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Gauge Invariance and Confinement in a Generalized NJL Model

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Abstract

We discuss the quark-antiquark loop diagrams of an extended version of the Nambu−Jona-Lasinio model that includes a description of confinement. We show how the constraints of gauge invariance affect the calculations of such diagrams, if the $q\bar{q}$ system has the quantum numbers of the rho or omega mesons. In the NJL model without confinement, gauge invariance may be maintained by using a regularization scheme that respects that symmetry. Alternatively, gauge invariance constraints may be imposed by introducing a substraction procedure when evaluating the $q\bar{q}$ loop integrals (vacuum polarization diagrams). We discuss the merits of such subtraction procedures in the presence of a confining interaction. We also provide a calculation of the vacuum polarization diagrams of the NJL model that includes the effects of confinement and also satisfies the constraints imposed by gauge invariance. Our analysis provides some justification for the subtraction procedure used in several studies of the NJL model. We also show, in a rather clear fashion, how confinement serves to change the analytic properties of the polarization tensors, so that the tensor for the theory with confinement has only a physical cut structure in the region of interest.

I. Introduction

In the application of the Nambu−Jona-Lasinio model to the study of hadron structure one calculates quark-antiquark loop diagrams of the type shown in Fig. 1 [1]. In this work we will concentrate on those diagrams that appear when the $q\bar{q}$ system has the quantum numbers $J = 1^-$, with isospin 0 or 1. These diagrams arise in the study of the omega and rho mesons when using the NJL model. For such studies it is important to include a model of confinement. We have shown how that may be done in an earlier work [2]. For example, if we include a ladder of confining interactions, as shown in Fig. 1b, we may define polarization tensors that do not have the unitarity cuts that would have appeared, in the absence of confinement, if both the quark and antiquark go on mass shell [3].

With reference to Fig. 1a, we define a tensor [2]

$$-iJ^{\mu\nu}(q) = (-1)n_c n_f \mathrm{Tr} \int \frac{d^4k}{(2\pi)^4} \left[\gamma^\mu iS(k)\gamma^\nu iS(k+q)\right] \quad , \tag{1.1}$$

where $n_c = 3$ is the number of colors and $n_f = 2$ is the number of flavors. Further, $S(k) = [\not{k} - m + i\epsilon]^{-1}$, where m is the constituent quark mass. Often, the integral of Eq. (1.1) is regulated by including a cutoff, Λ_E. After passing to a Euclidean momentum space, one imposes the condition $k_E^2 < \Lambda_E^2$ [1]. (Typically, $\Lambda_E \sim 1$ GeV is used.) That kind of regularization destroys the gauge invariance of the result. Therefore, the following scheme has been adopted [1]. In Eq. (1.1), γ^μ is replaced by $\hat{\gamma}^\mu = \gamma^\mu - \not{q}q^\mu/q^2$ and γ^ν is replaced by $\hat{\gamma}^\nu = \gamma^\nu - \not{q}q^\nu/q^2$. This allows one to write

$$J^{\mu\nu}(q) = -\tilde{g}^{\mu\nu}(q)J(q^2) \quad , \tag{1.2}$$

where

$$\bar{g}^{\mu\nu}(q) = g^{\mu\nu} - q^\mu q^\nu / q^2 \quad . \tag{1.3}$$

This procedure imposes the condition $q_\mu J^{\mu\nu}(q) = J^{\mu\nu}(q) q_\nu = 0$. However, we must also have $J(0) = 0$ to avoid generating a mass for the photon [4,5]. That may be accomplished by making a subtraction, so that $J(q^2)$ is replaced by $J(q^2) - J(0)$. This is the procedure used by Weise and collaborators in their extensive applications of the NJL model to the study of hadron structure [1]. (Note that, in their work, it was necessary to neglect $\mathrm{Im}J(q^2)$, since those authors did not include a model of confinement.) One may avoid making a subtraction by using a regularization scheme that respects gauge invariance. For example, Friedrich and Reinhardt use a proper-time regularization scheme and obtain vacuum polarization diagrams of the correct structure in their study of rho-omega mixing [4]. (A more complete discussion of this matter may be found in Ref. [5].)

In our work dealing with confinement, we have used a Euclidean-momentum-space cutoff for spacelike $q^2 (q^2 < 0)$ and a cutoff for each of the three-momenta in the loop integrals, when $q^2 > 0$ [2]. The (Minkowski-space) cutoff, Λ_3, used for $q^2 > 0$, was chosen so that the vacuum polarization integrals calculated for $q^2 > 0$ and $q^2 < 0$ were (approximately) continuous at $q^2 = 0$. (See Fig. 2.) In our earlier work, we calculated the tensor

$$-i\hat{J}^{\mu\nu}(q) = n_c n_f \mathrm{Tr} \int \frac{d^4k}{(2\pi)^4} \left[\hat{\Gamma}^\mu(q,k)\, S(k)\, \hat{\gamma}^\nu\, S(k+q)\right] \quad , \tag{1.4}$$

where $\hat{\Gamma}^\mu(q,k)$ was the vertex function that arose when we summed a ladder of confining interactions [2,3]. Our calculation was made such that $q_\mu \hat{\Gamma}^\mu(q,k) = 0$. Thus, we could define

$$\hat{J}^{\mu\nu}(q) = -\tilde{g}^{\mu\nu}(q)\hat{J}(q^2) \quad . \tag{1.5}$$

In the limit that the current masses of the up and down quarks are equal, the tensors, $\hat{J}^{\mu\nu}(q)$, calculated for the rho and omega mesons are the same. Therefore, we can identify $\hat{J}_{(\rho)}(q^2) = \hat{J}_{(\omega)}(q^2) = \hat{J}(q^2)$. In Fig. 2, we show values of $\hat{J}_{(\rho)}(q^2)$ calculated in Ref. [6]. (The dotted line in the figure interpolates between the calculations made for $q^2 > 0$ and $q^2 < 0$ to yield a continuous curve.) In Fig. 2, we see that $\hat{J}_{(\rho)}(0) \neq 0$. We may make a subtraction to obtain $\hat{J}_{(\rho)}(0) = 0$, as discussed above. This subtraction does not change the momentum-space bosonization procedure we have used in our work. For example, we may define a T matrix in the omega channel,

$$T_\omega(q^2) = \frac{G_\omega}{1 - G_\omega \hat{J}_{(\omega)}(q^2)} \quad , \tag{1.6}$$

where we have neglected reference to the Dirac and isospin matrices, for simplicity. (In Eq. (1.6), we have summed a string of $q\bar{q}$ loop integrals. The simplicity of the NJL model allows us to provide a simple result for that sum.) In Eq. (1.6), G_ω is a parameter of the Lagrangian of our extended NJL model,

$$\begin{aligned} \mathcal{L}(x) = {} & \bar{q}(x)\left(i\partial\!\!\!/ - m_q^0\right)q(x) - \frac{G_S}{2}\left[\left(\bar{q}(x)q(x)\right)^2 + \left(\bar{q}(x)i\gamma_5\vec{\tau}q(x)\right)^2\right] \\ & - \frac{G_V}{2}\left[\left(\bar{q}(x)\gamma^\mu\vec{\tau}q(x)\right)^2 + \left(\bar{q}(x)\gamma_5\gamma^\mu\vec{\tau}q(x)\right)^2\right] \\ & - \frac{G_\omega}{2}\,\bar{q}(x)\gamma^\mu q(x) + \mathcal{L}_{conf}(x) \quad . \end{aligned} \tag{1.7}$$

Here, $\mathcal{L}_{conf}(x)$ refers to the model of confinement that we have introduced [2,3]. In a momentum-space bosonization scheme, we may put [7]

$$\frac{G_\omega}{1 - G_\omega \hat{J}_{(\omega)}(q^2)} = - \frac{g^2_{\omega qq}(q^2)}{q^2 - m^2_\omega} \quad . \tag{1.8}$$

Equation (1.8) serves as a definition of the momentum-dependent coupling constant $g_{\omega qq}(q^2)$. (The widths of the omega and rho mesons may also be considered. However, we do not enter into that discussion here.) We now consider the subtraction procedure, replacing $\hat{J}_{(\omega)}(q^2)$ by $\hat{J}_{(\omega)}(q^2) - \hat{J}_{(\omega)}(0)$, for example. If we simultaneously replace G_ω^{-1} by $\bar{G}_\omega^{-1} \equiv G_\omega^{-1} - \hat{J}_{(\omega)}(0)$, we see that $g^2_{\omega qq}(q^2)$ and m^2_ω are unchanged. (On the other hand, replacing G_ω by $\bar{G}_\omega$ does significantly increase the (dimensionless) meson decay constant, g^ω, that has an empirical value of about 15.2 [6].)

In the present work, we wish to investigate the merits of the subtraction procedure, while maintaining our model of confinement. To that end, we will introduce a regularization procedure (Pauli-Villars) that respects gauge invariance and show how our model of confinement may be implemented in this case. That analysis is carried out in Section II. In Section III we provide further details concerning our model of confinement, while Section IV provides some additional discussion.

II. Gauge Invariance and Confinement in an Extended NJL Model

First, let us consider the calculation of the vacuum polarization tensor in QED. We will follow the discussion of Kaku [8]. Kaku defines the tensor

$$\Pi_{\mu\nu,m}(q) = -e^2 \mathrm{Tr} \int \frac{d^4k}{(2\pi)^4} \gamma_\mu \frac{1}{\not{k} - m + i\epsilon} \gamma_\nu \frac{1}{\not{k} - m + i\epsilon} \quad , \tag{2.1}$$

where the dependence on the fermion mass is made explicit. He then defines

$$\tilde{\Pi}_{\mu\nu}(q) = \Pi_{\mu\nu,m}(q) - \Pi_{\mu\nu,M}(q) \quad , \tag{2.2}$$

where M is the Pauli-Villars regulator mass. Further, we have the definition

$$\tilde{\Pi}_{\mu\nu}(q) = (g_{\mu\nu}q^2 - q_\mu q_\nu)\,\tilde{\Pi}_1(q^2) \quad . \tag{2.3}$$

The result obtained for $\tilde{\Pi}_1(q^2)$ is

$$\tilde{\Pi}_1(q^2) = \frac{ie^2}{12\pi^2}\left\{ -\ln\left[\frac{M^2}{m^2}\right] + \frac{1}{3} + 2\left[1 + \frac{2m^2}{q^2}\right]\left[\left[\frac{4m^2}{q^2} - 1\right]^{1/2} \cot^{-1}\left[\frac{4m^2}{q^2} - 1\right]^{1/2} - 1\right]\right\} \quad , \tag{2.4}$$

for $0 < q^2 < 4m^2$ [8].

Now let us assume that we regulate $J(q^2)$ of Eq. (1.2) using the Pauli-Villars method. We may call the result $\tilde{J}(q^2) = J_m(q^2) - J_M(q^2)$. The relation between $\tilde{J}(q^2)$ and $\tilde{\Pi}_1(q^2)$ is then

$$\tilde{J}(q^2) = i\,\frac{n_c n_f}{e^2}\, q^2\, \tilde{\Pi}_1(q^2) \quad . \tag{2.5}$$

At this point we have not introduced the effect of confinement. (Note that the condition $\tilde{J}(q^2) = 0$ corresponds to the requirement that $\tilde{\Pi}_1(q^2)$ not have a simple pole at $q^2 = 0$.) Equations (2.4) and (2.5) yield a result for $\tilde{J}(q^2)$ valid in the region $0 < q^2 < 4m^2$,

$$\tilde{J}(q^2) = -\frac{n_c n_f}{12\pi^2} q^2 \left\{ -\ln\left[\frac{M^2}{m^2}\right] + \frac{1}{3} + 2\left[1 + \frac{2m^2}{q^2}\right]\left[\left[\frac{4m^2}{q^2} - 1\right]^{1/2} \cot^{-1}\left[\frac{4m^2}{q^2} - 1\right]^{1/2} - 1\right]\right\} , \tag{2.6}$$

with $\tilde{J}(0) = 0$, as expected.

In the case that $q^2 > 4m^2$, we may introduce

$$x = \left[1 - \frac{4m^2}{q^2}\right]^{1/2} , \tag{2.7}$$

and obtain

$$\mathrm{Re}\,\tilde{J}(q^2) = -\frac{n_c n_f}{12\pi^2} q^2 \left\{ -\ln\left[\frac{M^2}{m^2}\right] + \frac{1}{3} + 2\left[1 + \frac{2m^2}{q^2}\right]\left[\frac{x}{2}\ln\left[\frac{1+x}{1-x}\right] - 1\right]\right\} , \tag{2.8}$$

and

$$\mathrm{Im}\,\tilde{J}(q^2) = \frac{n_c n_f}{12\pi} q^2 \left[1 + \frac{2m^2}{q^2}\right]\left[1 - \frac{4m^2}{q^2}\right]^{1/2} . \tag{2.9}$$

(In Eq. (2.8), $x < 1$.)

For $q^2 \leq 0$, we may define

$$y = \left[1 + \frac{4m^2}{|q^2|}\right]^{1/2} \tag{2.10}$$

and write

$$\tilde{J}(q^2) = -\frac{n_c n_f}{12\pi^2} q^2 \left\{ -\ln\left[\frac{M^2}{m^2}\right] + \frac{1}{3} + 2\left[1 + \frac{2m^2}{q^2}\right]\left[\frac{y}{2}\ln\left[\frac{y+1}{y-1}\right] - 1\right] \right\} \quad (2.11)$$

From Eq. (2.10), we see that $y \geq 1$. Note that $\tilde{J}(q^2)$ is real for $q^2 < 4m^2$. We also note that

$$\tilde{\Pi}_1(0) = \frac{ie^2}{12\pi^2}\left[-\ln\frac{M^2}{m^2}\right] . \quad (2.12)$$

(In QED, the dependence on $\ln(M^2/m^2)$ is removed when introducing the renormalized charge. The NJL model is not renormalizable and the various observables depend upon the regulator mass.)

We now want to show how these results are modified when we introduce confinement. In particular, our model of confinement will make the polarization tensor real for $q^2 < 4M^2$.

III. Numerical Results

It is useful to introduce the notation $\hat{J}_m^{\mu\nu}(q)$ for the tensor defined in Eq. (1.4). Recall that m is the constituent quark mass. (Note that we may replace $\hat{\gamma}^\mu$ by γ^μ, if we are using a regularization scheme that preserves gauge invariance.) Let us define

$$\bar{J}^{\mu\nu}(q) = \hat{J}_m^{\mu\nu}(q) - J_M^{\mu\nu}(q) \quad , \tag{3.1}$$

where $J_M^{\mu\nu}(q)$ is obtained from Eq. (1.1) upon replacing m by M. (The tensor defined in Eq. (3.1) is depicted in a schematic fashion in Fig. 3a.) Note that we do not include confinement in the calculation of $J_M^{\mu\nu}(q)$.

It is useful to rewrite Eq. (3.1):

$$\bar{J}^{\mu\nu}(q) = \left[J_m^{\mu\nu}(q) - J_M^{\mu\nu}(q)\right] + \left[\hat{J}_m^{\mu\nu}(q) - J_m^{\mu\nu}(q)\right] \quad , \tag{3.2}$$

where we have added and subtracted $J_m^{\mu\nu}(q)$. It is also useful to define

$$\bar{J}^{\mu\nu}(q) = -\tilde{g}^{\mu\nu}(q)\bar{J}(q^2) \quad . \tag{3.3}$$

For completeness, we also write

$$\tilde{J}^{\mu\nu}(q) = J_m^{\mu\nu}(q) - J_M^{\mu\nu}(q) \quad , \tag{3.4}$$

$$= -\tilde{g}^{\mu\nu}(q)\tilde{J}(q^2) \quad . \tag{3.5}$$

The value of $\tilde{J}(q^2)$ was given in Eqs. (2.5)-(2.9), where we gave the results for the Pauli-Villars regularization procedure, without reference to confinement. The second bracketed term of Eq. (3.2) may be written as [2]

$$-i\left[\hat{J}_m^{\mu\nu}(q) - J_m^{\mu\nu}(q)\right] = (-1)n_c n_f \mathrm{Tr}\int \frac{d^4k}{(2\pi)^4}\left[(\Gamma^\mu(q,k) - \gamma^\mu)\, iS_m(k)\gamma^\nu\, iS_m(q+k)\right] , \quad (3.4)$$

where $S_m(k) = [\not{k} - m + i\epsilon]^{-1}$. The contributions to the integral in Eq. (3.4) are finite only in a limited region, since $\Gamma^\mu(q,k) \to \gamma^\mu$ (in the frame where $\vec{q} = 0$), if $|\vec{k}|$ is large. To see this in more detail, let us consider some results of Ref. [2], obtained by first performing the integration over k^0 in the calculation of $\hat{J}(q^2)$. We record Eq. (3.14) of Ref. [2]:

$$\hat{J}(q^2) = -n_c n_f \int \frac{d^3k}{(2\pi)^3}\left[\frac{m}{E(\vec{k})}\right]^2 \left\{ \frac{a_1^{+-}\Gamma_1^{+-} + a_2^{+-}\Gamma_2^{+-}}{q^0 - 2E(\vec{k}) + i\epsilon} - \frac{a_1^{-+}\Gamma_1^{-+} + a_2^{-+}\Gamma_2^{-+}}{q^0 + 2E(\vec{k})} \right\} , \quad (3.5)$$

where $E(\vec{k}) = (\vec{k}^2 + m^2)^{1/2}$, and where the Γ^{+-} and Γ^{-+} parametrize the confinement vertex calculated there. In Eq. (3.5), $a_1^{+-} = a_1^{-+} = -2\vec{k}^2/3m$ and $a_2^{+-} = a_2^{-+} = 4E^2(\vec{k})/3m^2$. The corresponding integral, without confinement, is obtained from Eq. (3.5) by putting $\Gamma^{+-} = \Gamma^{-+} = m/\vec{k}^2$ and $\Gamma_2^{+-} = \Gamma_2^{-+} = 1$ [2]. (Note that the arguments of the various Γ's are q^0 and $|\vec{k}|$ in the frame where $\vec{q} = 0$.) For $|\vec{k}| > 0.7$ GeV, we have $\Gamma^{+-} = \Gamma^{-+} = m/\vec{k}^2$ and $\Gamma_2^{+-} = \Gamma_2^{-+} = 1$. Thus, the integral over $|\vec{k}|$ in Eq. (3.4) only receives contributions if $|\vec{k}| \leq 0.7$ GeV. (That is, we may extend the integral over large values of $|\vec{k}|$, once $|\vec{k}| > 0.7$ GeV, without changing the result.)

Note that $\hat{J}(q^2)$ is real and that we may drop the $i\epsilon$ in the denominator of the first term of Eq. (3.5). That follows, since $\Gamma_1^{+-}(q^0, |\vec{k}|)$ and $\Gamma_2^{+-}(q^0, |\vec{k}|)$ are equal to zero at the on-mass-shell point, where $q^0 = 2E(\vec{k})$. We see that the cut that would have been present for $q^2 > 4m^2$ has been eliminated, when using our model of confinement.

IV. Discussion

It is useful to compare the results obtained in Section III for $\bar{J}(q^2)$ with a subtracted function. Let us define

$$\hat{J}_{sub}(q^2) = \hat{J}_{(\rho)}(q^2) - \hat{J}_{(\rho)}(0) \quad . \tag{4.1}$$

The function $\hat{J}_{(\rho)}(q^2)$ was shown in Fig. 2 and was taken from Ref. [6]. (Note that, in the calculation of $\hat{J}_{(\rho)}(q^2)$, we used the regularization scheme that involved the cutoffs Λ_E and Λ_3. Thus, this function makes no reference to the Pauli-Villars regularization procedure that was used when defining $\bar{J}_{(\rho)}(q^2) \equiv J_m(q^2) - J_M(q^2)$, or $\bar{J}_{(\rho)}(q^2) \equiv \hat{J}_m(q^2) - J_M(q^2) = [J_m(q^2) - J_M(q^2)] + [\hat{J}_m(q^2) - J_m(q^2)] = \bar{J}(q^2) + [\hat{J}_m(q^2) - J_m(q^2)]$.) The function $\hat{J}_{sub}(q^2)$ is shown in Fig. 4 as a solid line. We see that $\hat{J}_{sub}(0) = 0$, as follows from Eq. (4.1). In Fig. 4, we also show the function $\bar{J}(q^2)$, calculated with $M = 0.6$ GeV and $m = 0.260$ GeV. The calculated value of $\bar{J}(0)$ was equal to -0.004 GeV^2. That quite small nonzero value has its origin in the various approximations used in calculation of the confining vertex, $\Gamma^\mu(q,k)$. In Fig. 4, we have made $\bar{J}(q^2)$ consistent with the gauge invariance constraint, $\bar{J}(0) = 0$, by adding 0.004 GeV^2 to the calculated values. The fact that the two curves of Fig. 4 are quite similar lends some justification for the subtraction procedure that was used when a regularization scheme that did not respect gauge invariance was introduced [1,2].

To understand the role of confinement in the calculation of the vacuum polarization diagrams, we compare $\hat{J}_{sub}(q^2)$ and Re$\bar{J}(q^2)$ in Fig. 5. There, we see the rapid rise in the value of Re$\bar{J}(q^2)$ near the threshold value of $q^2 = 4m^2 = 0.27$ GeV^2. Since $\hat{J}_{sub}(q^2)$ contains the effects of confinement, the unitarity $q\bar{q}$ cut is absent. Therefore, there is no rapid rise seen for $q^2 \simeq 4m^2$ in the case of $\hat{J}_{sub}(q^2)$. (As noted earlier, $\bar{J}(q^2)$ will have an imaginary part for

$q^2 > 4M^2 = 1.54$ GeV^2, since we did not include confinement in the calculation of those diagrams where m was replaced by the regulator mass, M.)

In summary, we have shown that a subtraction procedure used previously appears to be in accord with a more fundamental approach to the maintenance of gauge invariance in the regularization of the integrals of the NJL model. [See Fig. 4.] We have seen, rather clearly, how our model of confinement removes the unphysical quark-antiquark unitarity cut. We have also seen how the q^2-dependence of the integrals considered here undergoes significant modification in the presence of confinement. [See Fig. 5, where we compare Re$\tilde{J}(q^2)$ and $\hat{J}_{sub}(q^2)$.]

<u>Acknowledgement</u>

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Figure Captions

Fig. 1 a) The fundamental loop-integral of the NJL model that serves to define the tensor $-iJ^{\mu\nu}(q)$ is shown. Here the quark propagators are $S(k) = [\not{k} - m + \epsilon]^{-1}$, where m is the constituent quark mass.

b) The vertex function, $\Gamma^{\mu}(q, k)$, that sums a ladder of confining interactions is shown as a filled triangular area. The driving term is γ^{μ} in the case of the omega meson and $\gamma^{\mu}\tau_3$ in the case of the rho.

c) The loop integral that defines the tensor $-i\hat{J}^{\mu\nu}(q)$ is shown. Note that $\hat{J}^{\mu\nu}(q)$ is real, since the vertex function vanishes if both quarks go on mass shall simultaneously.

Fig. 2 Values of $\hat{J}(q^2) = \hat{J}_{(\rho)}(q^2)$ are shown. (This figure may be found in Ref. [6].) For $q^2 < 0$, confinement was neglected and the calculation was done using a Euclidean momentum space, with $\Lambda_E = 1.0$ GeV. For $q^2 > 0$, confinement was taken into account. That calculation was made in Minkowski momentum space with a cutoff on all three-momenta in the loop integral of $\Lambda_3 = 0.702$ GeV [6]. The dotted curve serves to interpolate between the two calculations.

Fig. 3 a) The figure serves to define $-i\bar{J}^{\mu\nu}(q)$ as the difference between a quark loop integral (including confinement) and the loop integral with the Pauli-Villars regulator mass. [See Eq. (3.1).] (Propagators with the mass M are shown as lines with a large filled circle imposed. We have not included confinement for such propagators.)

b) The figure depicts Eq. (3.2) in a schematic fashion. The value of the first bracket in Fig. 3b may be obtained using the procedures described in Ref. [8]. (See Section II for analytic expressions for the first bracket.) Note that the calculation of the second bracket does not require regularization. (See Section III.)

Fig. 4 The dotted curve represents $\bar{J}(q^2)$ of Eq. (3.1), while the continuous line represents the value of $\hat{J}_{sub}(q^2) = \hat{J}_{(\rho)}(q^2) - \hat{J}_{(\rho)}(0)$. The calculation of $\bar{J}(q^2)$ was made for $m = 0.260$ GeV and $M = 0.6$ GeV. [Note that $\bar{J}(q^2)$ has an imaginary part for $q^2 > 4M^2$, since we did not include the confinement vertex in the integrals containing the regulator mass M.] Values of $\hat{J}_{(\rho)}(q^2)$ were taken from Ref. [6], where the cutoffs, Λ_E and Λ_3, were used in the regularization procedure.

Fig. 5 The dashed curve shows the value of $\tilde{J}(q^2) = J_m(q^2) - J_M(q^2)$. [See Eqs. (2.6)-(2.11).] Only Re$\tilde{J}(q^2)$ is shown. The continuous curve represents $\hat{J}_{sub}(q^2) = \hat{J}_{(\rho)}(q^2) - \hat{J}_{(\rho)}(0)$ and is the same as the corresponding curve shown in Fig. 4. This figure shows the effects of confinement in the calculation of the vacuum polarization diagrams. (Note that Im$\tilde{J}(q^2)$ is nonzero for $q^2 > 4m^2$.) The calculation of $\tilde{J}(q^2)$ and $\bar{J}(q^2)$ was made with $m_q = 0.26$ GeV and $M = 0.6$ GeV.

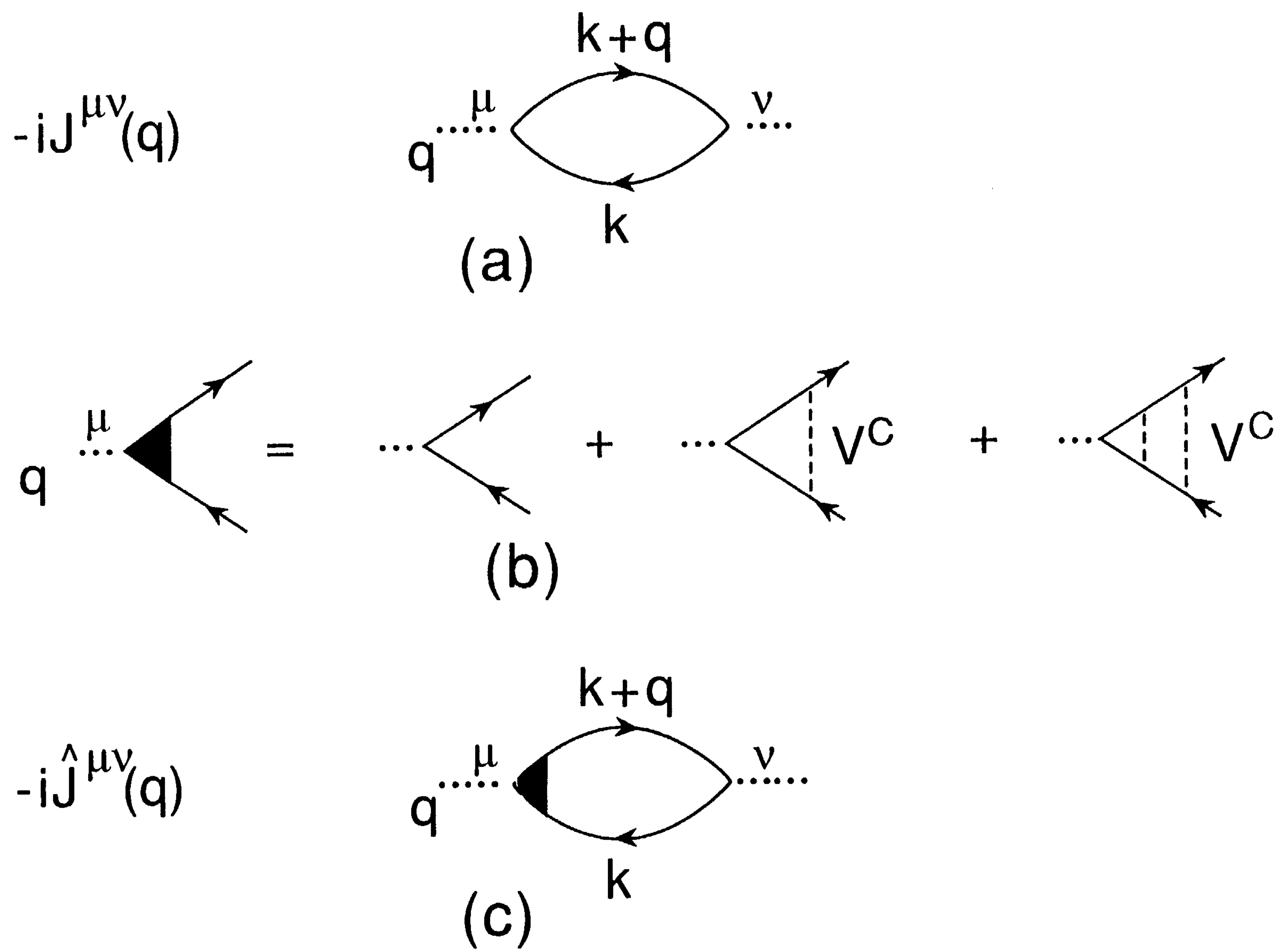


FIG. 1

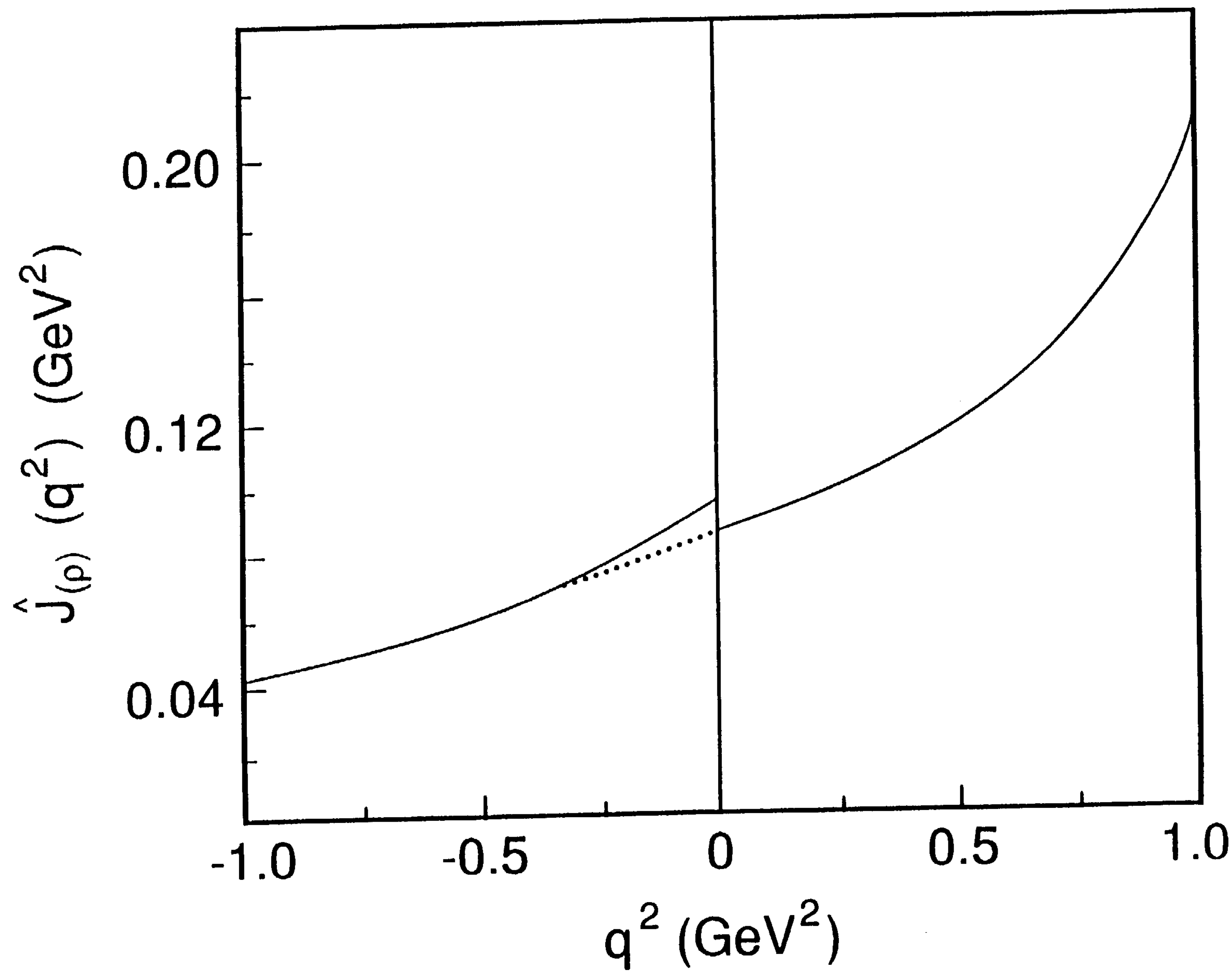


FIG. 2

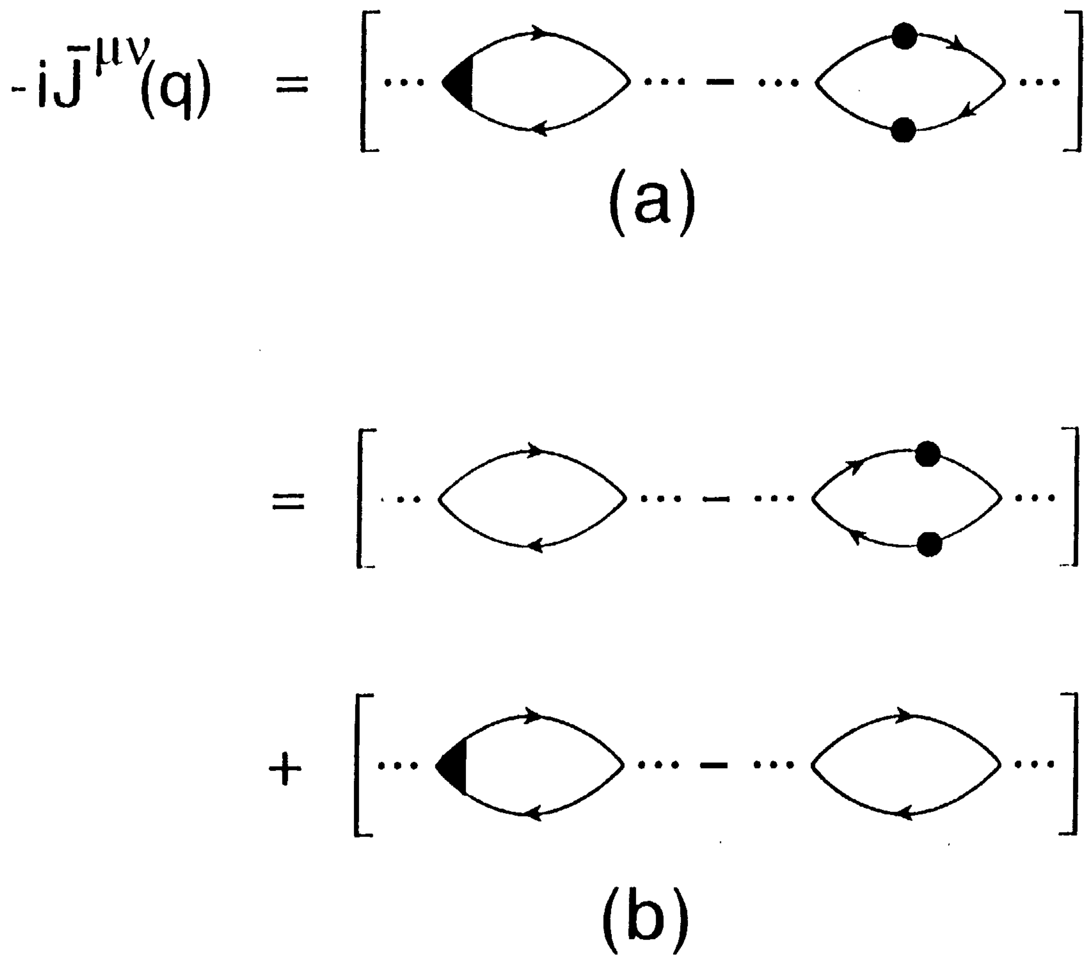
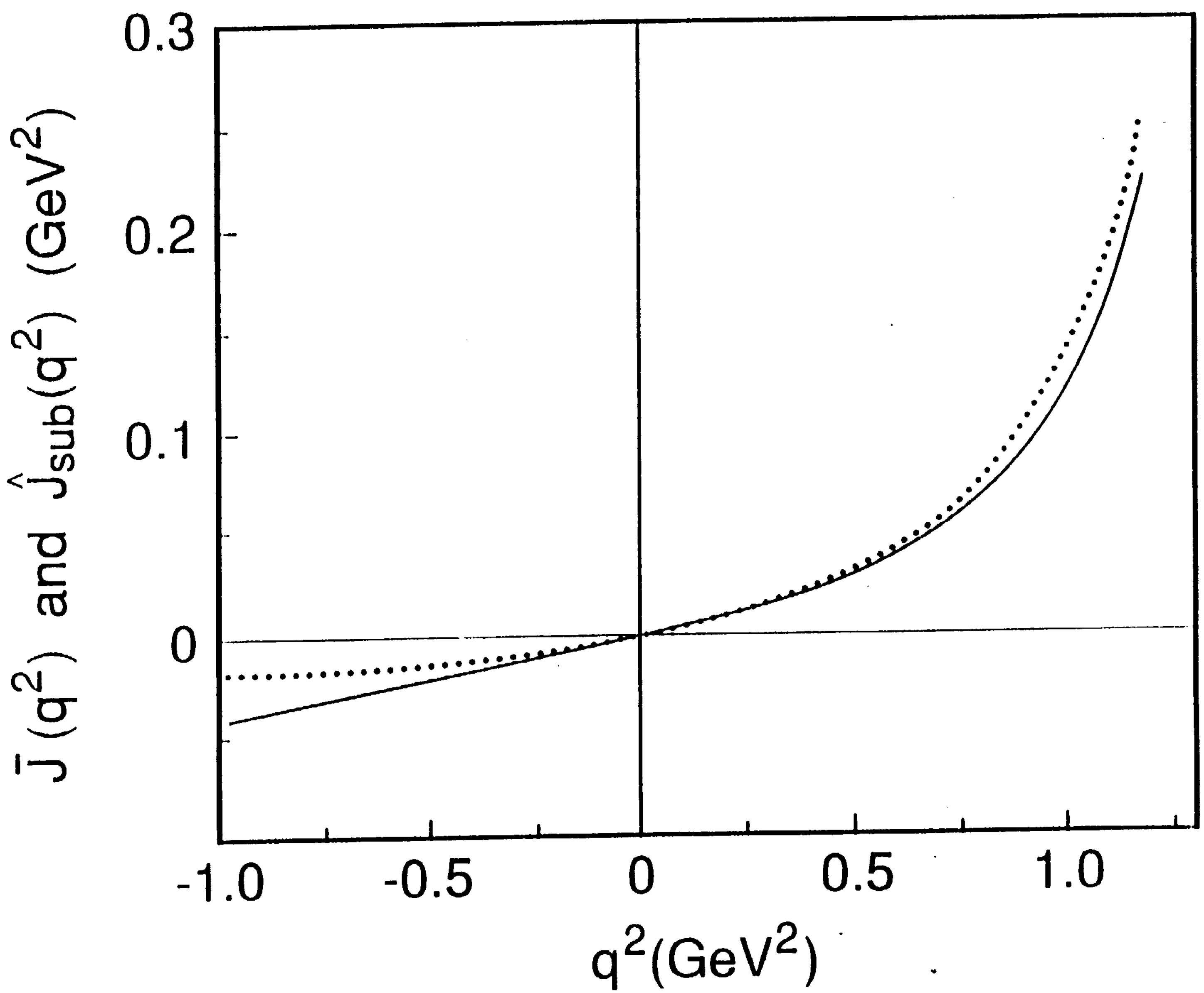


FIG. 3



0.3
0.2
0.1
0
-1.0
-0.5
0
0.5
1.0
$q^2(\mathrm{GeV}^2)$
$\bar{J}(q^2)$ and $\hat{J}_{sub}(q^2)$ (GeV^2)

FIG. 4

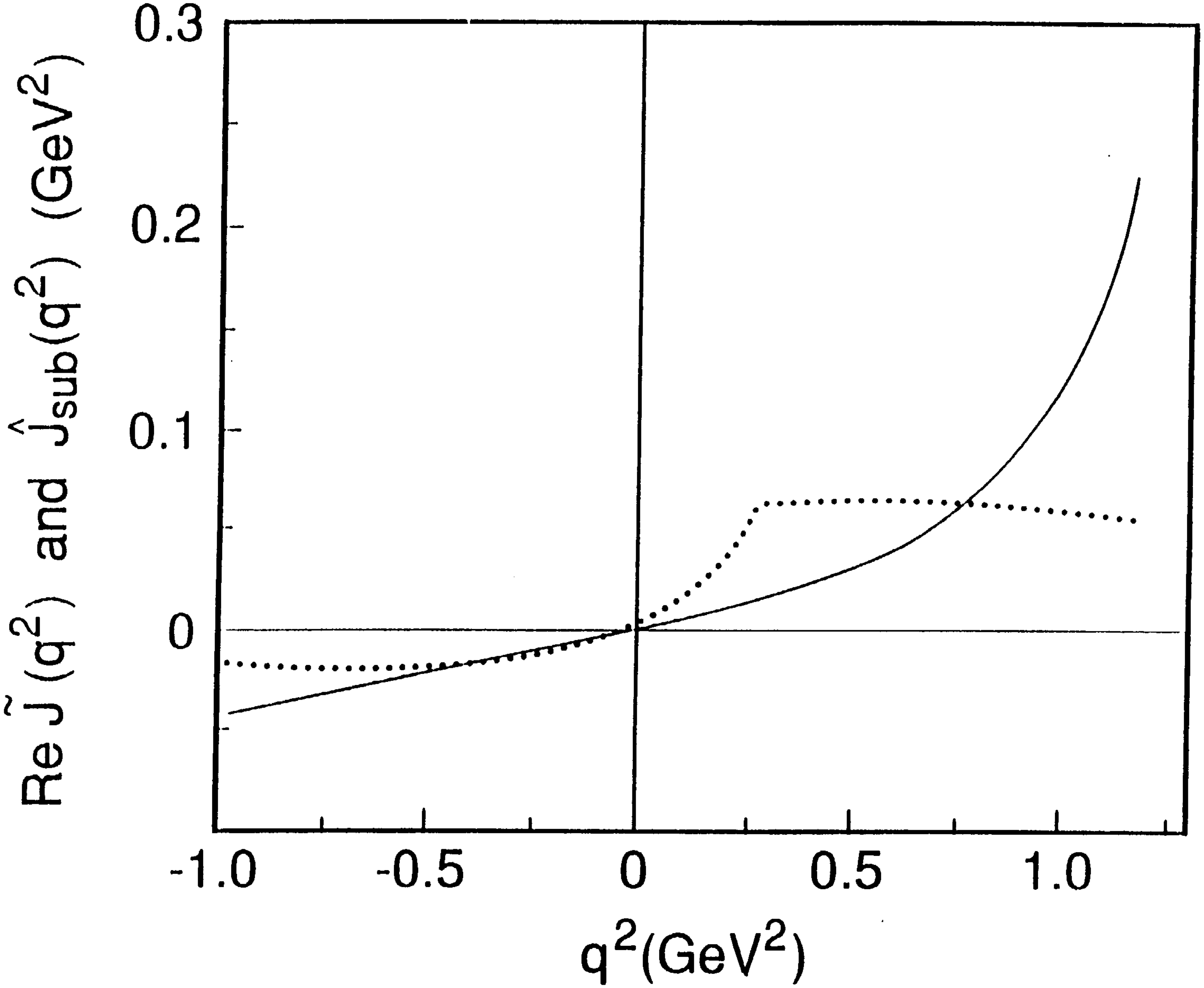


FIG. 5