p+2$-dimensional Minkowski space.

The integrability conditions for the geometric Killing spinors defined above is

$$[\hat{\nabla}_a, \hat{\nabla}_b]\eta(x) = 0 \qquad \Longrightarrow \qquad R_{ab}{}^{cd} = 4(c_1)^2(\delta_a{}^c\delta_b{}^d - \delta_a{}^d\delta_b{}^c) \tag{3}$$

$$[\hat{\nabla}_\alpha, \hat{\nabla}_\beta]\varsigma(y) = 0 \qquad \Longrightarrow \qquad R_{\alpha\beta}{}^{\gamma\delta} = -4(c_2)^2(\delta_\alpha{}^\gamma\delta_\beta{}^\delta - \delta_\alpha{}^\delta\delta_\beta{}^\gamma) \tag{4}$$

and it is satisfied by the geometries of the Anti-deSitter space and sphere. The full Killing spinor is

$$\epsilon(x, y) = \eta(x)\varsigma(y) \tag{5}$$

and it forms a Dirac spinor in a given dimension d. On the other hand the Killing spinor of unbroken supersymmetry of the p-brane solution near the horizon is defined by the corresponding supersymmetry transformation of the gravitino. If supersymmetric Killing spinor near the horizon can be identified with the geometric one of the maximally supersymmetric product space, we have an enhancement of supersymmetry:

$$\delta\Psi_M = 0 \qquad \Longrightarrow \qquad \hat{\nabla}_M\epsilon(x, y) = 0 \Longrightarrow \begin{pmatrix} \hat{\nabla}_\mu\eta(x) = 0 \\ \hat{\nabla}_\alpha\varsigma(y) = 0 \end{pmatrix} . \tag{6}$$

To study this issue we will work out some details of d=5 N=2 supergravity coupled to N=2 vector multiplets in the framework of very special geometry, see Sec. 2. This formulation of the d=5 theory is particularly adapted to d=11 supergravity compactified on Calabi-Yau manifold characterized by the intersection number C_{ABC}. In Sec. 3 we will identify the supersymmetric Killing spinors admitted by black holes near the horizon with the geometric Killing spinors of $AdS_2 \times S^3$. We will also perform detailed derivation of the area formula of the five-dimensional black holes [9] in terms of the volume of S^3 which we evaluate in terms of the charges and intersection number matrix. In Sec. 4 we will consider a special case of the N=2 theory interacting with one vector multiplet, which comes from the truncation of N=4, d=5 supergravity. This will allow us to embed some of the known black holes into supersymmetric theories and study the enhancement of supersymmetry near the horizon. We also study the N=4 theory interacting with arbitrary number n of N=4 vector multiplets as a particular example of N=2 theory interacting with $n+1$ N=2 vector multiplets and intersection matrix $C_{oij} = \eta_{ij}$, where η_{ij} is the Lorenzian metric on $(1, n)$. The duality invariant area-entropy formula for the N=4 theory is truncated to N=2 theory as it was done before in four-dimensional theory in [10]. In this way one describes the theory with very special geometry $O(1,1) \times O(1,n)/O(n)$. In the discussion section we explain the relation between the enhancement of supersymmetries and finiteness of the area of the horizon and speculate about other cases as yet not known.

2 Supersymmetric very special geometry in d=5

The action for d=5, N=2 supergravity coupled to N=2 vector multiplets has been constructed by Günaydin, Sierra and Townsend [11]. The bosonic part of the action has been adopted to the very special geometry [12] and the compactification of 11d supergravity down to five dimensions on Calabi-Yau 3-folds [13] with Hodge numbers $(h_{(1,1)}, h_{(2,1)})$ and topological intersection form

C_{IJK}. For our purpose, it will be extremely useful to adopt the full action and the supersymmetry transformation laws of [11] to that of very special geometry. In what follows we will give the detailed derivation of some formulae reported in [9].

The fields of the theory are: $e_\mu{}^m$, $A_\mu{}^I$, ϕ^i, $\psi_{\mu r}$, $\lambda_r{}^i$ where $I = 0, 1, \ldots, h_{(1,1)} - 1$, $i = 1, \ldots, h_{(1,1)} - 1$, and $r = 1, 2$. The index r on the spinors is raised and lowevered with the symplectic metric ϵ_{rs} and will be omitted.

The N=2 d=5 supersymmetric Lagrangian describing the coupling of vector multiplets to supergravity is determined by one function which is given by the intersection form on a CY 3-fold:

$$\mathcal{V} = \frac{1}{6} C_{IJK} X^I X^J X^K \ . \tag{7}$$

The action is

$$\begin{aligned} e^{-1}\mathcal{L} &= -\frac{1}{2}R - \frac{1}{2}\bar{\psi}_\mu \Gamma^{\mu\nu\rho} D_\nu \psi_\rho - \frac{1}{4} G_{IJ} F_{\mu\nu}{}^I F^{\mu\nu J} - \frac{i}{2}\bar{\lambda}^i (g_{ij}\Gamma^\mu D_\mu + \Gamma^\mu \partial_\mu \phi^k \Gamma^l_{kj} g_{li})\lambda^j \\ &- \frac{1}{2} g_{ij}\partial_\mu \phi^i \partial^\mu \phi^j - \frac{i}{2}\bar{\lambda}_i \Gamma^\mu \Gamma^\nu \partial_\nu \phi^i + \frac{1}{4}\left(\frac{3}{4}\right)^{2/3} t_{I,i} \bar{\lambda}^i \Gamma^\mu \Gamma^{\lambda\rho} \psi_\mu F^I_{\lambda\rho} \\ &+ \frac{i}{16 \cdot 6^{1/3}}\Big(g_{ij} t_I - 9 C_{JKL} t^J_{,i} t^K_{,j} t^L_{,k} t_I^k\Big) - \frac{3i}{16 \cdot 6^{1/3}} t_I \Big(\bar{\psi}_\mu \Gamma^{\mu\nu\rho\sigma} \psi_\nu F^I_{\rho\sigma} + 2\bar{\psi}^\mu \psi^\nu F^I_{\mu\nu}\Big) \\ &+ \frac{e^{-1}}{48} \epsilon^{\mu\nu\rho\sigma\lambda} C_{IJK} F^I_{\mu\nu} F^J_{\rho\sigma} A^K_\lambda + \ldots \ , \end{aligned} \tag{8}$$

where we use dots for 4-fermionic terms. Here $t^I = t^I(\phi)$ are the special coordinates subject to the conditions

$$t^I t_I = 1, \qquad C_{IJK} t^I t^J t^K = 1 \tag{9}$$

and t_I are the "dual coordinates"

$$t_I = C_{IJK} t^J t^K \equiv C_{IJ} t^J \ , \qquad t^I = C^{IJ} t_J \ . \tag{10}$$

We have introduced a notation

$$C_{IJ} \equiv C_{IJK} t^K \ , \qquad C^{IJ} C_{JK} = \delta^I{}_K \ . \tag{11}$$

The metric is derived from the prepotential (7) through the relation ($\partial_I \equiv \frac{\partial}{\partial X^I}$)

$$G_{IJ} = -\frac{1}{2}\partial_I \partial_J (\ln \mathcal{V})|_{\mathcal{V}=1} \tag{12}$$

and X^I is related to t^I by

$$X^I = 6^{1/3} t^I|_{\mathcal{V}=1} \ . \tag{13}$$

Finally, the metric g_{ij} is given by

$$g_{ij} = G_{IJ} X^J{}_{,i} X^K{}_{,j} = -3 C_{IJK} t^I t^J{}_{,i} t^K{}_{,j} \ . \tag{14}$$

We can express G_{IJ} in terms of t^I through the equations

$$G_{IJ} = -\frac{6^{1/3}}{2}(C_{IJ} - \frac{3}{2} t_I t_J) \tag{15}$$

and the inverse metric acting on vectors is

$$G^{IJ} = -\frac{2}{6^{1/3}}(C^{IJ} - 3t^I t^J) \ , \qquad G^{IK}G_{KJ} = \delta^I{}_J \ . \tag{16}$$

We will need in what follows the relation between the derivative of the special coordinate $(t^I)_{,i}$ and that of the dual coordinate $t_{I,i}$. Using the fact that C_{IJK} is a symmetric numerical tensor we have

$$t_{I,i} = 2C_{IJK}(t^J)_{,i}\, t^K = 2C_{IJ}(t^J)_{,i} \ , \qquad (t^J)_{,i} = \frac{1}{2}C^{IJ}t_{I,i} \ . \tag{17}$$

Differentiating eqs. (9) we get

$$C_{IJK}t^I t^J (t^K)_{,i} = 0 \ , \qquad t_I (t^I)_{,i} = t_{I,i}t^I = 0 \ . \tag{18}$$

The supersymmetry transformation laws are:

$$\begin{aligned}
\delta e_\mu{}^m &= \frac{1}{2}\bar\epsilon\Gamma^m\psi_\mu \ , \\
\delta\psi_\mu &= D_\mu(\hat\omega)\epsilon + \frac{i}{8\cdot 6^{1/3}}t_I\Big(\Gamma_\mu{}^{\nu\rho} - 4\delta_\mu{}^\nu\Gamma^\rho\Big)\hat F_{\mu\nu}{}^I\epsilon \\
&+ \frac{1}{12}g_{ij}\left(\frac{1}{4}\Gamma_{\mu\nu\rho}\epsilon\bar\lambda^i\Gamma^{\nu\rho}\lambda^j - \Gamma_{\mu\nu}\epsilon\bar\lambda^i\Gamma^\nu\lambda^j - \Gamma^\nu\epsilon\bar\lambda^i\Gamma_{\mu\nu}\lambda^j + 2\epsilon\bar\lambda^i\Gamma_\mu\lambda^j\right) \ , \\
\delta A^I_\mu &= 6^{1/3}\left(\frac{1}{2}t^I{}_{,i}\bar\epsilon\Gamma_\mu\lambda^i + i\bar\psi\epsilon t^I\right) \ , \\
\delta\lambda_i &= \delta\phi^k\,\Gamma_{ki}{}^l\,\lambda_l + \frac{1}{4}\left(\frac{3}{4}\right)^{2/3}t_{I,i}\Gamma^{\mu\nu}\epsilon F^I_{\mu\nu} - \frac{i}{2}g_{ij}\Gamma^\mu\partial_\mu\phi^j\epsilon \\
&+ \frac{3i}{16}t^I{}_{,i}\,t^J{}_{,j}\,t^K{}_{,k}\left(-3\epsilon\bar\lambda^j\lambda^k + \Gamma^\mu\epsilon\bar\lambda^j\Gamma^\mu\lambda^k + \frac{1}{2}\Gamma_{\mu\nu}\epsilon\bar\lambda^j\Gamma^{\mu\nu}\lambda^k\right) \ , \\
\delta\phi^i &= \frac{i}{2}\bar\epsilon\lambda^i \ .
\end{aligned} \tag{19}$$

3 Near horizon geometry and supersymmetry

Double-extreme black holes (which have 1/2 of unbroken supersymmetry and have constant moduli [14]) in five dimension have the geometry of the extreme Tangherlini solution [15]. It is a 5d analog of the extreme Reissner-Nordstrom metric.

$$ds^2 = -\left(1 - (\frac{r_0}{r})^2\right)^2 dt^2 + \left(1 - (\frac{r_0}{r})^2\right)^{-2} dr^2 + r^2 d\Omega_3^2 \tag{20}$$

and

$$2\sqrt{-g}\, G_{IJ}F^J_{tr} = q_I \ , \qquad \phi^i = \text{const} \ . \tag{21}$$

The horizon is at $r = r_0$ where the parameter r_0 defining the horizon as well as the constant values of moduli depend on charges of the vector fields and on the topological intersection form C_{IJK}. Our study of the near horizon geometry will allow us to determine this dependence.

Note that the area of the horizon of the black hole is given by the volume of the 3-dimensional sphere

$$A = 2\pi^2 r_0^3 \ .$$

The area formula of 5d black holes was found in [9] from the observation that the unbroken supersymmetry near the black hole horizon requires that the central charge $Z \sim t^I q_I$ has to be extremized in the moduli space, i.e. near the horizon $\partial_i Z = 0$. This leads to the area formula in the form $A \sim (q_I q_J C^{IJ}|_{\partial_i Z=0})^{3/4}$, where C^{IJ} is the inverse of $C_{IJ} = C_{IJK} t^K$. In the derivation of this area formula it was assumed that the unbroken supersymmetry of the black hole solution is enhanced near the horizon. This will be proved now. It will be also explained why the enhancement of supersymmetry near the horizon requires the extremization of the central charge for describing the area formula. By exactly solving the Killing equations for the near horizon geometry we will be able to justify the area formula suggested in [9] and find the explicit area formula including the numerical factor in front of it.

Near the horizon at $r \to r_o$ and one can exhibit the $AdS_2 \times S^3$ geometry using $\hat{r} = (r - r_0) \to 0$

$$ds^2 = -(\frac{2\hat{r}}{r_0})^2 dt^2 + (\frac{2\hat{r}}{r_0})^{-2} d\hat{r}^2 + r_0^2 d\Omega_3^2 \ . \tag{22}$$

Since we deal with the product space we may use $a = 0, 1$ for the coordinates of the AdS_2-space and $\alpha = 2, 3, 4$ for the coordinates of the 3-sphere (in tangent space). The vector field ansatz near the horizon becomes

$$2(r_0)^3 \ G_{IJ} F^J_{ab} = \epsilon_{ab} q_I \ . \tag{23}$$

Let us use this ansatz in the fermionic part of supersymmetry transformations with all vanishing fermions and constant moduli. We start with gaugino and keep only relevant terms

$$\delta\lambda_i = \frac{1}{4}\left(\frac{3}{4}\right)^{2/3} t_{I,i} \Gamma^{\mu\nu} \epsilon F^I_{\mu\nu} = 0 \ . \tag{24}$$

We study the possibility that the zero mode of this equation is given by the full size spinor ϵ without linear constraints on it, i.e. that the unbroken supersymmetry is indeed enhanced near the horizon. This is possible, provided that

$$t_{I,i} G^{IJ} q_J = 0 \ . \tag{25}$$

The enhancement of unbroken supersymmetry near the horizon which can be deduced from the gaugino part of supersymmetry, can be represented as the condition of the minimization of the central charge, as found in [9]. Let us derive this here in a more detailed way. The gravitino transformation

$$\delta\psi_\mu = D_\mu(\omega)\epsilon + \frac{i}{8 \cdot 6^{1/3}} t_I \Big(\Gamma_\mu{}^{\nu\rho} - 4\delta_\mu{}^\nu \Gamma^\rho\Big) F_{\mu\nu}{}^I \epsilon = 0 \tag{26}$$

shows that the graviphoton field strength is given by the linear combination of vector fields and moduli $t_I F_{\mu\nu}{}^I$, and therefore the central charge is proportional to $t^I q_I$. From eq. (25) we get

$$t_{I,i}(C^{IJ} - 3t^I t^J) q_J = 0 \ . \tag{27}$$

Using eqs. (17) and (18) we can conclude that

$$t_{I,i}(C^{IJ} - 3t^I t^J) q_J = 2(t^J)_{,i} q_J = 0 \qquad \Longrightarrow \qquad \partial_i Z = 0 \ . \tag{28}$$

Thus we have derived the condition of minimization of the central charge $Z = t^I q_I$ in the moduli space from the requirement that the gaugino supersymmetry transformation for constant moduli has the full size spinor ϵ as a zero mode, i.e. from the condition of enhancement of supersymmetry near the black hole horizon. The central charge has to be independent of ϕ^i. Let us consider some useful identities of the real special geometry which are valid only near horizon. Note that

$$g^{ij}\partial_i Z \partial_j Z = g^{ij} t^I{}_{,i} t^J{}_{,j}\, q_I q_J \equiv \Pi^{IJ} q_I q_J = 0 \ . \tag{29}$$

Using eq. (18) we find that

$$\Pi^{IJ} t_I = 0 \tag{30}$$

and we may look for the combination which is orthogonal to t_I in the form $\Pi^{IJ} = l(C^{IJ} - t^I t^J)$. To get the coefficient l we use $g_{ij} = 6^{2/3} t^I{}_{,i} t^J{}_{,j} G_{IJ}$ and contract it with g^{ij}

$$g_{ij} g^{ij} = h_{(1,1)} - 1 = 6^{2/3} \Pi^{IJ} G_{IJ} \ , \tag{31}$$

which leads to $l = -\frac{1}{3}$. Thus we conclude that near the horizon where the central charge is moduli independent we have an identity

$$\Big((C^{IJ} - t^I t^J) q_I q_J \Big)_{\partial_i Z = 0} = 0 \ . \tag{32}$$

What remains to be done to make the extremization of the central charge consistent is to check what happens with the gravitino: does the gravitino transformation rule admit the Killing spinor of the full supersymmetry in our background? And what are the conditions of that? Using the ansatz for the vector fields near the horizon (23) and taking into account that we have a product space we get the following form of the gravitino transformations (26).

$$\begin{aligned} \delta\psi_a = (\hat{\nabla}_a)\eta \ &\equiv \ (\nabla_a + c\tilde{\gamma}_a)\eta = 0 \ , \qquad a = 0, 1, \\ \delta\psi_\alpha = (\hat{\nabla}_\alpha)\varsigma \ &\equiv \ (\nabla_\alpha - \frac{c}{2}\gamma_\alpha)\varsigma = 0 \ , \qquad \alpha = 2, 3, 4, \end{aligned} \tag{33}$$

where

$$\tilde{\gamma}_a = i\epsilon_{ab}\gamma^b \ , \qquad \Gamma^a = \gamma^a \otimes I \ , \qquad \Gamma_\alpha = i\gamma_0\gamma_1 \otimes \gamma_\alpha \ , \tag{34}$$

and

$$c = -\frac{1}{6^{2/3}} \frac{t^I q_I}{r_0^3} \ , \tag{35}$$

and the γ-matrices satisfy

$$\{\tilde{\gamma}_a, \tilde{\gamma}_b\} = 2\eta_{ab} \ , \qquad \eta_{ab} = (-,+) \ , \qquad \tilde{\gamma}_{[ab]} = -\gamma_{[ab]} \ , \tag{36}$$

$$\{\gamma_\alpha, \gamma_\beta\} = 2\delta_{ab} \ , \qquad \alpha = 2, 3, 4. \tag{37}$$

This is a special example of the general case presented in the introduction. We have identified the supersymmetric Killing spinor with the geometric Killing spinor. The integrability conditions for the geometric Killing spinors defined above are

$$[\hat{\nabla}_a, \hat{\nabla}_b]\eta(x) = 0 \qquad \Longrightarrow \qquad R_{ab}{}^{cd} = \frac{4}{6^{4/3}} \frac{(t^I q_I)^2}{r_0^6} (\delta_a{}^c \delta_b{}^d - \delta_a{}^d \delta_b{}^c) \ . \tag{38}$$

$$[\hat{\nabla}_\alpha, \hat{\nabla}_\beta]\varsigma(y) = 0 \qquad \Longrightarrow \qquad R_{\alpha\beta}{}^{\gamma\delta} = -\frac{1}{6^{4/3}} \frac{(t^I q_I)^2}{r_0^6} (\delta_\alpha{}^\gamma \delta_\beta{}^\delta - \delta_\alpha{}^\delta \delta_\beta{}^\gamma) \ . \tag{39}$$

This can be contracted to give us the Ricci tensors on AdS_2 and on S^3:

$$R_{ab} = \frac{4}{6^{4/3}} \frac{(t^I q_I)^2}{r_0^6} \eta_{ab} \ , \qquad R_{\alpha\beta} = -\frac{2}{6^{4/3}} \frac{(t^I q_I)^2}{r_0^6} \delta_{\alpha\beta} \ , \tag{40}$$

with the result that the radii of AdS_2 and S^3 are related:

$$R_{(2)} = -\frac{4}{3} R^{(3)} \ . \tag{41}$$

This relation restricts the properties of geometric Killing spinors. They would exist without any relation between these two product geometries. However, supersymmetric Killing spinors require relation between geometries. The curvature can be calculated also directly from the metric (22):

$$R_{ab} = \frac{4}{r_0^2} \eta_{ab} \ , \qquad R_{\alpha\beta} = -\frac{2}{r_0^2} \delta_{\alpha\beta} \ . \tag{42}$$

Comparing eqs. (40) with eqs. (42) we can express r_0 via the values of the moduli near the horizon and electric charges as follows:

$$r_0^4 = \frac{1}{6^{4/3}} \left((t^I q_I)_{\partial iZ=0} \right)^2 \ . \tag{43}$$

This gives us the area of the horizon

$$A = 2\pi^2 r_0^3 = \frac{\pi^2}{3} \left\{ (t^I t^J q_I q_J)_{\partial iZ=0} \right\}^{3/4} \ . \tag{44}$$

Near the horizon at $\partial iZ = 0$ we can rewrite it as

$$A = \frac{\pi^2}{3} \left\{ (C^{IJ} q_I q_J)_{\partial iZ=0} \right\}^{3/4} \tag{45}$$

using identity (32).

Thus we have combined the supersymmetry analysis with the analysis of the geometry. In this way we have confirmed the structure of the area formula of five dimensional black holes in N=2 theories obtained in [9] and found the exact numerical coefficient in front of it for supersymmetric five dimensional black holes with finite area of the horizon in the N=2 theory.

4 Truncation of N=4 supergravity to N=2

In some cases it is useful to consider those N=2 theories which can be obtained by truncation from N=4 theories. We will first study the truncation of pure N=4 supergravity and later generalize the result to the case of N=4 supergravity interacting with arbitrary number of vector multiplets as it was done before in 4d theories in [10].

We focus on a a special example of the double extreme five dimensional black hole known as Strominger-Vafa black hole [6]. The Lagrangian which allows a supersymmetric embedding of this

black hole is obtained most easily by the truncation of N=4 supergravity in d=5 constructed by Awada and Townsend [17]. The bosonic part of the action is

$$\begin{aligned} e^{-1}\mathcal{L} &= -\frac{1}{2}R - \frac{1}{4}e^{\frac{2}{3}\phi}F_{\mu\nu}{}^{ij}F_{ij}{}^{\mu\nu} - \frac{1}{4}e^{-\frac{4}{3}\phi}G_{\mu\nu}G^{\mu\nu} \\ &- \frac{1}{6}(\partial_\mu\phi)^2 + \frac{e^{-1}}{4\sqrt{2}}\epsilon^{\mu\nu\rho\sigma\lambda}F_{\mu\nu}{}^{ij}F_{\rho\sigma ij}B_\lambda \ , \end{aligned} \tag{46}$$

where $i,j = 1,\ldots,4$ and $G_{\mu\nu}$ is the field strength of B_μ and $F_{\mu\nu}{}^{ij}$is the field strength of $A_\mu{}^{ij}$. The supersymmetry transformation laws are:

$$\begin{aligned} \delta e_\mu{}^m &= \frac{1}{2}\bar{\epsilon}^i\Gamma^m\psi_\mu \ , \\ \delta\psi_{\mu i} &= D_\mu(\hat{\omega})\epsilon_i + \frac{i}{6}(e^{\frac{\phi}{3}}F_{\rho\sigma ij} - \frac{1}{2\sqrt{2}}e^{-\frac{2}{3}\phi}G_{\rho\sigma}\Omega_{ij})\Big(\Gamma_\mu{}^{\rho\sigma} - 4\delta_\mu{}^\rho\Gamma^\sigma\Big)\epsilon^j + \cdots \ , \\ \delta A_\mu^{ij} &= -\frac{1}{\sqrt{3}}e^{-\frac{\phi}{3}}\left(\bar{\epsilon}^{[i}\Gamma_\mu\chi^{j]} + \frac{1}{4}\Omega^{ij}\bar{\epsilon}^k\Gamma_\mu\chi_k\right) - ie^{-\frac{\phi}{3}}\left(\bar{\epsilon}^{[i}\psi_\mu^{j]} + \frac{1}{4}\Omega^{ij}\bar{\epsilon}^k\psi_{\mu k}\right) \ , \\ \delta B_\mu &= \frac{1}{\sqrt{6}}(e^{\frac{2\phi}{3}}\left(\bar{\epsilon}^i\Gamma_\mu\chi_i - \frac{i\sqrt{3}}{2}\bar{\epsilon}^i\psi_{\mu i}\right) \ , \\ \delta\chi_i &= -\frac{i}{2\sqrt{3}}\Gamma^\mu\partial_\mu\phi\epsilon_i + \frac{1}{2\sqrt{3}}\Big(e^{\frac{\phi}{3}}F_{\rho\sigma ij} + \frac{1}{\sqrt{2}}(e^{-\frac{2\phi}{3}}G_{\rho\sigma}\Omega_{ij}\Big)\Gamma^{\rho\sigma}\epsilon^j + \cdots \ , \\ \delta\phi &= \frac{i\sqrt{3}}{2}\bar{\epsilon}^i\chi_i \ . \end{aligned} \tag{47}$$

Here Ω_{ij} is a symplectic matrix. The truncation of N=4 supergravity to N=2 supergravity interacting with one vector multiplet is carried out by keeping $\psi_{\mu i}, \chi_i$ with $(i = 1,2)$ only and $A_\mu{}^{12} = -A_\mu{}^{34} \equiv \frac{1}{2}A_\mu, B_\mu, g_{\mu\nu}$ and ϕ. By inspecting the supersymmetry transformations of the truncated fields it is not difficult to see that this is a consistent truncation. The bosonic action becomes[2]:

$$e^{-1}\mathcal{L} = -\frac{1}{2}R - \frac{1}{4}e^{\frac{2}{3}\phi}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}e^{-\frac{4}{3}\phi}G_{\mu\nu}G^{\mu\nu} - \frac{1}{6}(\partial_\mu\phi)^2 + \frac{e^{-1}}{4\sqrt{2}}\epsilon^{\mu\nu\rho\sigma\lambda}F_{\mu\nu}F_{\rho\sigma}B_\lambda \ . \tag{48}$$

The supersymmetry transformations of the fermionic fields with vanishing fermion are

$$\begin{aligned} \delta\psi_\mu &= D_\mu(\omega)\epsilon - \frac{i}{12}\Big(\Gamma_\mu{}^{\rho\sigma} - 4\delta_\mu{}^\rho\Gamma^\sigma\Big)(e^{\frac{\phi}{3}}F_{\rho\sigma} - \frac{1}{\sqrt{2}}e^{-\frac{2}{3}\phi}G_{\rho\sigma})\epsilon \ , \\ \delta\chi &= -\frac{i}{2\sqrt{3}}\Gamma^\mu\partial_\mu\phi\epsilon - \frac{1}{4\sqrt{3}}\Gamma^{\rho\sigma}\Big(e^{\frac{\phi}{3}}F_{\rho\sigma} + \sqrt{2}e^{-\frac{2\phi}{3}}G_{\rho\sigma}\Big)\epsilon \ , \end{aligned} \tag{49}$$

and we have omitted the symplectic index on the spinors (to avoid confusion with the index on ϕ^i in the previous section).

Our action can be compared with the action used in [6] if we rewrite it as follows

$$e^{-1}\mathcal{L} = \frac{1}{2}\left(-R - \frac{1}{4}e^{\frac{2}{3}\phi}F'_{\mu\nu}F'^{\mu\nu} - \frac{1}{4}e^{-\frac{4}{3}\phi}G'_{\mu\nu}G'^{\mu\nu} - \frac{1}{3}(\partial_\mu\phi)^2 + \frac{e^{-1}}{8}\epsilon^{\mu\nu\rho\sigma\lambda}F'_{\mu\nu}F'_{\rho\sigma}B'_\lambda\right) \ , \tag{50}$$

[2]This action is equivalent upon rescalings to the action of N=2 supergravity interacting with one vector multiplet as presented in the Appendix of [16].

where

$$F' = \sqrt{2}F \ , \qquad G' = \sqrt{2}G \ , \qquad B' = \sqrt{2}B \ . \tag{51}$$

We observe a discrepancy in the kinetic term for the scalar field which is however harmless since the solution has a constant moduli field. Thus the extreme black hole of Strominger-Vafa can be embedded into the N=2 supergravity interacting with one constant vector multiplet.

To make contact with the formalism of previous section dealing with the general case of the very special geometry, we make the following identifications: $i = 1, I = 0, 1$ and

$$\begin{aligned} i = 1, \qquad I = 0,1 \ , \qquad A_\mu{}^1 &\equiv A_\mu \ , \qquad A_\mu{}^0 \equiv B_\mu \ , \\ G_{11} = e^{\frac{2\phi}{3}} \ , \qquad G_{00} = e^{-\frac{4\phi}{3}} \ , \qquad C_{011} &= 2\sqrt{2} \ , \qquad g_{11} = 1 \ , \qquad \phi^1 = \frac{1}{\sqrt{3}}\phi \ , \\ q_1 = -\frac{8\sqrt{2}Q_F}{\pi} \ , \qquad q_0 &= 2\sqrt{2}Q_H \ . \end{aligned} \tag{52}$$

Comparing the supersymmetry transformations we must identify:

$$\frac{\partial t_1}{\partial \phi^1} = -\frac{1}{\sqrt{3}}\left(\frac{4}{3}\right)^{2/3} e^{\frac{\phi}{3}} \ , \qquad \frac{\partial t_0}{\partial \phi^1} = -\sqrt{\frac{2}{3}}\left(\frac{4}{3}\right)^{2/3} e^{-\frac{2\phi}{3}} \ . \tag{53}$$

To satisfy the relations of special geometry and in particular to have $(t^I)_{,i}t_I = 0$ we get

$$\begin{aligned} t_1 &= -\left(\frac{4}{3}\right)^{2/3} e^{\frac{\phi}{3}} \ , & t^1 &= \frac{2}{3}\left(\frac{3}{4}\right)^{2/3} e^{-\frac{\phi}{3}} \ , \\ t_0 &= \frac{1}{\sqrt{2}}\left(\frac{4}{3}\right)^{2/3} e^{-\frac{2\phi}{3}} \ , & t^0 &= \frac{\sqrt{2}}{3}\left(\frac{3}{4}\right)^{2/3} e^{\frac{2\phi}{3}} \ . \end{aligned} \tag{54}$$

The enhancement of supersymmetry near the horizon is provided by $\frac{\partial t_I}{\partial \phi} G^{IJ} q_J = 0$ and we get the fixed value of the moduli near the horizon:

$$e^{\phi} = -\frac{1}{\sqrt{2}}\frac{q_1}{q_0} \ , \qquad t^1 q_1 = \frac{1}{2^{1/6}3^{1/3}}\left(q_0 (q_1)^2\right)^{1/3} , \qquad t^0 q_0 = \frac{1}{2} t^1 q_1 \ . \tag{55}$$

Now we can express the combination $t^I q_I$ near the horizon required for the entropy as

$$t^I q_I = \frac{3^{2/3}}{2^{7/6}}\left(q_0 (q_1)^2\right)^{1/3} \tag{56}$$

and

$$r_0^6 = \frac{1}{6^2}\{(t^I q_I)_{\rm hor}\}^3 \qquad \Longrightarrow \qquad \frac{1}{2^{11/2}}(-q_0)^2 q_1 = \frac{8(Q_H Q_F)^2}{\pi^2} \ , \tag{57}$$

and

$$A = 2\pi^2 (r_0)^3 = \frac{\pi^2}{2}\left(\frac{q_0}{\sqrt{2}}\left[\frac{q_1}{\sqrt{2}}\right]^2\right)^{1/2} . \tag{58}$$

Thus we have reproduced the Strominger-Vafa area formula as an example of our general area formula

$$A = 2\pi^2 (r_0)^3 = \frac{\pi^2}{3}\left\{(C^{IJ} q_I q_J)_{\partial i Z = 0}\right\}^{3/4} \qquad \Longrightarrow \qquad 8\pi\sqrt{\frac{Q_H (Q_F)^2}{2}} \ , \tag{59}$$

and therefore this black hole solution fits into our consideration of the very special geometry.

For the general case of n N=4 vector multiplets with duality group $O(1,1)\times\frac{O(5,n)}{O(5)xO(n)}$ the formula for the largest eigenvalue of the central charge, extremized in the moduli space was presented in [10]. This translates into the N=4 duality invariant area formula

$$A = 8\pi\sqrt{\frac{Q_H(Q_F)^2}{2}}\ , \tag{60}$$

where Q_H is the singlet charge and $(Q_F)^2$ is the $O(5,n)$ Lorenzian norm of the other $5+n$ charges Q_F .

Upon truncation to $N=2$ theory this gives a very special geometry with C_{IJK} intersection of the form $(I,J=0,1,\ldots,n+1)$ and $(i=1,\ldots n+1)$

$$C_{IJK} => \begin{pmatrix} C_{0ij}=\eta_{ij} \\ 0 \ \text{ otherwise} \end{pmatrix}\ , \tag{61}$$

where η_{ij} is the Lorenzian metric of $O(1,n)$ and the very special geometry is $O(1,1)\times\frac{O(1,n)}{O(n)}$. Upon truncation the area has the same form as in (60)

$$A = 8\pi\sqrt{\frac{Q_H(Q_F)^2}{2}} = 8\pi\sqrt{\frac{Q_H(Q_i\eta_{ij}Q_j)}{2}}\ , \tag{62}$$

where again Q_H is the singlet charge and $(Q_F)^2$ is the $O(1,n)$ Lorenzian norm of the other $1+n$ charges Q_i.

5 Discussion

Thus we have given a complete description of 5d static black holes near the horizon which can be embedded into N=2 and N=4 supergravity with arbitrary number of vector multiplets. They all show enhancement of supersymmetry near the black hole horizon. In N=2 as well as in N=4 case the unbroken supersymmetry (1/2 in N=2 and 1/4 in N=4) is doubled. As the result in all cases the dimension of the Killing spinor of unbroken supersymmetry is the dimension of the Dirac spinor admitted by the anti-deSitter space and the sphere.

The significance of the enhancement of supersymmetry near the horizon is related to the fact that in all cases when we have found finite horizon area of supersymmetric solutions there was also the enhancement of supersymmetry present. This concerns all four- and five-dimensional static black holes where we deal with the near-horizon geometries $AdS_2\times S^2$ and $AdS_2\times S^3$ respectively. The area in both cases is given by the volume of the S^2 and S^3 sphere.

In more general situations when the near horizon geometry is given by $AdS_{p+2}\times S^{d-p-2}$ the area of the horizon is given by the volume of the S^{d-p-2} sphere times the volume of the torus of dimension p, which for the supersymmetric solutions shrinks to zero near the horizon. As the

result, in the class of solutions with $AdS_{p+2} \times S^{d-p-2}$ near-horizon geometries one can not expect finite area of the horizon per unit volume except for black holes. Examples of such configurations with enhancement of supersymmetry and still vanishing area of the horizon include: $d = 10, p = 3$; $d = 11, p = 2, p = 5$. This geometric observation matches the recent analysis [19] of the cases of the vanishing entropy in which a duality invariant expression in terms of integral charges does not exist. In all these situations the extremum of the central charge does not occur at finite rational values of the moduli.

Few more configurations with near-horizon geometry $AdS_{p+2} \times S^{d-p-2}$ are known [18]. They include $AdS_3 \times S^2$ describing a 5d magnetic string and $AdS_3 \times S^3$ describing a 6d self-dual string. It has not been established yet whether they have enhancement of supersymmetry near the horizon. A calculation of the type which we have performed here for 5d black holes is required to identify the supersymmetric Killing spinors with the geometric ones for the 5d and 6d strings.

From the perspective of this study it would be interesting to have a more careful look into configurations with $AdS_2 \times S^{d-2}$ geometries near the horizon. These would have finite non-vanishing area related to the volume of the S^{d-2} sphere. Such configurations of the Reissner-Nordstrom-Tangherlini-type are known to solve equations of motion of Einstein-Maxwell theory in any dimension.[3] However in $d > 5$ they do not seem to have a supersymmetric embedding, at least such embeddings have not been found so far.

In conclusion, we have studied the mechanism of enhancement of unbroken supersymmetry near 5d black hole horizon and we have found the entropy-area formula for solutions of N=2 supergravity interacting with arbitrary number of vector multiplets.

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