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DECOHERENCE OF HOMOGENEOUS AND ISOTROPIC METRICS IN THE PRESENCE OF MASSIVE VECTOR FIELDS*

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Abstract

Retrieval of classical behaviour in quantum cosmology is usually discussed in the framework of minisuperspace models in the presence of scalar fields together with the inhomogeneous modes either of the gravitational or of the scalar fields. In this work we propose alternatively a model where the scalar field is replaced by a massive vector field with global $U(1)$ or SO(3) symmetries.

The emergence of the classical properties from the quantum mechanics formalism is still largely an open problem. Some progress has, however, been achieved through the so-called decoherence approach. On fairly general grounds, decoherence can be regarded as a procedure where one considers the system under study to be part of a more complex world and which interacts with other subsystems, usually referred to as "environment". This interaction leads to the suppression of the quantum interference effects.

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These ideas have been developed with some depth in the context of minisuperspace models in quantum cosmology (see [1] and references therein). Most of the literature concerning the emergence of classical behaviour from quantum cosmological minisuperspace models considers scalar fields and as environment the inhomogeneous modes either of the gravitational or of the scalar fields. We propose alternatively a model where the self-interacting scalar field is replaced by a massive vector field with U(1) or SO(3) global symmetries [1]. Preliminary work on the system with $SO(3)$ non-Abelian global symmetry, whose classical cosmology has been studied in Ref. [2], shows that the ingredients necessary for the process of decoherence to take place are present [3]. Notice that the presence of a mass term is an essential feature as this breaks the conformal symmetry of the spin-1 field action which leads to a Wheeler-DeWitt equation where gravitational and matter degrees of freedom decouple [4].

The action of our model consists of a Proca field coupled with gravity [2]:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2k^2}(R-2\Lambda)+\frac{1}{4e^2}Tr(F_{\mu\nu}F^{\mu\nu})+\frac{1}{2}\ m^2Tr(A_\mu A^\mu)\right]-\frac{1}{k^2}\int_{\partial M}d^3x\sqrt{h}K, \tag{1}$$

where $k^2 = 8\pi M_P^{-2}$, M_P being the Planck mass, e is a gauge coupling constant, m the mass of the Proca field and with $h_{ij}(i,j=1,2,3)$ being the induced metric on the three-dimensional boundary ∂M of M, $h=\det(h_{ij})$ and $K=K^\mu_\mu$ is the trace of the second fundamental form on ∂M.

In quantum cosmology one is concerned with spatially compact topologies and we will consider the Friedmann-Robertson-Walker (FRW) ansatz for the $R\times S^3$ geometry

$$ds^2 = \sigma^2 a^2(\eta)\left[-N(\eta)^2 d\eta^2 + \sum_{i=1}^{3}\omega^i\omega^i\right]\ , \tag{2}$$

where $\sigma^2 = 2/3\pi$, η is the conformal time, $N(\eta)$ and $a(\eta)$ being the lapse function and the scale factor, respectively and ω^i are left-invariant one-forms in $SU(2)\simeq S^3$.

Aiming to obtain solutions of the Wheeler-DeWitt equation satisfied by the wave function $\Psi[h_{ij}, A^{(a)}_\mu]$ we expand the spatial metric as:

$$h_{ij} = \sigma^2 a^2 \quad (\Omega_{ij}+\epsilon_{ij}) \tag{3}$$

with Ω_{ij} being the metric on the unit S^3 and ϵ_{ij} a perturbation which can be expanded in in scalar harmonics ${\mathcal{D}^J}_N^M(g)$, which are the usual $(2J+1)$-dimensional $SU(2)$ matrix representation, and spin-2 hyperspherical harmonics $Y^{2}_{m}{}^{LJ}_{MN}(g)$ on S^3 [5].

The massive vector field

$$A = A^{ab}_m\omega^m_s\mathcal{T}_{ab} = A^{ab}_m\sigma^m_i\omega^i\mathcal{T}_{ab}\ , \tag{4}$$

where ω^m_s denote the one-forms in a spherical basis with $m=0,\pm1$, σ^m_i denotes a 3 x 3 matrix and $\mathcal{T}_{ab}$ are the $SO(3)$ group generators, can be expanded in spin-1 hyperspherical harmonics as [1],[6]:

$$A_0(\eta,x^j) = \sum_{JMN} \alpha^{abJM}_{\ \ \ \ N}(\eta)\mathcal{D}^{J\ M}_{\ N}(g)\mathcal{T}_{ab} = 0 + \sum_{J'M'N'} \alpha^{abJ'M'}_{\ \ \ \ N'}(\eta)\mathcal{D}^{J'\ 'M'}_{\ 'N'}(g)\mathcal{T}_{ab}, \quad (5)$$

$$\begin{aligned} A_i(\eta,x^j) &= \sum_{LJNM} \beta^{abMN}_{LJ}(\eta)\ Y^{1LJ}_{mNM}(g)\sigma^m_i\ \mathcal{T}_{ab} \\ &= \frac{1}{2}\left[1+\sqrt{\frac{2\overline{\alpha}}{3\pi}}\chi(\eta)\right]\epsilon_{aib}\mathcal{T}_{ab} + \sum_{L'J'N'M'} \beta^{abM'N'}_{L'J'}(\eta)\ Y^{1L'J'}_{mN'M'}(g)\sigma^m_i\ \mathcal{T}_{ab}, \quad (6) \end{aligned}$$

where $\overline{\alpha} = e^2/4\pi$ and $\chi(\eta)$ a time-dependent scalar function. A_0 is a scalar on each fixed time hypersurface, such that it can be expanded in scalar harmonics $\mathcal{D}^{J\ M}_{\ N}(g)$. The expansion of A_i is performed in terms of the spin-1 spinor hyperspherical harmonics, $Y^{1\ LJ}_{m\ MN}(g)$. Longitudinal and the transversal harmonics correspond to $L=J$ and $L-J=\pm 1$, respectively. The rhs of eqs. (5) and (6) correspond to a decomposition in homogeneous and inhomogeneous modes. For this decomposition one has used the ansatz for the homogeneous modes of the vector field which is compatible with the FRW geometry as discussed in Ref. [2].

From action (1) one can work out the effective Hamiltonian density obtained from the substitution of the expansions (2)-(6). To second order in the coefficients of the expansions and in all orders in a, one obtains the following effective Hamiltonian density for the system with SO(3) global symmetry (for the Abelian case one drops the last four terms) [1]:

$$\begin{aligned} \mathcal{H}^{eff} &= -\frac{1}{2M_P^2}\pi_a^2 + M_P^2\left(-a^2+\frac{4\Lambda}{9\pi}\ a^4\right) + \sum_{J,L}\ \frac{4}{3\pi}\ m^2\ a^2\ \beta^{abNM}_{LJ}\ \beta^{abLJ}_{NM} \\ &+ \sum_{|J-L|=1}\left[\overline{\alpha}\pi\Pi_{\beta^{abLJ}_{NM}}\Pi_{\beta^{abLJ}_{NM}} + \beta^{abNM}_{LJ}\beta^{abLJ}_{NM}(L+J+1)^2\right] \\ &+ \sum_J\left\{\overline{\alpha}\pi + \left[(-1)^{4J}\left(\frac{16\pi^2 J(J+1)}{2J+1}\right)\ \frac{3\pi}{4m^2}\right]\ \left[1+\frac{1}{a(t)}\right]\right\}\Pi_{\beta^{abJJ}_{NM}}\Pi_{\beta^{abJJ}_{NM}} \\ &+ \pi_\chi^2 + \frac{\overline{\alpha}}{3\pi}\left[\chi^2-\frac{3\pi}{2\overline{\alpha}}\right]^2 + \sum_{J,L}\frac{4}{\overline{\alpha}\pi}\ \left[1+\sqrt{\frac{2\overline{\alpha}}{3\pi}}\chi\right]^2\ \beta^{abNM}_{LJ}\ \beta^{abLJ}_{NM} \\ &+ 4\pi a^2 m^2\left[1+\sqrt{\frac{2\overline{\alpha}}{3\pi}}\chi\right]^2, \end{aligned} \quad (7)$$

where the canonical conjugate momenta of the dynamical variables are given by

$$\pi_a = \frac{\partial\mathcal{L}^{eff}}{\partial\dot{a}} = -\frac{\dot{a}}{N}\ ,\quad \pi_\chi = \frac{\partial\mathcal{L}^{eff}}{\partial\dot{\chi}} = \frac{\dot{\chi}}{N}\ ,\quad \pi_{\beta^{abNM}_{LJ}} = \frac{\partial\mathcal{L}^{eff}}{\partial\dot{\beta}^{abNM}_{LJ}} = \frac{\dot{\beta}^{abNM}_{LJ}}{2N\pi\overline{\alpha}}\ , \quad (8)$$

$$\pi_{\beta^{abNM}_{JJ}} = \frac{\partial\mathcal{L}^{eff}}{\partial\dot{\beta}^{abNM}_{JJ}} = \frac{\dot{\beta}^{abNM}_{JJ}}{2N\pi\overline{\alpha}} - \frac{1}{2\pi\overline{\alpha}}\frac{a}{N}\alpha^{abJM}_{\ \ \ \ N}(-1)^{2J}\sqrt{\frac{16\pi^2 J(J+1)}{2J+1}}\ , \quad (9)$$

$\mathcal{L}^{eff}$ denoting the effective Lagrangian density arising from (1) and the dots representing derivatives with respect to the conformal time.

The Hamiltonian constraint, $\mathcal{H}^{eff} = 0$, gives origin to the Wheeler-DeWitt equation after promoting the canonical conjugate momenta (8),(9) into operators:

$$\pi_a = -i\frac{\partial}{\partial a} \ , \quad \pi_\chi = -i\frac{\partial}{\partial \chi} \ , \quad \pi_{\beta^{abNM}_{LJ(JJ)}} = -i\frac{\partial}{\partial \beta^{abLJ(JJ)}_{NM}} \ , \quad \pi_a^2 = -a^{-P}\frac{\partial}{\partial a}\left(a^P\frac{\partial}{\partial a}\right) \ , \tag{10}$$

where in (10) the last substitution parametrizes the operator order ambiguity with p being a real constant. The Wheeler-DeWitt equation is obtained imposing that the Hamiltonian operator annihilates the wave function $\Psi[a, \beta^{abNM}_{LJ}, \beta^{abNM}_{JJ}, \chi]$.

A solution of the Wheeler-DeWitt equation which corresponds to a classical behaviour of its variables on some region of minisuperspace will have an oscillatory WKB form as

$$\Psi[a, A^{ab}_\mu] = e^{iM_P^2 S(a)} C(a)\psi(a, A^{ab}_\mu) \ . \tag{11}$$

In order to make predictions concerning the behaviour of the scale factor, a, for the U(1) case (see Ref. [1] for the discussion of the non-abelian case with SO(3) global symmetry) one uses a coarse-grained description of the system working out the reduced density matrix associated to (11)

$$\rho_R = \sum_{n,n'} e^{iM_P^2[S_{(n)}(a_1) - S_{(n')}(a_2)]} C_{(n)}(a_1)C_{(n')}(a_2)\mathcal{I}_{n,n'}(a_2, a_1), \tag{12}$$

where

$$\mathcal{I}_{n,n'}(a_2, a_1) = \int \psi_{(n')}(a_2, A^{ab}_\mu)\psi^*_{(n)}(a_1, A^{ab}_\mu) d[A^{ab}_\mu] \ . \tag{13}$$

The subindex (n) labels the WKB branches. The term $\mathcal{I}_{n,n'}(a_2, a_1)$ contains the environment influence on the system.

The decoherence process is sucessful if the non-diagonal terms ($n \neq n'$) in (12) are vanishingly small. Hence, there will be no quantum interference between alternative histories if $\mathcal{I}_{n,n'} \propto \delta_{n,n'}$. Once that is achieved one can analyse the correlations in each classical branch ($n = n'$). This can be done by looking at the reduced density matrix or the to corresponding Wigner functional:

$$F_{W,(n)}(a, \pi_a) = \int_{-\infty}^{+\infty} d\Delta [S'_{(n)}(a_1)S'_{(n)}(a_2)]^{-\frac{1}{2}} e^{-2i\pi_a\Delta} e^{iM_P^2[S_{(n)}(a_1) - S_{(n)}(a_2)]}\mathcal{I}_{n,n}(a_2, a_1) \tag{14}$$

where $\Delta = \frac{a_1 - a_2}{2}$. A correlation among variables will correspond to a strong peak about a classical trajectory in the phase space. The decoherence process is necessary as the Wigner function associated to (12) does not have a single sharp peak even for a WKB Wigner function as (14); such a peak (and a clearly classical WKB evolution) is found only among the $n = n'$ terms. If the conditions to achieve an effective diagonalization of (14) are met then the interference between the different classical behaviours is *also* highly suppressed. Furthermore, $\mathcal{I}_{n,n'}(a_2, a_1)$ will be damped for

$|a_2 - a_1| \gg 1$ and the reduced density matrix associated with (14) will be diagonal with respect to the variables a [1].

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