

M -Theory on a Calabi-Yau Manifold

Sergio Ferrara^{a,b,†}, Ramzi R. Khuri^{a,c} and Ruben Minasian^a

^a *CERN, CH-1211, Geneva 23, Switzerland*

^b *Physics Department, UCLA, Los Angeles, CA, USA*

^c *Physics Department, McGill University, Montreal, PQ, H3A 2T8 Canada*

We compactify M -theory on a Calabi-Yau manifold to five dimensions by wrapping the membrane and fivebrane solitons of the eleven-dimensional supergravity limit around Calabi-Yau two-cycles and four-cycles respectively. We identify the perturbative and non-perturbative BPS states thus obtained with those of heterotic string theory compactified on $K3 \times S^1$. Quantum aspects of the five-dimensional theory are discussed.

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1. Introduction

The existence of an underlying eleven-dimensional theory (the so-called M -theory [1–6]) whose low-energy limit is eleven-dimensional supergravity is crucial to the establishment of the various string/string dualities recently studied [7–9,1,10,6]. In this framework, the five seemingly distinct string theories arise as weak coupling limits of the various compactifications of the eleven-dimensional M theory, in which the membrane and fivebrane that naturally arise are either wrapped around or reduced on the compactified directions.

Although string/string dualities have best been seen in $D = 6$, the generalization of electric/magnetic duality of super Yang-Mills field theories requires an $N = 2$ duality in $D = 4$, which entails a duality (second quantized mirror symmetry [11]) between the heterotic string on $K3 \times T^2$ and the type IIA string on a Calabi-Yau threefold [12]. Several checks of the latter have recently been carried out [13,14]. It was further observed that this duality can be lifted to five dimensions to relate the heterotic string on $K3 \times S^1$ and M -theory on a Calabi-Yau [15]. This can be seen as the decompactification limit of the $D = 4$ theory when the CY volume becomes large. When the CY manifold is a $K3$ fibration [16,17], classical calculations in M -theory can be matched with one-loop calculations on the heterotic side. Further evidence in support of this duality can be seen through the matching of string and point-like fundamental and solitonic states and through one-loop tests along the lines of [8,18,2]. The fundamental heterotic string state arises from the M -theory fivebrane wrapped around a four-cycle in the CY space, while the point-like solitonic state resulting from the wrapping of the heterotic fivebrane around $K3 \times S^1$ arises from the M -theory membrane wrapped around a two-cycle in the CY space. A further reduction to four-dimensions yields the usual electric/magnetic duality. Connections are also made with ten-dimensional type IIA membrane/fourbrane duality and to six-dimensional heterotic/heterotic duality.

An outline of this paper is as follows: In section 2 we identify the perturbative BPS states in $D = 5$ obtained from M -theory compactified on a Calabi-Yau manifold, the string-like and point-like states arising as pairs of electric/magnetic duals. In section 3 we show that for M -theory compactified on a specific Calabi-Yau manifold with $h_{(1,1)} = 3$ and $h_{(2,1)} = 243$ this electric/magnetic duality follows from six-dimensional heterotic/heterotic duality. Furthermore, these string and point-like states can also arise from the heterotic string or the heterotic fivebrane compactified on $K3 \times S^1$. A new vector-gravity interaction is derived in section 4, providing a one-loop test of the five-dimensional duality. Finally,

in section 5, we discuss quantum aspects of the duality in five dimensions. In particular, we show that the gauge and gravitational anomalies of the bulk lagrangian in presence of string-like excitations are cancelled by the anomalous variation of a boundary term of a chiral worldsheet string action.

2. *M*-Theory on Calabi-Yau

It has often been the case that the first manifestation of a duality is the exchange of perturbative and non-perturbative states, represented by fundamental and solitonic classical solutions (see [19] and references therein). In establishing the dualities between *M*-theory and the various string theories, it is necessary to investigate the states obtained after compactification from the solitonic membrane and fivebrane solutions of the eleven-dimensional supergravity low-energy limit [9]. In compactifying *M*-theory on a Calabi-Yau manifold to an $N = 2$ supersymmetric theory in five dimensions [20,21], the membrane and fivebrane wrapped around two- and four-cycles of the Calabi-Yau space give rise to BPS states in $D = 5$.¹

In [15], the conjecture was made that the effective theory of heterotic string theory compactified on $K3 \times S^1$ is dual to eleven-dimensional supergravity compactified on a Calabi-Yau threefold. This theory is also equivalent to type *IIA* string theory compactified on the same Calabi-Yau threefold, in an appropriate large volume limit. Quantum effects in five dimensions were also studied [15].

Following [15], point-like (electric) states are obtained in $D = 5$ by wrapping the membrane from *M*-theory around two-cycles in the Calabi-Yau space. Denote two-cycles and four-cycles respectively by $C^{2\Lambda}$ and $C_{4\Lambda}$, where $\Lambda = 1, \dots, h_{(1,1)}$. The charges of these states are obtained from the charge of the membrane by

$$e_\Lambda = \int_{C_{4\Lambda} \times S^3} G_7, \tag{2.1}$$

where $G_7 = \frac{\delta \mathcal{L}}{\delta F_4}$, where $F_4 = dA_3$ is the field strength of the three-form antisymmetric tensor field.

¹ Note that “wrapping” a p -brane around a manifold entails simultaneously compactifying spacetime and its worldvolume on that manifold, while “reducing” a p -brane on a manifold entails no worldvolume compactification. So a string wrapped around S^1 , for example, yields a point-like object in the lower dimension, while a string reduced on S^1 remains a string in the lower dimension.

String-like (magnetic) states in $D = 5$ arise by wrapping the fivebrane around four-cycles in the Calabi-Yau space. The charges of these states are then obtained from the charge of the fivebrane by

$$m^\Lambda = \int_{C^{2\Lambda} \times S^2} F_4. \quad (2.2)$$

These states contribute to the point-like and string-like central charges in $D = 5$ via [15]

$$\begin{aligned} Z_e &= \sum_{\Lambda} t^\Lambda e_\Lambda, \\ Z_m &= \sum_{\Lambda} t_\Lambda m^\Lambda, \end{aligned} \quad (2.3)$$

where t^Λ are the $D = 5$ special coordinates and $t_\Lambda = C_{\Lambda\Sigma\Delta} t^\Sigma t^\Delta$ are the “dual” coordinates, $C_{\Lambda\Sigma\Delta}$ being the CY topological intersection matrix.

Since the membrane and fivebrane are electric/magnetic duals in eleven dimensions, the above point-like and string-like states are dual to each other in the electric/magnetic sense and correspond to point-like and string-like soliton solutions [9].

A further test of this duality can be performed in a straightforward manner as follows: a given point-like solution, when viewed as a solution of the point-like supergravity theory in $D = 5$, should appear to be singular and require the addition of a sigma-model source action to compensate the singularity. From the dual (string) viewpoint, the point-like solution should appear nonsingular. Similarly, a string solution should appear singular from the point of view of the string theory in $D = 5$ but nonsingular from the dual, point-like viewpoint.

Singularity of a solution in a given theory is tested by probing the solution with a test-probe which is a fundamental object of the theory [22]. If the probe reaches the origin in finite proper time, the solution is deemed singular with respect to the theory. If the probe takes an infinite proper time to reach the source, then the solution is considered nonsingular, as no singularity can be observed in finite proper time. For example, the point-like solution obtained by wrapping the membrane around a two-cycle should appear singular when viewed by a test point-object of the point-like theory in $D = 5$, but nonsingular when viewed by a test string of the dual string theory in $D = 5$.

In fact, the singularity criteria for the electric/magnetic dual objects at hand can be seen to be satisfied immediately in $D = 5$, since all objects in question are point-like or strings, and it was shown in [19] that provided at least one of the two objects in question is either a string or a point, then it is self-singular and mutually nonsingular with its dual.

3. Five-Dimensional Duality

In a recent paper [6], heterotic string/string duality was examined from the point of view of M -theory, where it was argued that the $E_8 \times E_8$ heterotic string compactified on $K3$ with equal instanton numbers in the two E_8 's is self-dual, a result which can be seen by looking in two different ways at eleven-dimensional M -theory compactified on $K3 \times S^1/Z_2$. One weakly coupled heterotic string is obtained by wrapping the $D = 11$ membrane around S^1/Z_2 , while the dual heterotic string, also weakly coupled, is obtained by reducing the $D = 11$ fivebrane on S^1/Z_2 and then wrapping around $K3$. Each of these two strings is strongly coupled from the point of view of the dual one.

If we further compactify by reducing the first six-dimensional heterotic string on S^1 and wrapping the dual six-dimensional heterotic string on S^1 , we obtain on the one hand a string in five dimensions and on the other a dual, point-like object in five dimensions. We claim that, starting with a $K3$ vacuum in which the gauge symmetry is completely Higgsed, this $D = 5$ string can be identified with the M -theory fivebrane wrapped around a Calabi-Yau four-cycle, while the $D = 5$ point-like object can be identified with the M -theory membrane wrapped around a Calabi-Yau two-cycle for the specific Calabi-Yau manifold $X_{24}(1, 1, 2, 8, 12)$ with $h_{(1,1)} = 3$ and $h_{(2,1)} = 243$ [23,12]. In five dimensions, this model contains $n_V = h_{(1,1)} - 1 = 2$ vector multiplets (not counting the graviphoton) and $n_H = h_{(2,1)} + 1 = 244$ hypermultiplets.²

Evidence for this identification from one-loop anomaly tests will be shown below in section 4. For now, we simply note that, following [15], it is straightforward to match the perturbative and non-perturbative BPS states arising from the ten-dimensional compactification with the states displayed in the previous section and arising from the eleven-dimensional compactification.

This can be seen as follows: from the ten-dimensional point of view, the heterotic string compactified on $K3 \times S^1$ has the perturbative fundamental string state with charge

$$m_0 = \int_{K3 \times S^1 \times S^2} H_7, \quad (3.1)$$

where $H_7 = e^{-\phi} * H_3$, H_3 is the field strength of the two-form antisymmetric tensor field and ϕ is the ten-dimensional dilaton. This state has mass per unit length $M_0 = m_0 g_5^2$.

² In this paper, we don't consider the hypermultiplet sector of M -theory where the low-energy effective action in $D = 5$ does receive membrane and fivebrane instanton corrections [24].

Here the string is reduced on, not wrapped around, the S^1 . The corresponding classical solution is given by the fundamental string of [25]. This mass formula, which can be seen from central charge/supergravity considerations [15], can also be obtained by computing the ADM mass of the fundamental string solution. This state is associated with the $b_{\mu\nu}$ field and is dual to a vector in $D = 5$.

The string theory also possesses a perturbative electrically charged point-like H -monopole (dual to the magnetically charged H -monopole state of [26]) state with charge

$$e_1 = \int_{K3 \times S^3} H_7, \tag{3.2}$$

and with mass $M_1 = e_1 R g_5$, where R is the radius of the S^1 and g_5 is the five-dimensional string coupling constant. In this case, the string is wrapped around the S^1 . Again one obtains the same mass from either the central charge or the ADM mass of the solitonic solution. This state is associated with the $b_{\mu 6}$ field. The T -dual electrically charged point-like Kaluza-Klein state with charge e_2 and associated with the $g_{\mu 6}$ field has mass $M_2 = e_2 g_5 / R$. In this case, the corresponding electrically charged solution is given by the extremal Kaluza-Klein black hole solution of heterotic string theory [27].

The fundamental string state can be identified with one of the three states shown in the previous section arising from the M -theory fivebrane, while the H -monopole and Kaluza-Klein states can be identified with two of the three states shown in the previous section arising from the M -theory membrane.

The dual case is similar: the heterotic fivebrane wrapped around $K3 \times S^1$ has the nonperturbative (from the string point of view) point-like state with charge

$$e_0 = \int_{S^3} H_3, \tag{3.3}$$

and mass $M'_0 = e_0 / g_5^2$ [1,15]. Here the classical solution is simply the heterotic fivebrane of [28] wrapped around $K3 \times S^1$, and which is dual to the fundamental heterotic string.

One also gets from the heterotic fivebrane a nonperturbative magnetically charged string-like H -monopole state with charge

$$m_1 = \int_{S^1 \times S^2} H_3, \tag{3.4}$$

and mass per unit length $M'_1 = m_1 R / g_5$, where here the fivebrane is wrapped around the $K3$ but reduced on the S^1 . The solution in this case is the usual magnetically charged

H -monopole, which in $D = 5$ is a string [26]. The T -dual magnetically charged string-like Kaluza-Klein state with charge m_2 has mass per unit length $M'_2 = m_2/g_5 R$.

The point-like state can be identified with one of the three states shown in the previous section arising from the M -theory membrane, while the string-like H -monopole and Kaluza-Klein states can be identified with two of the three states shown in the previous section arising from the M -theory fivebrane³.

Note that each of the three pairs of electric/magnetic dual states obey Dirac quantization conditions. Note also that neither the membrane nor the fivebrane from M -theory is in itself sufficient to reproduce the perturbative spectrum of either the five-dimensional string or the dual five-dimensional point-like object. This becomes clear when one realizes that from the M -theory side, the membrane wrapped around a two-cycle yields only point-like states, while the fivebrane wrapped around a four-cycle yields only string-like states. On the other hand, from the heterotic compactification, both the string and point-like theories in $D = 5$ contain both string and point-like objects in their perturbative spectra. In particular, it follows that the $D = 5$ spectrum of Calabi-Yau string solitons yields the fundamental string states on the heterotic side as well as the non-perturbative heterotic string states obtained by wrapping the heterotic fivebrane on $K3$.

In reducing further to four-dimensions, one obtains the standard (point-like) electric/magnetic duality. This entails wrapping the string around another S^1 and reducing the point-like theory on S^1 .

This four-dimensional duality can also be seen to arise directly from type IIA membrane/fourbrane duality. We first reduce the membrane of M -theory on S^1 to get the type IIA membrane theory and then compactify to four-dimensions on a Calabi-Yau manifold by wrapping the membrane around a two-cycle. To get the dual point, we wrap the fivebrane of M -theory around S^1 to get the type IIA fourbrane theory and then compactify on a Calabi-Yau manifold by wrapping the fourbrane around a four-cycle.

The connections between the fundamental states of the various theories are shown in Fig.1.

³ In particular, since the point-like state coming from the heterotic fivebrane does not come from the M -theory fivebrane, it follows that the two fivebranes are not identical.

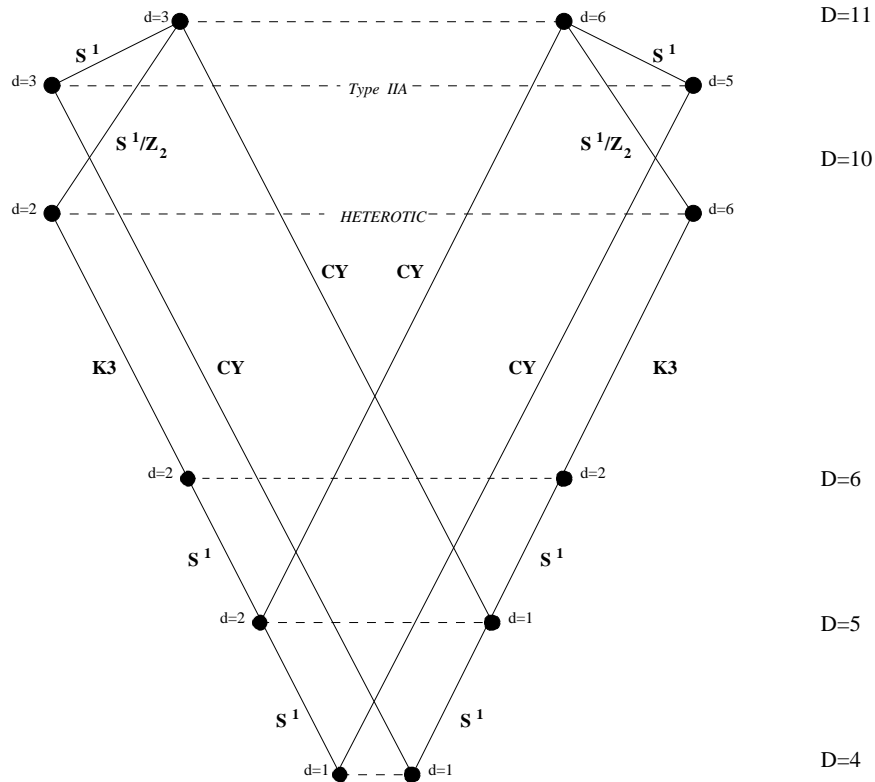


Fig.1 The ($N = 2$) “duality diamond”. The string and supergravity theories connected by lines in the diagram possess fundamental states identified under compactification. For example, the $D = 5$ fundamental string is obtained either from the heterotic string theory reduced on $K3 \times S^1$ or from the M -theory fivebrane wrapped around a Calabi-Yau four-cycle. Note, however, that the five-dimensional heterotic string theory possesses point-like perturbative states which come from the M -theory membrane.

4. One-Loop Results

The action of the eleven-dimensional supergravity limit of M -theory is given by

$$I_{11} = \frac{1}{2} \int_{M^{11}} d^{11}x \sqrt{-g} \left[R - \frac{1}{2} F_4 \wedge *F_4 - \frac{1}{6} A_3 \wedge F_4 \wedge F_4 \right]. \quad (4.1)$$

This action should be augmented by a term predicted by membrane/fivebrane duality [2]

$$I_{11}^{Lorentz} = \int_{M^{11}} A_3 \wedge \frac{1}{(2\pi)^4} \left[-\frac{1}{768} (\text{tr} R^2)^2 + \frac{1}{192} \text{tr} R^4 \right]. \quad (4.2)$$

The gravitational constant and the membrane and fivebrane tensions are set to one.⁴ The reduction of (4.1) to five dimensions is well known (see, *e.g.*, [20,15]), and in addition to

⁴ For a complete discussion of the tension quantization in 11 dimensions see [2].

$h_{(1,1)}$ vectors and $h_{(2,1)} + 1$ hypermultiplets has a topological coupling term [29]

$$I_5^{top} = -\frac{1}{12} C_{\Lambda\Sigma\Delta} \int_{M^5} A_1^\Lambda \wedge F_2^\Sigma \wedge F_2^\Delta. \quad (4.3)$$

The $U(1)$ fields are normalised so that they couple to integer charges. On the other hand, the reduction of (4.2) yields an interaction of the form

$$I_5^{Lorentz} \sim \int_{M^5} \alpha_\Lambda A_1^\Lambda \wedge \text{tr} R^2, \quad (4.4)$$

where

$$\alpha_\Lambda = \frac{1}{16(2\pi)^2} \int_{M^6} \omega_\Lambda \wedge \text{tr} R_0^2, \quad (4.5)$$

where $\Lambda = 1, \dots, h_{(1,1)}$ and $\omega_\Lambda \in h_{(1,1)}$. The α_Λ define the topological couplings, which for $X_{24}(1, 1, 2, 8, 12)$ are (24, 48, 92) (see, *e.g.*, [30]).

Thus we obtain a set of $h_{(1,1)}$ vector equations of motion

$$d(G_{\Lambda\Sigma} H_3^\Sigma) = -\frac{1}{4} \left[C_{\Lambda\Sigma\Delta} dA_1^\Sigma dA_1^\Delta + \frac{1}{24} \alpha_\Lambda \text{tr} R^2 \right], \quad (4.6)$$

where $\Lambda, \Sigma, \Delta = 1, \dots, h_{(1,1)}$ and $H_3^\Sigma = *F_2^\Sigma$. We follow the conventions of [20] in defining the metric $G_{\Lambda\Sigma}$ and intersection constants $C_{\Lambda\Sigma\Delta}$. As explained in [15], when the Calabi-Yau manifold is a $K3$ fibration, one of these vectors can be dualized to give a two-form field that can be identified with the $b_{\mu\nu}$ field of the heterotic string on $K3 \times S^1$. In the previous section, this claim was supported at the level of BPS states. (4.6) can be obtained from the fivebrane (tree-level) Bianchi identity, involving gravitational Chern-Simons corrections arising from a sigma-model anomaly on the fivebrane worldvolume, $dG_7 = -\frac{1}{2} F_4^2 + (2\pi)^4 \tilde{X}_8$ by decomposing the fields in the basis of cohomology on the Calabi-Yau manifold. From the heterotic point of view, we see that the fivebrane Bianchi identity yields the string Bianchi identity, involving the $b_{\mu\nu}$ field (tree-level), and $h_{(1,1)} - 1$ vector equations of motion (one-loop effect).

As a further test, let us compare the holomorphic functions F_1 for the heterotic string and for M -theory for the specific three-moduli CY manifold. In the M -theory case, the absence in the $D = 5$ spectrum of a scalar field corresponding to the two-form antisymmetric tensor with both internal indices implies that there are no non-perturbative corrections to the low-energy action describing the vector multiplet and in particular its gravitational coupling:

$$F_1^M = 24A_1^1 + 48A_1^2 + 92A_1^3. \quad (4.7)$$

This can be viewed as the decompactification limit of the four-dimensional type II topological function [31]

$$F_1^{II} = \frac{12\pi i}{12} c_2 \cdot \vec{J} + \text{non-perturbative corrections}, \quad (4.8)$$

where $c_2 \cdot \vec{J} = \alpha_\Lambda t^\Lambda = 24t_1 + 48t_2 + 92t_3$ and where the non-perturbative corrections are absent in $D = 5$.

On the heterotic side, the expression for our three moduli case is given by [32,14]

$$F_1^{het} = 24S_{inv} + \frac{2}{4\pi^2} \log(j(T) - j(U)) - \frac{b_{grav}}{8\pi^2} \log \eta^{-2}(T) \eta^{-2}(U), \quad (4.9)$$

where $b_{grav} = 2(n_H - (n_V + 1)) + 46 = 528$ for $n_H = 244$ and $n_V + 1 = 3$. $j(T)$ is the modular j -function, η is the Dedekind function and $S_{inv} = (1/4\pi)S$. In the large T limit this reduces to

$$F^{het} = \frac{1}{4\pi} (24S + 48T + 44U). \quad (4.10)$$

Employing the connections between heterotic and M moduli in the large moduli limit $t_1 = -iS; t_2 = -i(T - U); t_3 = -iU$ [33] ($T > U$ is assumed), one finds agreement in the large T limit between the tree-level M -theory result (4.7) and the one-loop heterotic expression (4.10) for the three moduli case. Agreement between the heterotic and type IIA holomorphic functions for the particular Calabi-Yau threefold $X_{12}(1, 1, 2, 2, 6)$ was found in [13]. This model does not, however, arise from six-dimensional heterotic/heterotic duality.

5. Anomaly Cancellation from Strings

It was pointed out in [34,5] that, in the presence of a fivebrane, a term representing the coupling of an anti-self dual three-form field strength T_3 on the fivebrane worldvolume is necessary to cancel the anomaly from the interaction $\int_{M^{11}} A_3 \wedge F_4 \wedge F_4$. This can be seen as follows. In the presence of a fivebrane with charge m ,

$$dF_4 = m\delta_V, \quad (5.1)$$

where δ_V is supported on the fivebrane worldvolume V (*i.e.* it integrates to 1 on the space transverse to the fivebrane). So, under $\delta A_3 = d\Lambda_2$,

$$\begin{aligned} \frac{1}{12} \delta \left(\int_{M^{11}} A_3 \wedge F_4 \wedge F_4 \right) &= \frac{1}{4} \int_{M^{11}} d\Lambda_2 \wedge F_4 \wedge F_4 \\ &= -\frac{m}{2} \int_V \Lambda_2 \wedge F_4. \end{aligned} \quad (5.2)$$

This anomaly needs to be cancelled by a term

$$\frac{m}{2} \int_V T_3 \wedge A_3, \quad (5.3)$$

where T_3 is the anti-self dual field three-form strength on the fivebrane worldvolume and $dT_3 = F_4$.

An analogous situation arises for the five-dimensional theory in the presence of string sources with charge

$$dF_2^\Sigma = m^\Sigma \delta_W, \quad (5.4)$$

where δ_W is supported on the string worldsheet. The topological term $I_5^{top} = (-1/12)C_{\Lambda\Sigma\Delta} \int_{M^5} A_1^\Lambda \wedge F_2^\Sigma \wedge F_2^\Delta$ is anomalous under $\delta A_1^\Lambda = d\lambda^\Lambda$:

$$\begin{aligned} \delta I_5^{top} &= -\frac{1}{4} C_{\Lambda\Sigma\Delta} \int_{M^5} d\lambda^\Lambda \wedge F_2^\Sigma \wedge F_2^\Delta \\ &= \frac{m^\Lambda}{2} C_{\Lambda\Sigma\Delta} \int_W \lambda^\Sigma \wedge F_2^\Delta. \end{aligned} \quad (5.5)$$

Another way of seeing this is to note that due to (5.4), (4.6) is inconsistent: taking an external derivative makes the left hand side vanish, while the right hand side is nonzero.

The remedy is to add to the action a term

$$\frac{1}{2} m^\Lambda C_{\Lambda\Sigma\Delta} \int_W T_1^\Sigma \wedge A_1^\Delta, \quad (5.6)$$

where T_1^Σ is a self-dual one-form field strength on the string worldsheet and $dT_1^\Delta = F_2^\Delta$. This term cancels the $U(1)^{h(1,1)}$ gauge anomalies of the bulk action in the presence of strings and arises as a part of a string worldsheet action analogous to D -brane action⁵ presented in [34]

$$I_2 = \frac{1}{4} d_{\Sigma\Delta} \int_W (T^\Sigma - \hat{*}A^\Sigma) \wedge (\hat{*}T^\Delta - A^\Delta), \quad (5.7)$$

where $d_{\Sigma\Delta} = C_{\Lambda\Sigma\Delta} m^\Lambda$; here A^Σ denote the pullbacks to the worldsheet of spacetime vectors and $\hat{*}$ is the dualization on the worldsheet.

Similarly, as in $D = 11$ [2,5], the interaction term of the form (4.4) $\int \alpha_\Lambda F_2^\Lambda \wedge \Omega_3$ which is covariant in the absence of strings, now develops an anomaly due to (5.4):

$$\delta I_5^{Lorentz} = \alpha_\Lambda m^\Lambda \int_W \epsilon R, \quad (5.8)$$

⁵ In eleven dimensions, fivebranes can be interpreted as D -branes of open membranes [34]. After compactification, this picture reduces to point-like intersections of strings in five dimensions.

where ϵ is the infinitesimal parameter of the diffeomorphism (as a reminder, $\text{tr}R^2 = d\Omega_3$ and $\delta\Omega_3 = d(\epsilon R)$). Again, the worldsheet anomaly and the one in the interaction in the bulk are expected to cancel.⁶ It is clear from this consideration that the resulting five-dimensional string is necessarily chiral on the worldsheet.⁷ This fact is also supported by the possible identification with the heterotic string compactified on $K3 \times S^1$ (for the suitable Calabi-Yau's); from section 4 it can be seen that obtaining (5.8) from the heterotic side requires both a tree-level and a one-loop calculation. A detailed calculation of the anomalous worldsheet action of the string excitations of M -theory will be given elsewhere.

We see that the five-dimensional theory mimics its eleven-dimensional “ancestor” in many ways, at the same time having the advantage of being coupled to only string and point-like objects. Thus more detailed study of these five-dimensional theories may help in understanding M -theory while allowing calculations to be carried out in the more familiar setting of string theory. Finally, while further reduction on a circle is fairly straightforward and yields $N = 2$ supersymmetric theories in $D = 4$, as displayed in Fig.1, one may hope to obtain dual $N = 1$ chiral theories following [6] by considering two different limits of M -theory compactified on $CY \times S^1/Z_2$.

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⁶ In presence of a string source (5.4) coming from the compactification of a fivebrane source on a CY manifold, supersymmetry is also anomalous [35]. We assume that the supersymmetry anomaly may be cancelled by a corresponding anomalous variation of the string worldsheet action. This non-invariance may still require additional terms both in the bulk lagrangian and in the worldsheet action.

⁷ Another way of seeing this is to note, following Witten [5], that in $D = 5$, introducing a string worldsheet given by $x_3 = x_4 = x_5 = 0$ breaks half of the spacetime supersymmetries and the surviving supersymmetries obey $\Gamma_3\Gamma_4\Gamma_5 = 1$ or, equivalently, in view of $\Gamma_1\Gamma_2\Gamma_3\Gamma_4\Gamma_5 = 1$, $\Gamma_1\Gamma_2 = 1$.

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