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Observation of the color coherence effect in sub-jet multiplicity of three-jet and four-jet events in e^+e^- annihilations

The AMY Collaboration

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Abstract

We have measured the ratio of the average sub-jet multiplicity in 3-jet to 2-jet events, R_{23}, and 4-jet to 2-jet events, R_{24}, using the e^+e^- annihilation data at $\sqrt{s} = 58$ GeV obtained from the AMY detector. We compare the measured values of R_{23} and R_{24} with the predictions of coherent and incoherent Monte Carlo models. R_{23} is also compared with the NLLA calculations to all-orders. The color coherence effect has been observed to contribute to the suppression of R_{23} and R_{24}. Also, the difference of the behavior between gluon jets in 3-jet and 4-jet events has been studied.

1 Introduction

A study of the average hadron multiplicity in jet events in e^+e^- annihilations explores the nature of quark- and gluon-induced jets and provides a check of perturbative QCD predictions.

According to a naive (lowest order) QCD expectation[1], in the asymptotically high energy limit the ratio of the average hadron multiplicity in gluon-jets to that in quark-jets, $\langle n\rangle_g/\langle n\rangle_q$, is equal to the ratio of their respective color charges $C_A/C_F = 9/4$, where $C_F = 4/3$ and $C_A = 3$ are the color factors for gluon radiation from quarks and gluons, respectively. This expectation is based on the assumption of local parton hadron duality (LPHD)[2] according to which the number of hadrons produced locally in a hard process is proportional to the number of partons emitted perturbatively from the primary(hard) partons. The experimentally observed value of $\langle n\rangle_g/\langle n\rangle_q$ is 1.27 ± 0.07[3] and is strongly suppressed with respect to the predicted value. This may suggest that the multiplicity of the gluon-jet is not enhanced to the extent expected from consideration of its higher color charge.

Theoretical attempts have been made to interpret this observed suppression. Dremin *et al.*[4] have predicted the ratio of the parton multiplicity of gluon-jets to quark-jets to be 1.84 ± 0.02 by solving exactly the QCD equations for the generating functions in the case of a fixed value of the strong coupling constant. Using a Monte Carlo model they have obtained a corresponding ratio of the hadron multiplicities to be 1.38 ± 0.02, roughly in agreement with the experimental value reported in Ref.[3]. Kimura *et al.*[5] have argued that previous measurements of $\langle n\rangle_g/\langle n\rangle_q$ can not be compared directly with its calculations since the former ones involve biases due to selection procedures of jets. They have calculated the parton multiplicity in well-collimated 2-jet and 3-jet events in the modified leading logarithm approximation using 3-jet events with a thrust value of 2/3. They have predicted the ratio $\langle n\rangle_g/\langle n\rangle_q$ to be about 2.0 under the LPHD assumption without identifying gluon-jets and quark-jets.

Difficulties in comparing hadron multiplicity in quark-jet and gluon-jet between calculations and measurements lie in the fact that hadron multiplicity is an infrared and collinear unsafe quantity and its calculation involves assumptions about the hadronization process. Further, selection procedures of quark-jet and gluon-jet employed in experiments are not free from ambiguity and are likely to affect the measurement of hadron multiplicity. To avoid these difficulties Catani *et al.*[6] have introduced the concept of sub-jet multiplicity in 2-jet and 3-jet events. The sub-jet multiplicity is infra-red and collinear safe and can be compared directly between experimental data and theoretical calculations. In addition, identification of gluon-jets and quark-jets is not necessary in the measurement of the sub-jet multiplicity in 3- and higher jet events.

The sub-jets or clusters inside of a jet are realized by defining first a jet resolution scale y_1 that determines the jet multiplicity of the events and then an additional resolution scale y_0 ($< y_1$) which determines the cluster(sub-jet) multiplicity. The variables y_0 and y_1 are values of the cut-off resolution parameter of the clustering algorithm. Catani *et al.*[6] have calculated the ratio of the average sub-jet multiplicity in 3-jet events to that in 2-jet events, $\langle M_3\rangle/\langle M_2\rangle$, including leading and next-to-leading logarithms to all orders in α_s. These calculations are referred to as the all-orders-NLLA calculations in the present paper.

According to a naive QCD prediction the ratio of average number of soft gluons emitted in 3-jet to 2-jet events, $\langle M_3-3\rangle/\langle M_2-2\rangle$, approaches $(2C_F+C_A)/2C_F = 17/8$ in the asymptotically high energy limit if we assume that 2-jet and 3-jet events consist of $q\overline{q}$ and $q\overline{q}g$ final states, respectively. The ratio $\langle M_3\rangle/\langle M_2\rangle$ is approximately equal to $\langle M_3-3\rangle/\langle M_2-2\rangle$ for large values of $\langle M_2\rangle$ and $\langle M_3\rangle$. Catani *et al.*[6] have found that the ratio $\langle M_3\rangle/\langle M_2\rangle$ approaches $17/8$ only in the limit of $\log(1/y_0) \gg \log(1/y_1)$ which is not attainable under present experimental conditions. They predict a parton level ratio of $\langle M_3\rangle/\langle M_2\rangle$ around or below 1.5 for the y_0 and y_1 values relevant to the center-of-mass energies of the current and future experiments.

Experimental studies have been carried out on the ratio of sub-jet multiplicity of 3-jet to 2-jet events in e^+e^- annihilations by L3[7] and OPAL[8]. Suppression of the ratios $\langle M_3\rangle/\langle M_2\rangle$ and $\langle M_3-3\rangle/\langle M_2-2\rangle$ has been observed in these measurements. Comparing their experimental data with the all-orders-NLLA calculations and a model without the soft gluon coherence effect, L3[7] has suggested that this coherence effect is necessary to describe their measurement of $\langle M_3\rangle/\langle M_2\rangle$. Recently OPAL[8] has compared their experimental data of $\langle M_3\rangle/\langle M_2\rangle$ and $\langle M_3-3\rangle/\langle M_2-2\rangle$ with Monte Carlo models and all-orders-NLLA calculations. They have obtained good agreement of their data with the NLLA calculations and with coherent Monte Carlo models and have suggested that coherence effects are present in their data.

One of the sources of the suppression of $\langle M_3\rangle/\langle M_2\rangle$ suggested by Catani *et al.*[6] is destructive interference among soft gluons[10] which is also known as the color coherence effect. The color coherence effect results in restricted soft gluon emission in certain regions of phase space during parton shower evolution. This effect suppresses the interjet multiplicity between gluon and quark(antiquark) jets in 3-jet events more severely than that between quark-jets and antiquark-jets in 2-jet events[6]. This can contribute, therefore, to the suppression of $\langle M_3\rangle/\langle M_2\rangle$. Boudinov *et al.*[11] have compared the predictions of QCD Monte Carlo calculations with LEP and lower energy e^+e^- data and argued that there is no evidence for the sensitivity of the hadronic spectra to soft gluon coherence. Recently evidence for this effect has been obtained by measuring energy-energy correlation asymmetry and particle-particle correlation asymmetry in e^+e^- annihilations[12] and measuring kinematic correlations between soft and leading jets in $p\overline{p}$ collisions[13].

In the present paper, we study the contribution of the color coherence effect to the suppression of $\langle M_3-3\rangle/\langle M_2-2\rangle$. This quantity has higher sensitivity to a change of y_0 compared to $\langle M_3\rangle/\langle M_2\rangle$ in the region close to y_1. We compare our experimental data of $\langle M_3-3\rangle/\langle M_2-2\rangle$ with the predictions of coherent and incoherent Monte Carlo models and with the all-orders-NLLA calculations. We report the first measurement of the ratio of the average sub-jet multiplicity in 4-jet to that in 2-jet events, $\langle M_4-4\rangle/\langle M_2-2\rangle$. Comparison is also made between the experimental data of $\langle M_4-4\rangle/\langle M_2-2\rangle$ and the predictions of these Monte Carlo models. Additionally, in order to study the difference of the behavior between gluon-jets in 3-jet and 4-jet events, the experimental data of an observable $\Re = (\langle M_4\rangle - \langle M_2\rangle)/(\langle M_3\rangle - \langle M_2\rangle)$ are compared with the predictions of these Monte Carlo models and the naive QCD prediction.

2 Experiment

We have used the e^+e^- annihilation data obtained from the AMY detector at the TRISTAN storage ring. The center-of-mass energy of the data sample used in our analysis is 58 GeV. This data sample corresponds to an integrated luminosity of $297.15\ pb^{-1}$.

A detailed description of the AMY detector appears elsewhere[14]. It is worthwhile, however, presenting a few special features of the detector components used in this analysis. The central drift chamber (CDC) which is used for charged particle detection is comprised of 25 axial and 15 stereo layers and provides a momentum resolution of $0.7\% \times p_t$ (GeV/c). The cylindrical electromagnetic shower counter (SHC) which is used for neutral particle detection is an assembly of 20 layers of alternate lead and gas proportional tubes and provides an energy resolution of $23\%/\sqrt{E(GeV)}$ + 6%. Charged and neutral particles are detected in the CDC and SHC over the region $|\cos\theta| \leq 0.87$ and $|\cos\theta| \leq 0.75$, respectively.

3 Data sample

3.1 Event selection

Measurements of the momentum of charged tracks are made in the CDC and, of the energy deposited by photons, in the SHC. A track with at least 8 axial and 5 stereo hits that fit to a helix is defined as a charged track in the CDC. An acceptable cluster in the SHC is required to have an energy deposit of at least 0.3 GeV. The events are then subjected to the following event selection criteria. An event is regarded as an acceptable multi-hadronic event if:

1. There are five or more charged tracks in the CDC each satisfying the following conditions,
    (a) $|\cos\theta| \leq 0.85$,
    (b) point of origin is away from the interaction point by at most 5 cm radially and ±15 cm in the z direction,
2. E_{vis}, defined as the sum of energy of the charged particles (assumed to be pions) in the CDC and the energy deposited by neutral clusters in the SHC, is more than half of the center-of-mass energy,
3. The longitudinal momentum imbalance is less than $0.4 \times E_{vis}$ GeV/c,
4. There is at least 5 GeV of total energy deposit in the SHC.

A shower with an energy deposit of less than 1 GeV in the SHC and within 2° of a fitted track in the CDC is not regarded as an independent cluster.

A total of 30,042 events pass the above selection criteria and comprise the data sample for the present study.

3.2 Monte Carlo simulation

In the present analysis we have used three Monte Carlo event simulators, the LUND7.3 (JETSET)[15], HERWIG5.7[16] and ARIADNE4.04[17]. Each of these models consists of two broad stages, *i.e.* *parton generation* and *hadronization*. In the parton generation stage, the first two simulators use a branching model commonly called the *parton shower model*, whereas the third one uses the *color dipole model*[18]. In the hadronization stage, the HERWIG model uses the *cluster hadronization model*, whereas the other two use the LUND *string fragmentation model*.

The parton generation stage includes either a probabilistic branching of partons or parton emission off dipoles formed from a pair of partons. Both cases result in parton showering. The color coherence effect is taken into account in the *parton shower model* by employing *angular ordering* in which the parton emission angle in the showering process successively decreases and in the *color dipole model* by requiring decreasing p_t in successive emissions. The restriction of the angle or p_t imposed on the emission of successive partons in these models approximates the coherent emission of partons. The hadronization phase follows the parton shower converting the partons into hadrons. This process is non-perturbative and is applicable only in the form of phenomenological models.

We have used the LUND model both with and without angular ordering. We call the former the LUND *coherent* and the latter the LUND *incoherent* model. Parameters for the LUND *incoherent* and HERWIG models have been tuned using the global event shape distributions of *thrust*, p_t^{in}, p_t^{out} and the mass-squared difference between broad-jets and slim-jets. In the case of the LUND *coherent* and ARIADNE models, the default parameters set by the models' authors are used. All of the above models have been found to reproduce the event shape distributions of the experimental data quite well.

4 Analysis

4.1 Sub-jet multiplicity

The DURHAM ($k_\perp$) clustering algorithm[19] is used to form jets in the selected events. The resolution parameter for the $k_\perp$ algorithm is given by

$$y_{ij} = \frac{2\,min(E_i^2, E_j^2)(1-\cos\theta_{ij})}{E_{vis}^2} \,. \qquad (1)$$

where E_i and E_j are the energies of any pair of particles/clusters i and j, respectively and θ_{ij} is the angle between them.

In the present analysis we use two cut-off resolution parameters, y_1 and y_0 as mentioned in Sec. 1. According to Ref. [6], the NLLA calculations are most reliable for $y_1 \sim \mathcal{O}(10^{-2})$ and tend to be less reliable for larger y_1 values. Considering this we have chosen four y_1 values, 0.007, 0.01, 0.02 and 0.03 which correspond to cut-off transverse momenta between two jets of 4.9 GeV/c, 5.8 GeV/c, 8.2 GeV/c and 10.0 GeV/c, respectively.

For a particular value of y_1 we determine the jet multiplicity in each event. The observed jet fractions are 49.6%, 37.7% and 11.2% for 2-jet, 3-jet and 4-jet events, respectively at $y_1 = 0.007$. After having selected 2-jet and 3-jet samples we vary y_0 in a range from 4×10^{-3} down to 10^{-5} (corresponding to a cut-off transverse momentum between two sub-jets of 0.18 GeV/c) and obtain the sub-jet multiplicities $\langle M_n - n\rangle$ for $n = 2, 3$ averaged over all events in the n-jet event sample. The quantities, $\langle M_n - n\rangle$, may be interpreted as the average number of emitted soft gluons in n-jet events. The ratio R_{23} of the average sub-jet multiplicity in 3-jet events to that in 2-jet events for a given pair of resolution parameters is then defined as:

$$R_{23}(y_1, y_0) = \frac{\langle M_3 - 3\rangle}{\langle M_2 - 2\rangle} \,. \qquad (2)$$

The same analysis method is employed for the ratio of the average sub-jet multiplicity in 4-jet events to that in 2-jet events, $R_{24}(y_1, y_0) = \langle M_4 - 4\rangle / \langle M_2 - 2\rangle$.

The observable $\Re(y_1, y_0) = (\langle M_4\rangle - \langle M_2\rangle)/(\langle M_3\rangle - \langle M_2\rangle)$ is analyzed in the same way as R_{23} and R_{24}. This observable can be interpreted as the ratio of the number of soft gluons radiated from two primary gluons in 4-jet events to that from one primary gluon in 3-jet events assuming 3-jet and 4-jet events are comprised of $q\bar{q}g$ and $q\bar{q}gg$ final states, respectively.

4.2 Correction factor

Since the experimental data are biased by detector effects, initial state radiation(ISR) and final state radiation(FSR), they cannot be directly compared with theoretical and Monte Carlo calculations. Hence, we have corrected the experimental raw data for each value of y_0 for given values of y_1. We have prepared two types of Monte Carlo data samples using the LUND *coherent* model, one with a complete simulation of the AMY detector and the other without it. These data are referred to as *simulated* and *generated* data, respectively. The *generated* data are calculated without ISR and FSR and without the event selection criteria. The *simulated* data, on the other hand, are calculated with ISR and FSR and with the event selection criteria. In these calculations all the hadrons with lifetime larger than 1 ns are used, as given in Ref. [20].

Taking into account the changes in the detector conditions during our experiment, we have calculated *simulated* data for seven sets of detector conditions. We have corrected the experimental raw data for these different conditions using corresponding *simulated* data samples as shown below:

$$R_{2n}(y_1, y_0)_{corrected} = CF(y_1, y_0) \times R_{2n}(y_1, y_0)_{raw\ data} \;;\; n = 3, 4 \qquad (3)$$

where $CF(y_1, y_0)$ is a correction factor given by

$$CF(y_1, y_0) = \frac{(Generated\ data)_{without\ ISR\ and\ FSR,\ without\ event\ selection\ criteria}}{(Simulated\ data)_{with\ ISR\ and\ FSR,\ with\ event\ selection\ criteria}} \,. \qquad (4)$$

The corrected experimental data of R_{2n}, obtained for different detector conditions, have been merged together. The experimental raw data of $\Re = (\langle M_4\rangle - \langle M_2\rangle)/(\langle M_3\rangle - \langle M_2\rangle)$ have been corrected in the same way as R_{23} and R_{24}.

The values of CF for y_1=0.007 range from 1.012 to 1.078 for R_{23}, from 1.017 to 1.090 for R_{24} and from 0.980 to 0.995 for $\Re$ and their total errors range from 0.009 to 0.040 for R_{23}, from 0.013 to 0.065 for R_{24} and from 0.008 to 0.040 for $\Re$.

5 Results

5.1 Results

The experimental data have been compared with predictions of the Monte Carlo models for R_{23}, R_{24} and $\Re$ and with the all-orders-NLLA calculations for R_{23} for

y_1 values of 0.007, 0.01, 0.02 and 0.03. At $y_1 = 0.007$ we have obtained the best agreement with the coherent (LUND *coherent*, HERWIG and ARIADNE) Monte Carlo models and with the all-orders-NLLA calculations.

The corrected experimental data for $R_{23} = \langle M_3 - 3\rangle / \langle M_2 - 2\rangle$ are plotted in Fig. 1 as a function of y_0 in the y_0 range from 10^{-5} to 4×10^{-3} for $y_1 = 0.007$ and compared with the predictions of coherent (LUND *coherent*, HERWIG and ARIADNE) and incoherent (LUND *incoherent*) models.

Figure 2 shows the dependence of R_{23} on y_1 in the range of y_1 from 0.007 to 0.03 for the experimental and Monte Carlo data. In figures (a), (b) and (c), values of R_{23} are plotted as a function of y_1 for $y_0 = 4 \times 10^{-4}$, 1×10^{-3} and 2×10^{-3}, respectively.

In addition to R_{23}, we have obtained the sub-jet multiplicity ratio $R_{24} = \langle M_4 - 4\rangle / \langle M_2 - 2\rangle$. The corrected data for R_{24} for $y_1 = 0.007$ are shown in Fig. 3 in comparison with the Monte Carlo predictions.

Following the all-orders-NLLA calculations by Catani *et al.*[6], we have calculated R_{23} at $\sqrt{s} = 58$ GeV. Best agreement between the NLLA calculations and the experimental data has been obtained for $\Lambda = 300$ MeV and $y_1 = 0.007$ in the y_0 range of $7 \cdot 10^{-4} < y_0 < 4 \cdot 10^{-3}$, although variation of R_{23} in the range of Λ from 100 MeV to 500 MeV is within 1.8% around its value for Λ=300 MeV. In the present calculation the leading order expression for α_s has been used. In Fig. 1 the calculation for $\Lambda =$ 300 MeV and $y_1 = 0.007$ is also shown in comparison with the experimental data.

In Fig. 4, the corrected experimental data for $\Re = (\langle M_4\rangle - \langle M_2\rangle)/(\langle M_3\rangle - \langle M_2\rangle)$ for $y_1 = 0.007$ are shown in comparison with the four Monte Carlo predictions.

It should be noted that all the data of R_{23}, R_{24} and $\Re$ at successive values of y_0 are correlated to some extent since the sub-jet multiplicity is a cumulative observable.

5.2 Systematic errors

The major sources of systematic error in the present study are due to:

1. Detector simulation dependence,

2. Generator dependence,

3. Dependence on event selection criteria.

The first source of systematic error arises due to the use of detector simulators for the CDC and SHC and the different detector conditions that existed during the course of

the data taking. The second source is from the correction of the experimental data based on the LUND *coherent* model. The third source is due to the event selection procedures. In each of these cases, the systematic error has been obtained from the deviation of the corrected experimental data for each value of y_0 resulting from the variation of certain conditions.

The uncertainty due to the first source is from two factors. The first factor is due to the CDC and SHC simulators and the second one is due to different detector conditions. For the estimation of the first factor, we have used the LUND *coherent* model and have compared three sets of corrected experimental data, charged particles only, neutral particles only and both charged and neutral particles. Half of the maximum difference between any two sets have been taken as the systematic error. We have taken the standard deviation of the corrected experimental data obtained from seven sets of detector conditions to be the second factor. These two factors have been added in quadrature to obtain the systematic error due to this source. These range from 0.58% to 2.80% for R_{23}, from 1.51% to 6.87% for R_{24} and from 0.78% to 2.64% for $\Re$.

The systematic error due to the second source has been estimated by comparing the corrected data obtained by using the LUND *coherent* and HERWIG models both without ISR and FSR. The reason for not using ISR and FSR in this estimation is that the HERWIG model doesn't have an inherent provision for implementation of ISR and FSR. We have obtained the systematic errors in this case to be 0.56% ~ 3.03%, 0.13% ~ 1.38% and 0.26% ~ 1.85% for R_{23}, R_{24} and $\Re$, respectively.

The uncertainty due to the third source is from two factors. The first factor is due to the limited detector acceptance which may introduce biases on the determination of jet/sub-jet multiplicities. The second factor is due to the method of selecting the data sample. In order to estimate the systematic error due to the first factor, we have used a thrust angle cut of $50° < \theta_{thr} < 130°$ and have taken the deviation of the data with this cut from the data without it to be the systematic error. The second factor is estimated in the following way. For our analysis, after imposing track selection criteria, we select good events in two broad steps. Firstly, we impose event selection criteria as mentioned in Section 3.1 and secondly, on the basis of a track matching scheme, we find the charged tracks and neutral clusters from the CDC and SHC, respectively. The systematic error due to the second factor has been estimated by switching the sequence of these two steps. The overall systematic error due to the selection criteria dependence has been obtained by adding in quadrature the systematic errors due to the two factors. These are 0.02% $\sim$ 0.32% for R_{23}, 0.08% $\sim$ 1.61% for R_{24} and 0.02% $\sim$ 0.45% for $\Re$.

All other sources of systematic error have been disregarded owing to the benefit of using ratios such as R_{23}, R_{24} and $\Re$ where errors are likely to cancel.

The total systematic errors have been obtained by adding in quadrature the errors due to all sources discussed above. In Table 1, total systematic errors and statistical errors on R_{23}, R_{24} and $\Re$ are shown for upper and lower y_0 regions.

6 Discussion

Figure 1 shows that over most of the y_0 region the measured values of R_{23} agree well with the coherent (LUND *coherent*, HERWIG and ARIADNE) models and do not favour the incoherent (LUND *incoherent*) model. This indicates that the destructive interference among soft gluons emitted from the primary gluon in 3-jet events contributes to the suppression in the average sub-jet multiplicity ratio R_{23}. It should be noted that other sources, *e.g.* the finite energy effect, are also expected to contribute to this suppression. Our result is consistent with that of L3[7] and in agreement with that of OPAL[8]. Our observed value of R_{23}=1.200 $\pm$ 0.008 at $y_0 = 10^{-5}$ for $y_1 = 0.007$ is comparable with that, 1.222$\pm$0.011, obtained by OPAL[8] for same values of y_0 and y_1.

In order to confirm the disagreement between the experimental data and the LUND *incoherent* model prediction, we have checked the sensitivity of the calculated R_{23} value to the tunable parameters[21] of this model. We find that variation of the parameters within the range in which this model still reproduces event shapes reasonably, does not reduce the difference between experimental data and the incoherent model.

As seen in Fig. 2, the measured values of R_{23} agree with the coherent models over almost the whole region of y_1 and y_0 covered by this study. This indicates that the color coherence effect contributes to the suppression of R_{23} also for y_1 values between 0.01 and 0.03.

The contribution of the color coherence effect to the suppression of R_{24} for y_1 = 0.007 is seen in Fig. 3. The coherent models are in good agreement while the incoherent model is not favoured by the experimental data. At y_1 values between 0.01 and 0.03 we also observe agreement between the data and the coherent Monte Carlo models.

In the non-perturbative hadronization process, the relative rate of increase of $\langle M_3 - 3\rangle$ as a function of decreasing $\log y_0$, $\Delta\langle M_3(y_0) - 3\rangle/(-\Delta \log y_0)/\langle M_3 - 3\rangle$, is expected to be same as that in 2-jet events, provided that quarks and gluons hadronize in the same manner. In the y_0 region corresponding to the non-perturbative process, therefore, $R_{23}(y_0)$ is expected to be nearly constant with respect to y_0, although $\langle M_2\rangle$ and $\langle M_3\rangle$ increase monotonically with respect to decreasing y_0. On the other hand, in the perturbative parton showering process, quarks and gluons are expected to emit different numbers of partons due to their different color charges. Therefore, the relative rate of increase of $\langle M_3 - 3\rangle$ with respect to decreasing y_0 is expected to be different from that of $\langle M_2 - 2\rangle$. Consequently it is expected that R_{23} is not constant with respect to y_0. These expectations are consistent with Monte Carlo model predictions. Figure 1 shows that the measured values of R_{23} are nearly constant in the region of $y_0 < 7\times10^{-4}$ and increase with decreasing y_0 in the region of $y_0 > 7\times10^{-4}$. In the present paper, therefore, the regions of y_0 less than and greater than 7×10^{-4} (corresponding to a cut-off transverse momentum between sub-jets of about 1.5 GeV/c) are termed the *non-perturbative region* and the *perturbative region*, respectively.

In Fig. 1 observed values of R_{23} agree with the all-orders-NLLA calculations for Λ = 300 MeV and y_1 = 0.007 in the perturbative region under the LPHD assumption. The present result is consistent with that reported by OPAL[8] which shows excellent agreement between their data and the all-orders-NLLA calculations for Λ = 350 MeV and $y_1 = 0.007$.

Figure 4 shows that the experimental data of $\Re = (\langle M_4\rangle - \langle M_2\rangle)/(\langle M_3\rangle - \langle M_2\rangle)$ lie between 1.96$\pm$0.02 and 1.89$\pm$0.05 over the whole region of y_0. According to the naive QCD prediction based on color charge counting, we expect that $\Re\approx2$ if the behavior of emission of soft gluons from a primary gluon is same in 3-jet and 4- jet events and if the fraction of the $q\bar{q}q\bar{q}$ final state in 4-jet events (<10%)[22] is neglected. In this estimation, the 3-jet events are assumed to be comprised of $q\bar{q}g$ final state and the 4-jet events, of a mixture of $q\bar{q}gg$ and $q\bar{q}q\bar{q}$ final states at y_1 = 0.007. Our observation of the $\Re$ value is consistent with this expectation. This suggests that the behavior of emission of soft gluons from a primary gluon in 3-jet events is roughly same as that in 4-jet events. In Fig. 4, all the Monte Carlo models are in agreement with the experimental data. At y_1 values other than 0.007 we observe the same behavior of the data as that at $y_1 = 0.007$. In the figure the LUND *coherent* model predicts almost the same value of $\Re$ as the LUND *incoherent* model, because the color coherence effect doesn't affect the value of $\Re$ if the contribution of the color coherence effect to the suppression of the sub-jet multiplicity is same in 3-jet and 4-jet events.

7 Summary

The ratios of average sub-jet multiplicity in 3-jet to 2-jet events, R_{23} and 4-jet to 2-jet events, R_{24}, have been measured in e^+e^- annihilations at $\sqrt{s} = 58$ GeV using the AMY detector. The observed values of R_{23} and R_{24} have been compared with the predictions of coherent (LUND7.3 *coherent*, HERWIG5.7 and ARIADNE4.04) and incoherent (LUND7.3 *incoherent*) models.

It has been shown that the measured values of R_{23} agree well with the coherent models and not the incoherent one. This is evidence of the contribution of the color coherence effect to the observed suppression in R_{23}. The all-orders-NLLA calculations agree with the measured values of R_{23} in the perturbative region ($y_0 > 7 \times 10^{-4}$) for $\Lambda = 300$ MeV and $y_1 = 0.007$ under the LPHD assumption.

The contribution of the color coherence effect to the suppression of R_{24} has also been observed. Good agreement has been observed between measured values of R_{24} and the coherent models.

The study of the quantity $\Re$ suggests that the behavior of emission of soft gluons from the primary gluon in 3-jet events is almost the same as that in 4-jet events.

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Table 1: The systematic and statistical errors on R_{23}, R_{24} and $\Re$ for $y_1 = 0.007$

	y_0	Systematic errors(%)	Statistical errors(%)
R_{23}	$1\cdot10^{-5} \sim 7\cdot10^{-4}$	0.81∼1.87	0.36∼0.68
	$1\cdot10^{-3} \sim 4\cdot10^{-3}$	1.59∼4.13	0.79∼2.21
R_{24}	$1\cdot10^{-5} \sim 7\cdot10^{-4}$	1.54∼2.48	0.51∼0.99
	$1\cdot10^{-3} \sim 4\cdot10^{-3}$	2.18∼7.19	1.19∼3.72
$\Re$	$1\cdot10^{-5} \sim 7\cdot10^{-4}$	1.97∼3.22	1.47∼1.66
	$1\cdot10^{-3} \sim 4\cdot10^{-3}$	0.83∼1.95	0.87∼1.46

Figure captions

Fig. 1: The experimental data for $R_{23} = \langle M_3 - 3\rangle / \langle M_2 - 2\rangle$ are plotted as a function of y_0 for $y_1 = 0.007$ in comparison with the predictions of the LUND (*coherent/incoherent*), HERWIG and ARIADNE models and also with the all-orders-NLLA calculations for $\Lambda = 300$ MeV. Errors shown are statistical and systematic errors added in quadrature.

Fig. 2: The experimental data for $R_{23} = \langle M_3 - 3\rangle / \langle M_2 - 2\rangle$ are shown as a function of y_1 in comparison with the LUND *coherent* (dashed line), LUND *incoherent* (dot-dashed line), HERWIG (dotted line) and ARIADNE (spaced-dotted line) models for y_0 values (a) $4\cdot10^{-4}$, (b) $1\cdot10^{-3}$ and (c) $2\cdot10^{-3}$.

Fig. 3: The experimental data for $R_{24} = \langle M_4 - 4\rangle / \langle M_2 - 2\rangle$ are plotted as a function of y_0 in comparison with the predictions of LUND (*coherent/incoherent*), HERWIG and ARIADNE models for $y_1 = 0.007$.

Fig. 4: The experimental data for $\Re = (\langle M_4\rangle - \langle M_2\rangle)/(\langle M_3\rangle - \langle M_2\rangle)$ are shown in comparison with the predictions of LUND (*coherent/incoherent*), HERWIG and ARIADNE models for $y_1 = 0.007$.

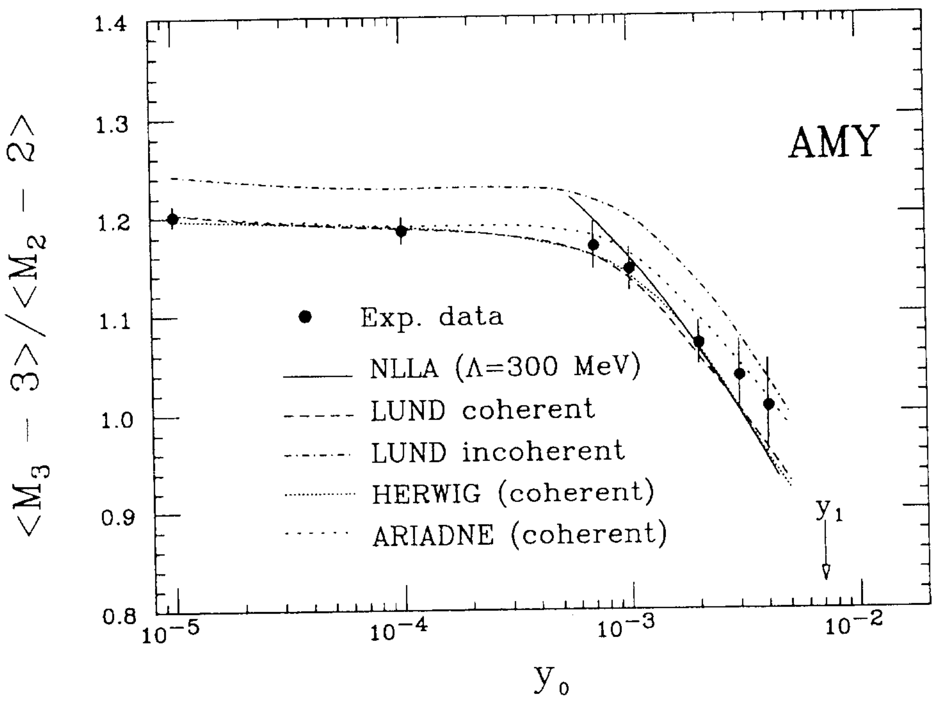
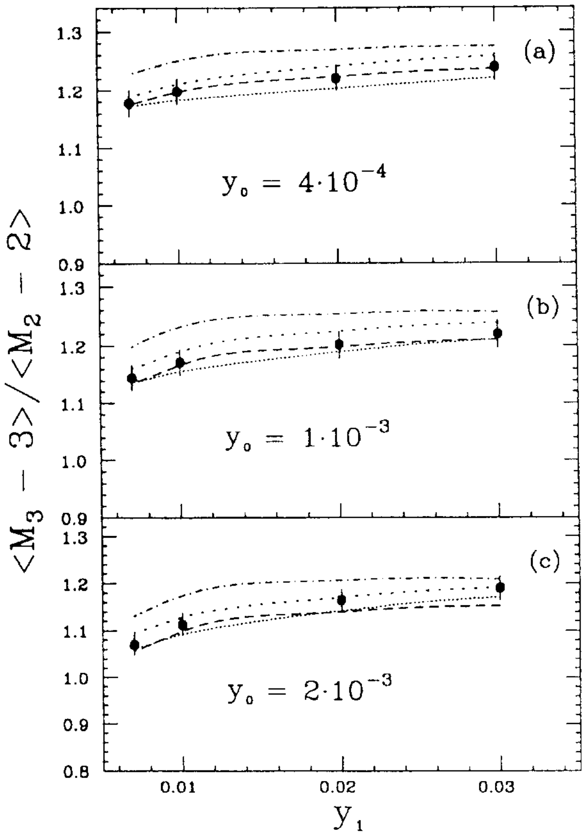
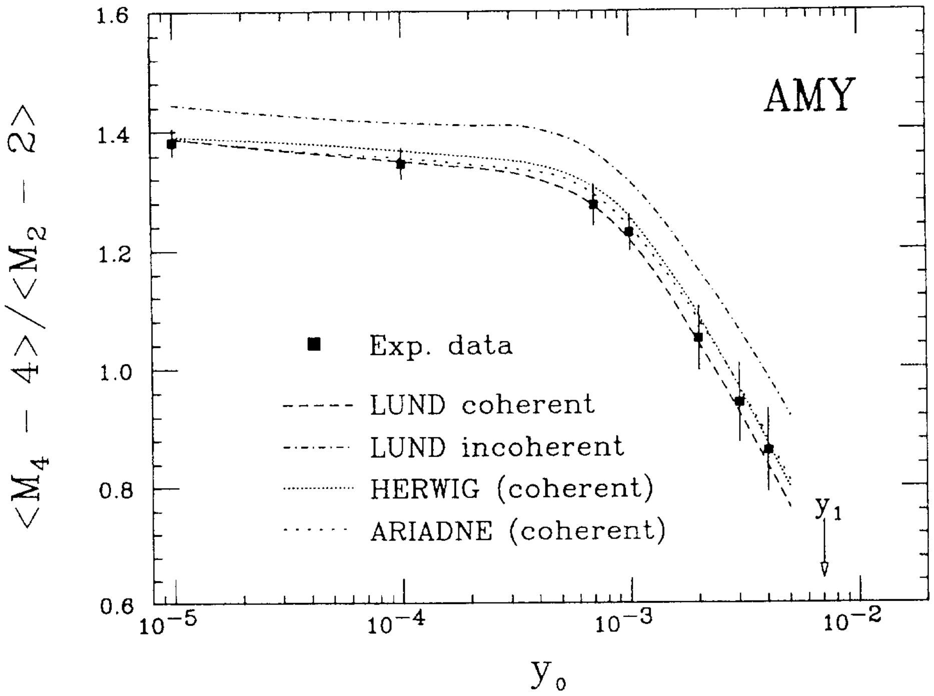


Fig. 1



$\langle M_3 - 3\rangle / \langle M_2 - 2\rangle$
(a)
$y_0 = 4\cdot10^{-4}$
(b)
$y_0 = 1\cdot10^{-3}$
(c)
$y_0 = 2\cdot10^{-3}$
1.3
1.2
1.1
1.0
0.9
0.8
0.01
0.02
0.03
y_1

Fig. 2



AMY
$\langle M_4 - 4\rangle / \langle M_2 - 2\rangle$
■ Exp. data
LUND coherent
LUND incoherent
HERWIG (coherent)
ARIADNE (coherent)
y_1
1.6
1.4
1.2
1.0
0.8
0.6
10^{-5}
10^{-4}
10^{-3}
10^{-2}
y_0

Fig. 3

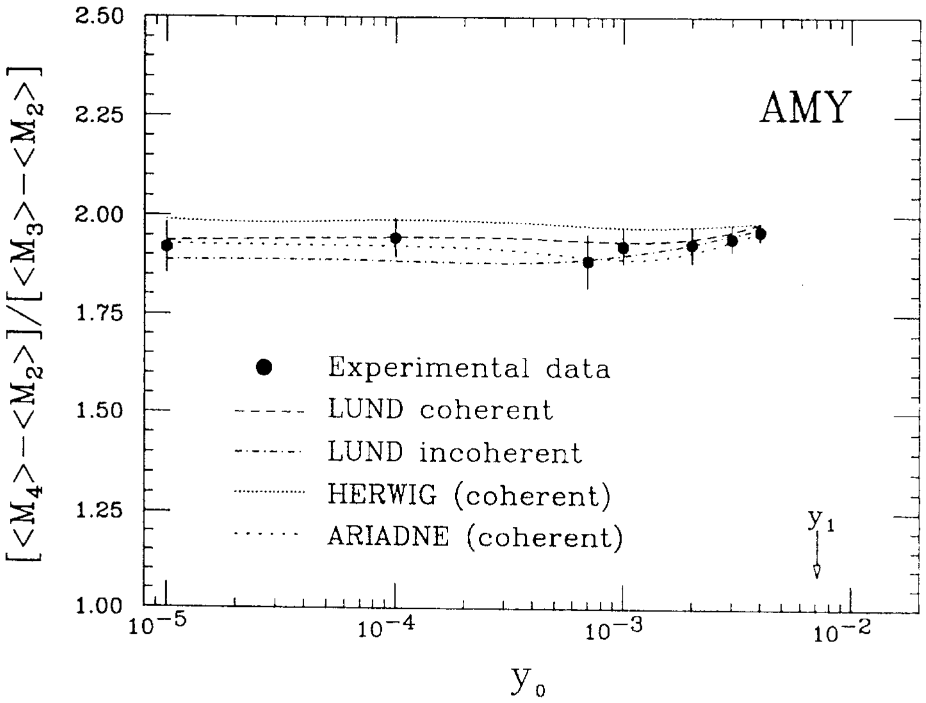

AMY
2.50
2.25
2.00
1.75
1.50
1.25
1.00
$[\langle M_4\rangle-\langle M_2\rangle]/[\langle M_3\rangle-\langle M_2\rangle]$
10^{-5}
10^{-4}
10^{-3}
10^{-2}
y_0
y_1
● Experimental data
LUND coherent
LUND incoherent
HERWIG (coherent)
ARIADNE (coherent)

Fig. 4