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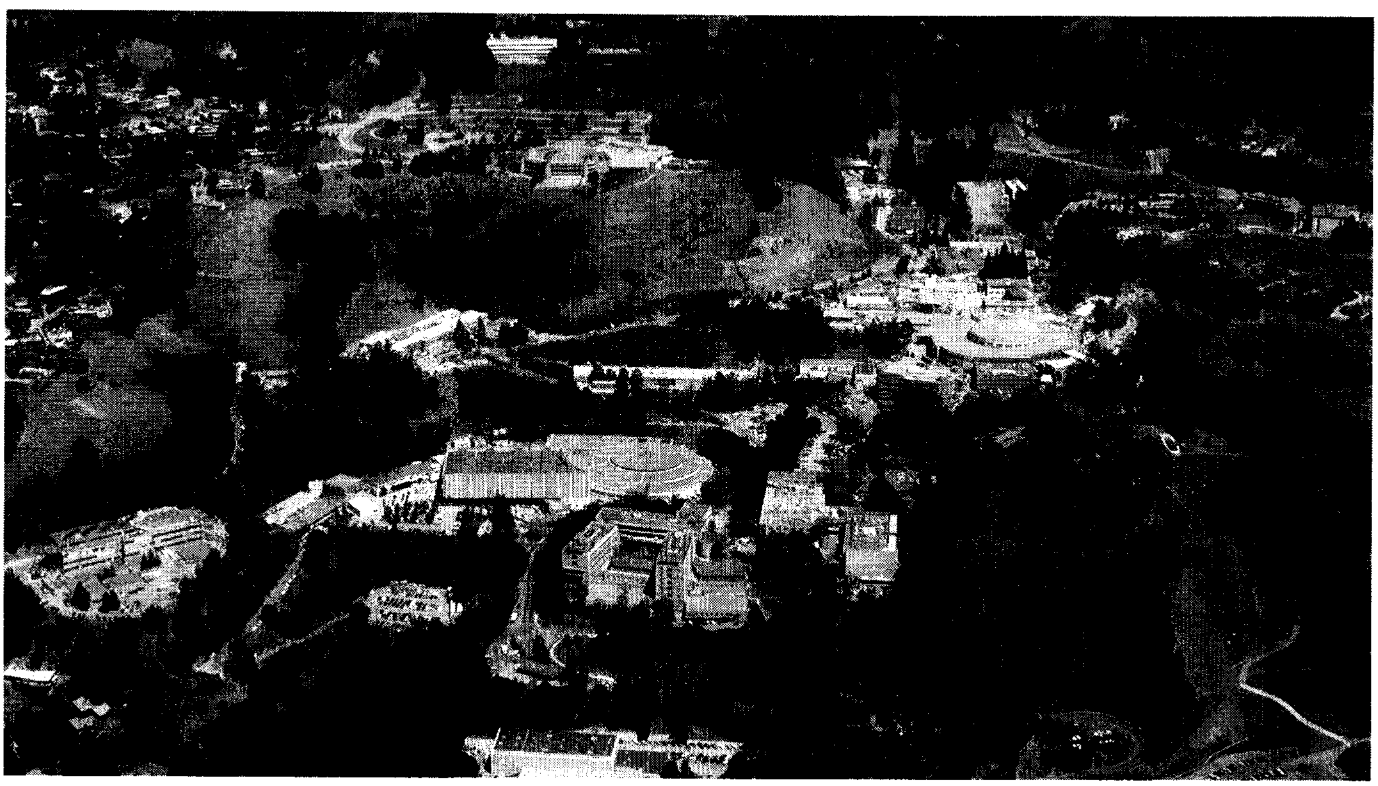

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Basics of QCD * †

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Abstract

These lectures provide an introduction to perturbative QCD and some of its current applications.

1 Introduction

In these lectures, I shall provide an introduction to perturbative QCD and to some of its applications. In the limited time available, I shall first concentrate on the basics of perturbative QCD and on the tools for calculations. After discussion the total hadronic cross-section and jet rates in e^+e^- annihilation, I shall discuss the QCD parton model. I will end with a discussion of how non-perturbative effects are parameterized by using heavy quark effective field theory as an example. There are many excellent references for the material in these lectures. Some recent sets of lecture notes and review articles should be consulted for more details and an alternative view.[1] [2].

2 The QCD Lagrangian.

The QCD Lagrangian describes the interactions of n_f flavors of quarks each of which has three colors (ψ_i) with an octet of gluon fields (G^a_μ) and may be written as follows:

$$-\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}+\sum_{i,j}\bar{\psi}_i(i/D_{\mu ij}-\delta_{ij}m_j)\psi_j \tag{1}$$

The sum on j runs over quark flavors and gluonic filed strength tensor is written as,

$$F^a\mu\nu=\partial_\mu G^a_\mu-\partial_\nu G^a_\mu-igf_{abc}G^b_\mu G^c_\mu \tag{2}$$

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where $t = \log(Q^2/\mu^2)$. Then Equation 16 has the solution

$$F(t, \alpha) = F(1, \alpha(t)) \tag{19}$$

Hence the only dependence on the scale Q or t is carried by $\alpha(t)$. We can expand β as a power series in α.

$$\beta = -b\frac{\alpha}{4\pi} - b'(\frac{\alpha}{4\pi})^2 + \ldots \tag{20}$$

Hence $\alpha(\mu^2)$ has the following form:

$$\alpha(\mu^2) = \frac{4\pi}{b\log(\mu^2/\Lambda^2)} + \cdots \tag{21}$$

Here $b = 11 - 2n_f/3$ where n_f is the number of quark flavors with mass less than μ. We can regard the fundamental parameter of QCD either as $\alpha(Q_0^2)$ or as the scale Λ. Notice that as μ becomes small, α becomes large. Therefore, perturbation theory cannot be used to discuss processes which involve momentum flows as small as a few times Λ. This is the reason that the on-shell subtraction scheme is impractical.

3 Processes in e^+e^- annihilation

As a specific example of QCD process, consider the total cross-section for $e^+e^- \to$ hadrons at center-of-mass energy $\sqrt{s}$. or the decay width of the Z boson into hadrons ($\Gamma \to hadrons$). If the coupling of the Z to fermions (ψ_i) is written as

$$Z^\mu \overline{\psi_i}(v_i\gamma_\mu + a_i\gamma_\mu\gamma_5)\psi_i \tag{22}$$

Then at lowest order in QCD (α_s^0), the cross-section for $e^+e^- \to hadrons$ at center of mass energy $\sqrt{s} = M_Z$ is calculated from the process $e^+e^- \to q\overline{q}$ and is given by

$$\sigma_h = \sigma_0 \tag{23}$$

$$= \frac{16G_F M_Z^4}{3\pi\Gamma_Z^2}\sum_{quarks}(v_i^2 + a_i^2) \tag{24}$$

where Γ_Z is the total decay width of the Z boson.

At next order in α_s two process are possible; $e^+e^- \to q\overline{q}$ and $e^+e^- \to q\overline{q} + gluon$. If we define x_1 and x_2 as the energy of the outgoing quark and anti-quark scaled by M_Z, *viz.* $x_1 = 2E_q/M_Z$ and $x_2 = 2E_{\overline{q}}/M_Z$., so that $0 \leq x_i \leq 1$, the cross section for the latter process can be written as

$$\sigma(q\overline{q}g) = \sigma_0\frac{2\alpha_s}{3\pi}\int dx_1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \tag{25}$$