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Nuclear and Atomic Physics of the Solar Neutrino Problem

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I discuss several nuclear and atomic physics issues that arise in the production and detection of solar neutrinos, such as the ^{7}Be(p,γ)^{8}B cross section and the implications of the GALLEX neutrino source experiment. I also touch on two more speculative issues, "salty water" experiments in Superkamiokande and mixing of the solar core on times characteristic of ^{3}He equilibration.

As this is a mixed audience, let me begin this brief review of some of the "microphysics" of the standard solar model with a primer on charged particle reactions at solar temperatures. The reactions driving the pp chain (see Fig. 1) are thought to be nonresonant, proceeding at center of mass energies (kT $\sim$ 2 keV in the solar core) well below the height of the Coulomb barrier. Competition with the Coulomb barrier heavily favors particles from the high energy tail of the Boltzmann distribution, in terms of effectiveness in generating reactions, leading to typical reaction energies $\sim$ 10 keV or higher. Even at such energies, however, direct laboratory measurements are not possible, being limited by counting rates to energies typically $\gtrsim$ 50 keV. Thus the usual approach to determining solar cross sections has been an extrapolation of higher energy laboratory measurements to the solar energies of interest via a nuclear model.

Traditionally, the cross section is expressed in terms of the astrophysical S-factor [1]

$$\sigma(E) = \frac{S(E)}{E} e^{-2\pi Z_1 Z_2 \alpha/\beta} \quad (1)$$

where Z_1 and Z_2 are the charges of the reacting particles, α is the finite structure constant, and $\beta = |\vec{v}_{cm}|/c$, where v_{cm} is the center-of-mass velocity. This removes from the cross section a factor describing the s-wave Coulomb interactions of point charges Z_1 and Z_2, so that the residual dependence of S(E) on the center-of-mass energy is relatively gentle. It is important to understand that this is merely a definition: theorists who then determine S(E) from the data take into account nuclear finite size effects, atomic screening, higher partial waves, and other corrections (see, e.g., Ref. [2]).

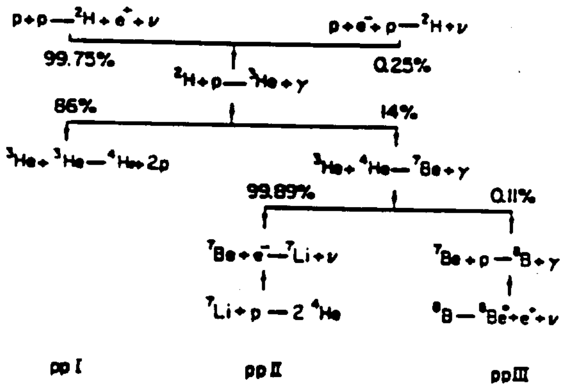

Figure 1. The solar pp chain

The nuclear models used in extrapolating the S-factor are generally not exact, even in the context of nonrelativistic point nucleons interacting through potentials. The nuclear many-body problem for A$\gtrsim$ 5 is still quite challenging. However great progress is being made with techniques such as Green's function Monte Carlo, so an exact nonrelativistic calculation of reactions like ^{7}Be(p, γ) may not be too far off. Generally, theory is used to describe the shape of S(E), but the normalization is taken from laboratory data to which theory is fitted.

There is quite good agreement among most solar modelers in choosing "best values" for the S-factors that govern the pp chain. If we use as an example the comparison between Bahcall and Pinnsoneault (BP) [3] and Turck-Chièze and Lopez (TCL) [4], there is a 5% difference in the choice of S_{33} (the S-factor for ^{3}He + ^{3}He), 5.0 vs. 5.24 Mev-b. The physics issue is whether one unfolds screening corrections from the laboratory data, which are thought to enhance the cross section somewhat at the lowest energies. There is also some difference in the choice of error bar for $S_{17}(0)$. The data sets for $S_{17}(E)$ agree in shape, but disagree in absolute normalization by about 25% (a problem that should be resolved experimentally). BP and TCL adopt the same central value (24.4 eV-b) of Johnson et al. [4], but differ in their choices of the associated error [5]. BP adopt the error bar recommended by Johnson et al. [4], who performed a recent reanalysis of S-factor fits to the data, while TCL use a more generous error to cover the full systematic differences between the data sets (which separately give S(0) values of ~25 eV-b and ~20 eV-b).

Theory tends to favor somewhat larger values of S_{17} (0) (~25 eV-b). For example, it has been recently argued that the ^{7}Be quadrupole moment and ^{7}Li properties reduce the spread of viable theories, favoring those with S(0) ~ 24.6 - 26.1 eV-b [6]. However it is also generally acknowledged that any theoretical value between 15-30 eV-b probably cannot be ruled out rigorously [7].

The value of $S_{17}(0)$ is certainly important to the solar neutrino problem as lower values reduce the ^{8}B neutrino flux discrepancy. Recently a new technique, ^{8}B breakup in the Coulomb field of a high Z target (^{208}Pb), was used to estimate $S_{17}(0)$. The Motobayashi et al. [8] result is 16.7 $\pm$ 3.2 eV-b. Despite the fact that the mean value is considerably below either 20 or 25 eV-b, a few people have advocated taking $S_{17}(0) \sim 17$ eV-b as a new "best value". I do not believe this is a reasonable choice. While the older experiments measure $S_{17}(E)$ directly, the Coulomb breakup does not, involving instead the virtual photon spectrum seen by ^{8}B as it passes through the field of ^{208}Pb. For example, while the solar reaction is dominated by E1 photons (with negligible contributions of M1 and E2), theory predicts that the Coulomb breakup reaction has a sizeable, interfering E2 amplitude. The calculation of that interference is not trivial, as it varies substantially depending on how the post-breakup Coulomb acceleration of the proton is modeled. While it appears that such dynamic effects reduce the importance of the E2 interference, the authors who reached this conclusion (Esbensen and Bertsch [9]) also offer the following caution: "Our model ... makes certain predictions which may or may not be realistic."

My conclusion is that the neglect of several data sets of direct measurements of $S_{17}(E)$ and supporting theory in favor of the Coulomb breakup value is not justified at present. The Coulomb breakup result has a substantial experimental error bar as well as a difficult-to-quantify theoretical error bar associated with the E2 amplitude, Coulomb reacceleration effects, and possible strong interaction contributions to the scattering.

Another important set of nuclear physics issues involves uncertainties in cross sections for solar neutrino detectors. One old issue-once thought to be fully resolved-has been settled just in the past year, the ^{37}Cl(ν,e$^-$)^{37}Ar cross section. Bahcall [10] originally pointed out that the model-dependent part of this cross section, the Gamow-Teller (GT) transitions, could be determined, under the assumption of isospin invariance of the nuclear force, from the analog β-decay ^{37}Ca(β^-)^{37}K. The β decay feeds excited levels in ^{37}K that decay by proton emission to ^{36}Ar. Thus by measuring the β-delayed proton spectrum, one can deduce the strengths of each of the GT transitions of interest.

However the measurements were made under the assumption that levels populated by allowed β decay of ^{37}Ca, the $1/2^+$,$3/2^+$, and $5/2^+$ levels in ^{37}K, would decay by delayed protons to the ground state of ^{36}Ar [11]. Eric Adelberger and I [12] reconsidered this issue when we noticed that the GT distribution derived in this way was substantially different from that deduced from forward-angle ^{37}Cl(p,n)^{37}Ar cross sections. We pointed out that the assumption made in the original β decay experiments was almost certainly in error, as $3/2^+$ and $5/2^+$ states in ^{37}K can decay by s-wave proton emission to the first excited 2^+ state in ^{36}Ar, but only by higher partial waves to the 0^+ ground state.

Garcia, Adelberger, and collaborators [13] have done a series of kinematically complete measurements of the β decay, measuring both the delayed protons and the coincident γ's from nuclear deexcitation in ^{36}Ar. The ISOLDE results of Garcia et al. now determine the ^{37}Cl cross section to an accuracy of 3%.

Fortuitously, the resulting change in the ^{37}Cl cross section, an increase of about 6% to $\sigma(^8\mathrm{B}) = 1.09 \cdot 10^{-42}\mathrm{cm}^2$ [13], is quite modest. While the error in the deduced GT distribution for ^{37}Cl was large (half of the GT strength had been missed), the misidentification of decays to the 2^+ state had two cancelling effects: while the β decay strength was underestimated, it was also displaced in energy by the excitation energy of the 2^+ state. The same technique, analog β decay producing

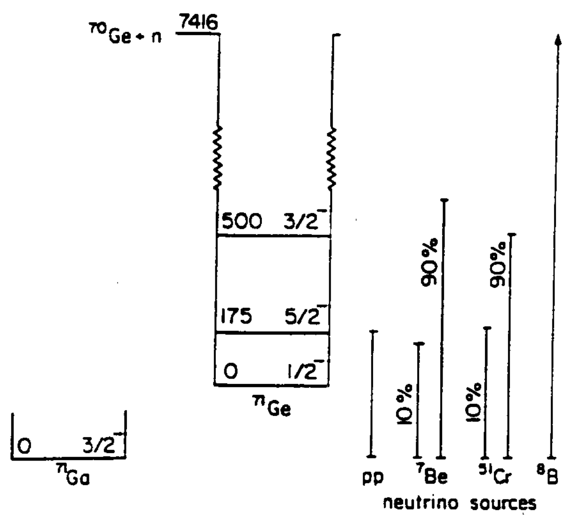

Figure 2. Level scheme for ^{71}Ge showing the excited states that contribute to absorption of pp, ^{7}Be, ^{51}Cr, and ^{8}B neutrinos. The ^{70}Ge + n break-up threshold is 7.4 MeV.

delayed protons, can in principle be used for another important target, the ^{40}Ar of ICARUS [14].

Hata and I [15] recently argued that the GALLEX [16] and SAGE [17] neutrino source experiments are an important new constraint on the nuclear physics of these detectors. ^{7}Be solar neutrinos can excite not only the ground state of ^{71}Ge($1/2^-$), but also two GT transitions to excited states at 175 keV ($5/2^-$) and 500 keV ($3/2^-$) (Fig. 2). Arguments that these excited state transitions were weak were based primarily on (p,n) cross sections. However the (p,n) and GT proportionality is often badly broken for weak transitions because of a spin-tensor interaction

$$[Y_2(\hat{r}) \otimes \sigma]_1 \tau_\pm \qquad (2)$$

that is known to contribute to (p,n) transition operators for forward-angle scattering. When we examined the particular case of ^{71}Ga(ν,e)^{71}Ge, we concluded that the excited state contribution for ^{7}Be neutrino absorption could plausible range anywhere from zero to a value comparable to that of the ground state: the spin-tensor matrix elements could be unusually large in this region of the fp-shell.

However, the GALLEX source experiment [16] places an important constraint [15] on these poorly known transition strengths

$$E\left[1 + 0.667\frac{BGT(5/2^-)}{BGT(gs)} + 0.218\frac{BGT(3/2^-)}{BGT(gs)}\right]$$
$$= 1.09 \pm 0.13 \qquad (3)$$

where E represents any deviation in the true GALLEX detector efficiency from that determined by the experimentalists. Under the assumption that the other tests of the GALLEX detector had confirmed that E$\sim$1.0, we used the source experiment results to constrain the unknown transition strengths. Fig. 3 shows that most of the region of possible GT values for the $5/2^-$ and $3/2^-$ states is eliminated by the calibration experiment. The results further tighten with the use of the final GALLEX source experiment results. As the ^{7}Be cross section was the largest potential nuclear physics uncertainty for this detector, the source experiment has been very important in greatly reducing this potential source of error. The source experiment result, as a constraint on the ^{7}Be cross section for ^{71}Ga, can now be folded into a general chi-square fit of neutrino fluxes to the experimental results. The results are given in Fig. 4.

In the last few mintes of my talk, I would like to describe briefly two rather exotic ideas. The first concerns the possibility of enhancing the experimental signals in Superkamiokande by adding a solute to the water. Superkamiokande, as presently envisioned, will not have the capability by itself to separate charged and neutral current events. This is in contrast to SNO, which has a charged current signal that provides spectral sensitivity and a neutral current channel that provides a total rate.

The idea I recently explored [18] was whether a solute could be added that would allow Superkamiokande to simultaneously obtain charged and neutral current spectral information. The presence of two channels

$$\nu_x + e^- \rightarrow \nu_x + e^-$$

$$\nu_e + (A, Z) \rightarrow e^- + (A, Z+1) \qquad (4)$$

distinguished, for example, by their angular distributions would allow the experimentalists to study the event ratio as a function of electron energy. This ratio is sensitive to heavy flavor neutrino contributions, and it is unaffected by systematic difficulties such as variations in the detector efficiency with energy.

While no ideal target emerged from this study, three possibilities appear quite interesting, ^{7}Li, ^{11}B, and ^{35}Cl. The ^{7}Li(ν,e)^{7}Be reation has an extraordinary cross section ($\sigma(^8\mathrm{B}) = 3.50 \cdot 10^{-42}$ cm^2) known to high accuracy [18] from studies of ^{7}Be β decay. As essentially all of the transition strength is carried by the ground and first excited states, the spectrum of produced electrons is unusually hard, an important feature in searches for MSW-induced ^{8}B neutrino spectrum distortions. The unfortunate aspect of ^{7}Li is that the Fermi and GT contributions to the cross section conspire to produce an isotropic distribution of electrons. Thus the ^{7}Li events could be distinguished from background only in energy bins where backgrounds are negligible. For the expected operating conditions of Superkamiokande [19], this is the region above 8 MeV (backgrounds less than 10%) or 9 MeV (backgrounds less than 1%), assuming Superkamiokande achieves the anticipated 200-fold improvement in detector radon content, relative to Kamioka II/III. These cuts, however, still leave a healthy count rate of 4450/year (3290/year) above 8 MeV (9 MeV), assuming a 5% solution of LiOH and an undistorted ^{8}B neutrino flux reduced to approximately 50% of the SSM value ($3 \cdot 10^6$/cm^2 sec). As shown in Fig. 5, the information such a charge-current channel would provide could greatly extend the power of Superkamiokande to distinguish competing solutions of the solar neutrino problem.

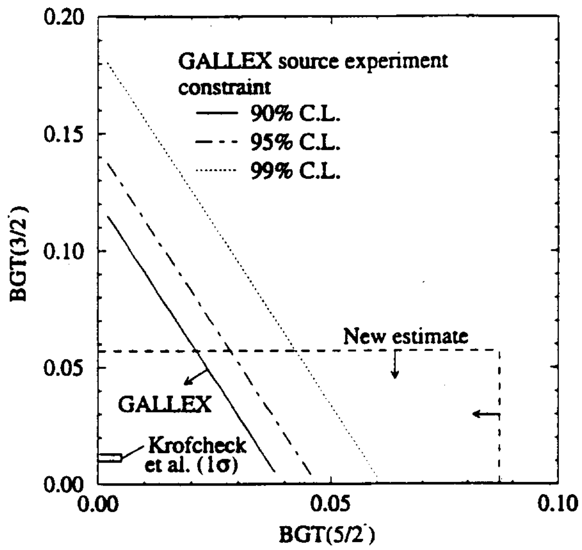

Figure 3. Constraints on the BGT values for the $5/2^-$ and $3/2^-$ excited states in ^{71}Ge imposed by the GALLEX source experiment. The rectangle is the region of possible values prior to the source experiment, as discussed in [15].

In contrast, ^{35}Cl provides a distinctive coincidence signal

$$^{35}\mathrm{Cl} + \nu_e \rightarrow ^{35}\mathrm{Ar} + \mathrm{e}^-$$

$$^{35}\mathrm{Ar} \rightarrow ^{35}\mathrm{Cl} + \nu_e + \mathrm{e}^+, \qquad (5)$$

a prompt electron followed by a delayed positron correlated in time ($\tau_{1/2}(^{35}Ar) = 1.77$ sec) and position. Although the maximum positron energy is only 5.38 MeV, Superkamiokande should be sensitive to low energy events above 2.5 MeV in coincidence mode. This coincidence signal has been studied in detail and compared to anticipated background rates in Superkamiokande [20]. The signal to background ratio turns out to be ~ 1. This is entirely satisfactory, as the electron angular distribution ($1 + 0.863\,\beta\cos\theta$) and exponential time correlation of the positron should allow a clean separation of signal from background in a maximum likelihood analysis.

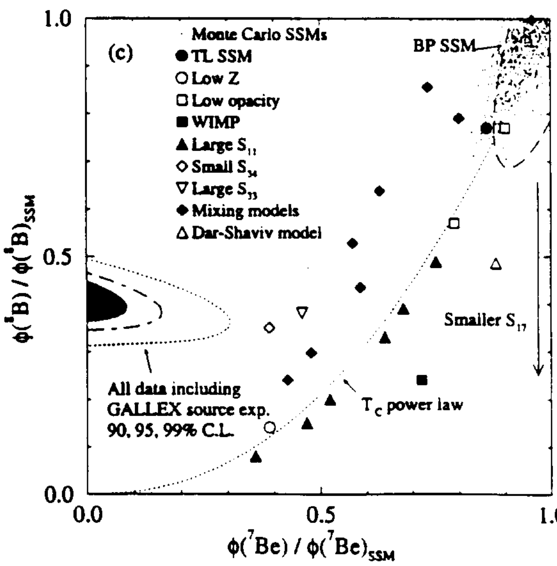

Figure 4. The ^{7}Be and ^{8}B fluxes, normalized by their SSM values, allowed by the ^{37}Cl, Kamioka II/III, SAGE/GALLEX, and GALLEX calibration experiments. The results of various SSM variations and nonstandard models are also shown [15].

The difficulty in this case, however, is the low counting rate due to the high threshold for $^{35}Cl(\nu,e^-)^{35}Ar$ and the limited efficiencies for detecting the low-energy electron and coincident positron. The resulting counting rate in Superkamiokande, assuming a 10% solution of ammonium chloride, is about 100/year, comparable to the present solar neutrino counting rate in Kamioka III.

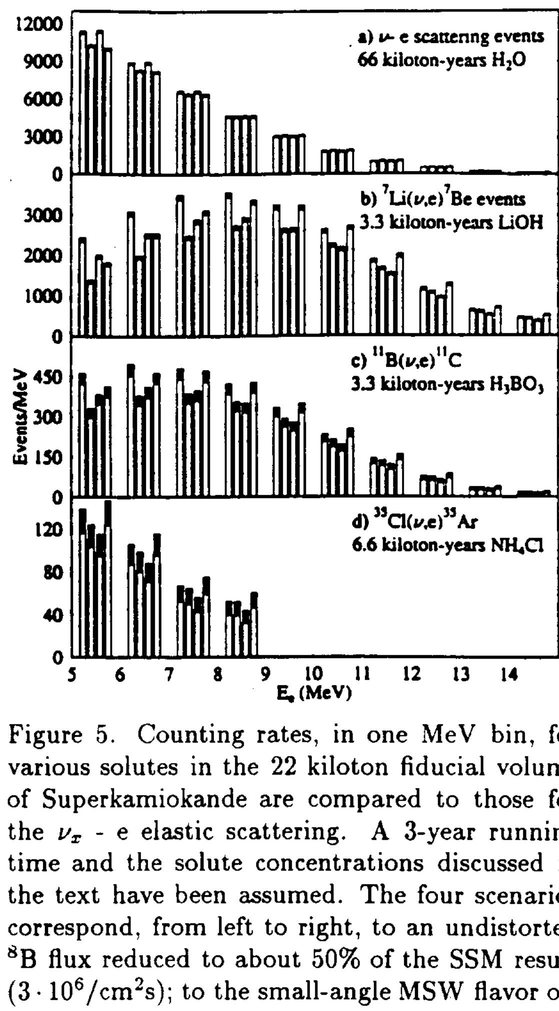

Figure 5. Counting rates, in one MeV bin, for various solutes in the 22 kiloton fiducial volume of Superkamiokande are compared to those for the ν_x - e elastic scattering. A 3-year running time and the solute concentrations discussed in the text have been assumed. The four scenarios correspond, from left to right, to an undistorted ^{8}B flux reduced to about 50% of the SSM result ($3 \cdot 10^6/cm^2s$); to the small-angle MSW flavor oscillation solution; to the large-angle flavor oscillation solution; and to the small-angle sterile oscillation solution. The parameters for each scenario have been adjusted to produce identical counting rates under the operating conditions of Kamioka II/III [18].

Despite the cleanliness of the $^{35}Cl(\nu,e)^{35}Ar$ signal, the low counting rate results in a significant loss in sensitivity to spectral distortions resulting from matter-enhanced solar neutrino oscillations. This is apparent from Fig. 5. Thus the conclusion of the study was that ^{7}Li appears to be the optimal choice.

Finally, let me quickly describe a rather intriguing calculation that Andrew Cumming and I have completed [21]. The pattern of solar neutrino fluxes deduced from the ^{37}Cl, Kamiokande II/III, and GALLEX/SAGE experiments is [22]

$$\phi(^8\text{B}) \sim 0.44\phi(^8\text{B})_{SSM}$$

$$\phi(^7\text{Be}) \sim 0 \tag{6}$$

$$\phi(\text{pp}) \sim \phi(\text{pp})_{SSM}$$

where SSM stands for the standard solar model. Given the reduced ^{8}B neutrino flux, the most puzzling aspect of these numbers is the low $\phi(^7\text{Be})/\phi(^8\text{B})$ flux ratio. Many investigators have noted that variations in the SSM, and many nonstandard solar models, predict neutrino fluxes that correlate very well with the solar core temperatures T_c produced in those models. The dependence of this flux ratio on T_c is

$$\frac{\phi(^7\text{Be})}{\phi(^8\text{B})} \sim T_c^{-10} \tag{7}$$

Thus a model with a low value of T_c, required by $\phi(^8\text{B})$, should produce a ratio substantially larger than the SSM value. Some investigators have argued, therefore, that the anomalous results of (6) rule out solar model explanation of the solar neutrino puzzle.

We decided to try to test this conclusion not in the conventional way - e.g., by modifying parameters in the SSM or by making a new physics assumption to produce a nonstandard model - but by taking a more phenomenological approach. Among the standard model properties that we preserved were the assumption of a steady-state sun, SSM microphysics (e.g., the standard values for nuclear cross sections and opacities), and the standard luminosity. However we did not require that the steady-state assumption applies locally, but only globally. That is, we allowed "catalytic" elements in the pp chain - elements produced and then consumed - to be transported before being destroyed. Thus the equilibrium density profile of such elements could differ substantially from that of the SSM. [Note that recent work [3] in the SSM also allows transport, the slow diffusion of ^{4}He and metals.]

Among the "catalytic" elements (d, ^{3}He, ^{7}Be) of the pp chain, only ^{3}He has an equilibration time scale long enough to be interesting, given any reasonable guess for nonstandard model mixing velocities. Our study involved allowing arbitrary changes in the ^{3}He profile, restricted only by the assumption that the functional form of the profile be rather simple and satisfy the steady-state assumption. [Note that the SSM ^{3}He profile for r$\lesssim$0.27 $R_\odot$ rises steeply with increasing r, as it varies as T^{-6}, T the local temperature.] We looked for solutions with a $\phi(^8\text{B})$ consistent with that of Eq. (6), and with a $\phi(^7\text{Be})/\phi(^8\text{B})$ ratio smaller than that of the SSM. A characteristic modification of the ^{3}He abundance resulted, with a value elevated well above (factor of 5-10) the SSM equilibrium value deep in the solar core, and depleted throughout the remainder of the core.

This construction is intended as an "existence proof" that steady state solar models producing the correct luminosity and employing the conventional microphysics can reproduce neutrino fluxes approximately consistent with Eq. (6): typical $\phi(^7\text{Be})$ values for such a profile are ~0.2 $\phi(^7\text{Be})_{SSM}$. I make no claims here that the result of this exercise would follow from a plausible model. The required flux adjustments are accomplished in a straightforward way: ^{3}He is burned principally at small r, where the abundance is substantially out of equilibrium. Because the ^{3}He + ^{3}He rate is quadratic in X_3, while ^{3}He + ^{4}He is linear, ppI cycle terminations are favored and ppII + ppIII substantially suppressed. However, the smaller amount of ^{7}Be that is produced by ^{3}He + ^{4}He then also burns at small r, and thus at elevated temperatures. This implies a small $\phi(^7\text{Be})/\phi(^8\text{B})$ ratio. Finally, the enhanced ppI termination rate requires a reduction in the temperature scale to produce the correct luminosity: this adjustment helps in achieving a reduced $\phi(^8\text{B})$.

There are aspects of this result, however, that perhaps merit further study: what mechanisms could produce such a ^{3}He profile? Two conditions - the requirement that ^{3}He burns at small r substantially out of equilibrium and that the model is steady state - suggests the rapid downward motion of filaments - or "fingers" - of ^{3}He-rich material from large r. If the velocity is chosen to be $\sim$ 10m/day, such a volume element will penetrate well into the solar core before burning. This velocity is established by the height of the ^{3}He maximum at small r, which in turn is determined by Eq. (6). The burning of ^{3}He will make the volume element buoyant, so that the motion will reverse at this point. If we assume that the upward motion is short-lived - that the volume element rather quickly returns to thermal (and possibly chemical) equilibrium, then the subsequent upward motion is slow, determined by the rate at which the fingers are displacing ^{3}He-depleted material from below. Quite remarkably, the resulting upward velocity is comparable to the local equilibration time, suggesting that the ^{3}He could be approximately replenished by the time the volume element is displaced to large $r \sim 0.2 R_\odot$.

These ideas are further developed in Ref. [21], where connections are made to the ^{3}He gradient instability of Dilke and Gough [23] and to ^{3}He production by red giants.

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