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Study of an Instability of the PEP-II Positron Beam (Ohmi Effect and Multipactoring)*

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The transverse instability in the Photon Factory has been observed and described by Ohmi[1] as an instability of a positron beam in a storage ring induced by photoelectrons. (I would like to mention that W. Stoeffl also pointed out that photoelectrons may produce an effective wake-field somewhere in the middle of 1994). The photoelectrons are produced by synchrotron radiation in the beam pipe wall and accelerated by the transverse electric field of the beam. A transverse kick given by the photoelectrons to the trailing bunches depends on the off-set of the previous bunches. The situation is, in general, analogous to the beam break-up[2] induced by the regular transverse wakefields, and to the recently described ion-induced transverse instability of an electron beam[3].

The processes defining the density distribution of the photoelectrons are quite complicated. Detailed description of the instability requires computer simulations, as has been done in Ohmi's original paper. It is useful to have a simplified model of the instability to get a quick estimate of the growth rate of the instability and the relative importance of the parameters. The model described below uses parameters of the LER of the PEP-II B-factory[4].

The paper is organized in the following way. First, Ohmi effect induced by direct flow of primary photoelectrons is studied for the PEP-II parameters. The production rate and kinematics take into account the antechamber of the LER. We discuss the effect of the secondary emission of electrons in the AL chamber, where the yield is larger than one. Resonance multipactoring is considered, and then the average density of the secondary electrons is estimated taking into account the space-charge effect and the interaction with the beam. We show that in the extreme case there is a self-consistent regime similar to the regime of the space-charge dominated cathode. Finally, the rate of ion production by accumulated electrons and the possibility of the ion induced pressure instability is discussed.

Kinematics of the radiation

Consider a short dipole $l_d < \sqrt{2b\rho}$ with length l_d, and bending radius ρ. A photon emitted at the azimuth θ upstream from the end of the dipole, $0 < \theta < l_d/\rho$, propagates downstream into the beam pipe of the straight section, covering the distance $l_\gamma = \rho \tan\theta + s/\cos\theta$ before it hits the beam pipe wall. The photon hits the wall of the beam pipe with radius b at distance

$$s \simeq b/\theta - \rho\theta/2, \tag{1}$$

counting downstream from the end of the dipole. The distance l_γ is larger than the distance $l_e = \rho\theta + s$ passed by the parent electron to the same moment. The delay is $\Delta l = l_\gamma - l_e \simeq \rho\theta^3/12 + b\theta/2$. The maximum delay is for the photons emitted at the entrance to the dipole, $\theta = l_d/\rho$, $(\Delta l)_{\max} \simeq b\sqrt{b/2\rho}$. For the LER parameters, $l_d = 0.45$ m, $\rho = 13.75$m, $b = 4.5$cm, $\gamma = 6. \times 10^3$, the delay is small compared to the bunch length σ_l: the maximum delay is 1.3 mm, while $\sigma_l = 1$ cm. Neglecting the delay, we assume that the primary photon and the parent electron remain on the same azimuth during the life time of the photon.

The photons are radiated uniformly along the orbit within a dipole. The energy radiated per unit time and the solid angle $d\Omega$ in the frequency range $d\omega$ is given by the well known formula

$$\frac{dW}{d\omega} = \frac{e^2}{\rho}\frac{\omega^2}{6\pi^3\omega_0^2}[\epsilon^2 K_{2/3}^2(\frac{\omega}{3\omega_0}\epsilon^{3/2}) + \epsilon\sin^2\alpha K_{1/3}^2(\frac{\omega}{3\omega_0}\epsilon^{3/2})]d\Omega, \tag{2}$$

where ω_0 is the revolution frequency, $\epsilon = (1/\gamma)^2 + \alpha^2$, α is the radiation angle with the trajectory plane, and $d\Omega = \cos\alpha d\alpha d\theta$.

The number of photons per unit azimuth, and the total number of the radiated photons per dipole are respectively:

$$\frac{dN_\gamma^{tot}}{d\theta} = \frac{5\alpha_0\gamma}{2\sqrt{3}}, \quad N_\gamma^{tot} = \frac{5}{2\sqrt{3}}\alpha_0\gamma\theta_d. \tag{3}$$

The angle α defines the vertical displacement of a photon $y = l_\gamma\alpha$. Eq. (1) relates s and θ giving the distribution of photons $dn/dt = dW/h\omega$:

$$\frac{dn}{dt} = \frac{\alpha_0}{6\pi^3}\frac{\omega}{\omega_0}\frac{d\omega dy ds}{\rho(s^2+2b\rho)}[\sqrt{s^2+2b\rho} - s][\epsilon^2 K_{2/3}^2(\frac{\omega}{3\omega_0}\epsilon^{3/2}) + \epsilon\sin^2\alpha K_{1/3}^2(\frac{\omega}{3\omega_0}\epsilon^{3/2})] \tag{4}$$

where $\omega_0 = c/\rho$, $\alpha_0 = e^2/(hc) = 1/137$, and $\epsilon = (1/\gamma)^2 + y^2/(s^2+2b\rho)$.

Let us estimate the number of photons which strike the edges of the antechamber $|y| > h_g/2$, where $h_g = 1.5$ cm is the antechamber full height for PEP-II. In the most of the following, we ignore the photoelectrons produced in the antechamber, assuming that they loose memory on the offset of the parent bunch while drifting to the beam pipe. The photoelectrons that may be accelerated by the field of the beam and affect the beam stability are generated mostly on the edges of the slot of the antechamber. The electrons on the upper and lower decks of the slot may be pulled into the beam pipe by the field of the beam leaking inside of the slot, but their number is relatively small.

The maximum distance photons can travel in the beam pipe is limited by the bend of the beam pipe in the downstream dipoles, $s_{\max} < 9$ m. In this case, $\epsilon = y^2/(s^2+2b\rho)$, and number of photons radiated by a positron per one dipole is

$$\frac{d^2n}{dsdy} = \frac{\alpha_0}{3\pi^2}\frac{\omega d\omega}{\omega_0^2\sqrt{s^2+2b\rho}}(\frac{y^2}{s^2+2b\rho})^2[1-\frac{s}{\sqrt{s^2+2b\rho}}][K_{2/3}^2(\frac{\omega}{3\omega_0}\epsilon^{3/2})+K_{1/3}^2(\frac{\omega}{3\omega_0}\epsilon^{3/2})]. \tag{5}$$

Integration over frequencies gives

$$\frac{d^2n}{dsdy} = \frac{\alpha_0\sqrt{3}}{\pi\rho}[1 - \frac{s}{\sqrt{s^2+2b\rho}}]\frac{\sqrt{s^2+2b\rho}}{y^2}. \tag{6}$$

At large $s^2 >> 2b\rho$,

$$n_\gamma(s,y) = \frac{d^2n}{dsdy} = \eta_\gamma\frac{b}{sy^2}, \tag{7}$$

where

$$\eta_\gamma = \frac{\alpha_0\sqrt{3}}{\pi} = 0.4 \times 10^{-2}. \tag{8}$$

This gives the number of photons hitting the edges of the antechamber downstream from a dipole

$$\frac{dN_\gamma}{ds} = \frac{4\alpha_0\sqrt{3}}{\pi}\frac{b}{sh_g}, \qquad N_\gamma = \frac{4\alpha_0\sqrt{3}}{\pi}\frac{b}{h_g}\ln(\frac{s_{\max}}{s_{\min}}). \tag{9}$$

The minimal distance $s_{\min}$ can be defined from the condition $(\omega/3\omega_0)y^3 < (s^2 + 2b\rho)^{3/2}$, otherwise the number of photons from Eq. (5) is exponentially small. The distance s cannot be too small because $y^2 > (h_g/2)^2$, and the photon energy should be large enough to produce a photoelectron: $\hbar\omega > I_0$, where $I_0 \simeq 4.5$ eV is the work function of a beam pipe wall. This limits

$$s > s_{\min} = (h_g/2)[\frac{I_0}{3\hbar\omega_0}]^{1/3}. \quad (10)$$

PEP-II parameters give $s_{\min} = 3.5$ m, larger than $\sqrt{2b\rho} = 1.1$ m.

Eq. (9) gives $N_\gamma = 0.045$ per positron per dipole. This is about 2.2% of the total number of photons given by Eq. (3) $N_\gamma^{tot} = 2.06$ for PEP-II parameters.

Kinematics of the photo-electrons

The average number of photoelectrons generated by N_b particles per bunch per unit length is

$$\frac{dN_e}{ds} = \eta_{e\gamma}\frac{N_\gamma}{(s_{\max} - s_{\min})}N_b \quad (11)$$

where the yield $\eta_{e\gamma}$ is the number of electrons per incident photon. This yield depends on the photon energy and the incident angle, which varies from 1.5 mrad at $s \simeq s_{\min}$ to 0.15 mrad at $s = s_{\max}$. Ohmi uses $\eta_{e\gamma} \simeq 0.1$ photo-electrons per photon. For the PEP-II parameters, $N_b = 8.3 \times 10^{10}$ at the beam current 3 A, $dN_e/ds = 6.8 \times 10^5$ cm^{-1}.

The initial photoelectrons have density $n_e(s,x,y,0) = \eta_{e\gamma}N_B n_\gamma(s,y)\delta(x-b)$ within the range $|y| > h_g/2$, $s_{\min} < s < s_{\max}$.

The photoelectrons have an initial energy on the order of a few eV-s. The transverse electric field of the parent bunch gives a kick to the photoelectrons

$$\Delta p(x) = \frac{2N_B e^2}{c}\frac{x}{x^2+y^2}, \qquad (\frac{v}{c})_{\max} = \frac{2N_B r_0}{b}, \quad (12)$$

where r_e is the electron classical radius. This corresponds to $(v/c)_{\max} = 1.0 \times 10^{-2}$, and energy $mv^2/2 = 26$ eV at 3A beam current.

This energy is large compared to the initial energy of photoelectrons coming out from the wall. Because the field lines are perpendicular to the wall, the photoelectrons get only horizontal kick from the parent bunch. The electrons are distributed vertically with the height on the order of $h_g = 1.5$ cm. An initial vertical velocity corresponding to the energy 1 eV would displace the electron vertically by the distance 2.5 mm $<< h_g$ when the next bunch arrives for the PEP-II bunch spacing $s_b = 120$ cm. The actual displacement is even smaller due to $\cos\theta$ distribution of the initial photo-electrons. We may, therefore, neglect the initial energy of the photo-electrons so that the distribution of the photo-electrons over the horizontal velocity after the parent bunch passed by is

$$\rho(v) = \int d\rho(z)\,\delta[v_x(z) - v]\,. \quad (13)$$

Here, $\rho(z)$ is the longitudinal density of the bunch, and $v_x(z) = v_{\max}\int_{-\infty}^{z} dz'\rho(z')$ is the velocity of a photoelectron due to radiation of positrons at the location z within a bunch. In Eq. (13) we neglect the time delay between a photon and the parent positrons.

Integration in Eq. (13) gives a uniform distribution within the range $0 < v < v_{\max}$: $\rho(v) = (1/v_{\max})\Theta(v_{\max} - v)$. As a result, the distribution of the electrons in the horizontal direction at moment t after the parent bunch is also uniform within a strip with length $\Delta x = v_{\max}t$:

$$n_e(s,x,y,t) = \eta_\gamma\eta_{e\gamma}\frac{N_b}{\Delta x(t)}\frac{b}{s(y-y_c^l)^2}. \quad (14)$$

The density is nonzero in the range $|y| > h_g/2$, $s_{\min} < s < s_{\max}$. Here we modified Eq.(8), introducing the offset y_c^l of the parent bunch centroid. The vertical distribution remains unchanged.

At the moment $t_1 = s_b/c$, when the next bunch comes to the same azimuth s, $\Delta x = v_{\max}s_b/c$. For PEP-II parameters at 3 A current, $\Delta x = 1.28$ cm, $\Delta x << b$. The density in this moment is independent on N_b. The maximum average density at $|y| = h_g/2$ is $n_{\max} = 3.6. \times 10^5$ cm^{-3} at PEP-II parameters. Later the strip is stretched because the kick is larger for the particles at the end of the strip that is closer to the beam. At the arrival of the next bunch, the strip is 2.2 cm long and the head of the strip is at a distance $x = 3.47$ cm from the wall, close to the beam. When the next bunch comes, the head of the strip is 0.25 cm away from the opposite wall and the bunch is 5.3 cm long. The centroid of the strip moves vertically from a position above beam line to the position below it before it hits the wall.

The head of another strip of electrons produced by the next bunch coincides with the tail of the strip of the previous bunch. The beam makes a continuous ribbon of photoelectrons flowing from the wall toward the beam with varying density due to the stretching of each strip.

Effect of the photoelectrons

Consider for the sake of simplicity the 1-D case, $y = y_c = 0$.

The photoelectrons give a kick to the positrons $dp_y/ds = -(\partial/\partial y)e^2U$, where $U(s,x,y)$ is the potential defined by the density Eq. (14), properly modified in time

$$\Delta U(s,x,y) = -4\pi n_e(s,x,y). \tag{15}$$

Taking into account the interaction of a bunch only with the group of photoelectrons closest to the beam, we get the equation of betatron oscillations in the vertical plane $x = 0$

$$\frac{d^2y}{ds^2} + k_\beta^2 y = \frac{r_e}{\gamma}\frac{\partial U(s,0,y)}{\partial y}, \tag{16}$$

where $k_\beta = \nu_y/R_0$ is defined by the vertical betatron tune ν_y and the average radius of the machine R_0.

Considering the potential of a strip of electrons with the length Δx and the centroid at x_c, we can estimate

$$\frac{\partial U(s,0,y)}{\partial y} = 2\int dx'dy' n_e(x',y')\frac{y-y'}{(x'-x_c)^2+(y-y')^2}, \tag{17}$$

where n_e is given by Eq. (14).The integration over y' is in the limits $|y'| > h_g/2$

The equation of motion takes the form

$$\frac{d^2y}{ds^2} + k_\beta^2 y = \frac{\Lambda}{s}J(y,y_c^l), \tag{18}$$

where $\Lambda = \eta_\gamma\eta_{e\gamma}N_b b r_e/\gamma$. The RHS of Eq. (18) is nonzero in the range $s_{\rm min} < s < s_{\rm max}$ after each dipole, and

$$J(y,y_c^l) = \frac{2}{\Delta x}\int\frac{dx'dy'}{(y'-y_c^l)^2}\frac{y-y'}{(x'-x_c)^2+(y-y')^2}. \tag{19}$$

Consider expansion of the RHS of Eq. (18) in y and y_c^l. The term driving the instability is proportional to the offset y_c^l. Other terms give a negligible small correction to the betatron tune and orbit distortion. The driving term is

$$J = \frac{8\pi}{\Delta x h_g^2}y_c^l \tag{20}$$

in the case $\Delta x >> x_c$, and $\Delta x \simeq h_g$, and J is smaller than this for large $x_c >> h_g/2$, $x_c \simeq \Delta x$ by a factor $\Delta x h_g/(\pi x_c^2)$.

The offset of the accelerating bunch affects the density distribution and is suppressed by a factor $\pi h_g/8b << 1$. Equations (18), and (20) give

$$\frac{d^2y}{ds^2} + k_\beta^2 y = -\lambda y_c^l, \tag{21}$$

where the averaged λ is

$$\lambda = 8\pi\eta_\gamma\eta_{e\gamma}\frac{N_b r_e b}{\gamma(s_{\rm max}-s_{\rm min})h_g^2\Delta x}\ln\frac{s_{\rm max}}{s_{\rm min}}. \tag{22}$$

The vertical component of the kicks changes the width of the distribution and the vertical offset of the centroid of the group of photoelectrons simultaneously without changing this result.

Oscillations of the first bunch in a train of bunches with the amplitude A_1, $y_1(s) = A_0 e^{ik_\beta s}$, excite oscillations of the following bunches with the amplitude of the n-th bunch growing with the distance s as

$$y_{n+1} \propto \frac{A_1}{n!} (\frac{\lambda s}{2k_\beta})^n. \tag{23}$$

The amplitude of the n-th bunch

$$A_{n+1} \propto \frac{A_1}{\sqrt{n}} e^{n \ln[e\lambda s/(2nk_\beta)]} \tag{24}$$

starts growing at $s > s_n = 2nk_\beta/(e\lambda)$, $e = 2.718...$ Consider the worst situation of a 3 A beam bunch, when the head of the strip is only 1 cm away from the beam and $\Delta x = 2.2$ cm. Although at a lower current this distance can decrease, the reduction of N_b does not make the situation worse.

For the PEP-II parameters, $\lambda = 0.6 \times 10^{-6}\ m^{-2}$ for $\eta_{e\gamma} = 0.1$, and average current 3 A. The amplitude of the second bunch becomes equal to the amplitude of the first bunch after 100 turns, or 0.73 msec and then propagates to the tail of the beam with the same rate from bunch to bunch.

Effect of the background electrons and multipactoring

The instability described above is due to interaction with a single group of photoelectrons.

The interaction with other groups of photoelectrons or with electrons existing in the beam pipe may give a similar effect. Such "background" electrons is the media interacting with the beam and the electron density defines the beam instability. The background electrons may be generated by different mechanisms: inelastic collisions of the positron beam with the residual gas, photoeffect on the residual gas, diffusion of photoelectrons from the antechamber, generation of electrons by the scattered synchrotron radiation, by secondary electron emission, etc.

The secondary electrons may be accelerated by the transverse field of the beam to large energies and, hitting the wall, produce new electrons. The number of fast

electrons can be enhanced at certain currents when secondary electrons are produced at the moment when another bunch is passing by. Such a beam induced multipactoring may lead to a substantial increase in the electron density, provided that the yield η_{ee} of the secondary electrons is more than one.

The interaction with the background electrons is the main concern in the dipoles where the magnetic field prevents the photoelectron from drifting toward the beam, because the Larmor radius of such electrons is of the order of 20 μm. The electrons move primarily in the vertical plane, because the longitudinal cross-field drift and the drift caused by the space charge are slow. The multipactoring of the background electrons in the dipoles may lead to large electron densities. It is worthwhile noting that instability of the KEK Photon Factory probably is the result of some processes in the dipoles.

The average number of photoelectrons per unit length from Eq. (11) generated by the primary synchrotron radiation of a bunch with $N_b = 8.3 \times 10^{10}$ positrons is $dN_{e\gamma}/ds = 6.8 \times 10^5$ cm^{-1}, and the initial density in a stripe of photoelectrons $n_{\max} = 3.6 \times 10^5$ cm^{-3}. The density of a stripe which centroid moved to the beam, is smaller, about $2. \times 10^5$ cm^{-3}. The question here is whether the density of the background electrons can be much larger. In a steady state regime, one could expect the equilibrium electron density given by the condition of neutrality: $n = N_B/(\pi b^2 s_b) = 1.5 \times 10^7$cm^{-3}. However, the background electrons are not in equilibrium, and the answer may depend on such details as the energy dependence of the yield of the secondary electrons.

Figure 1 shows dependence of the yield η_{ee} on the energy of the incident electron for Al oxide and the distribution of the secondary electrons. The distribution at large initial energy $E_0 >> E^{\eta}_{\rm min}$ has two maxima. The high energy peak is at $E \simeq E_0$ and corresponds to a backscattered incident electron. The width of the first maxima describing real secondary electrons is about 5 eV. Fig. 2 shows the yield for the different materials.

The kick from the transverse electric field of a bunch changes the velocity of an electron located at distance r from the beam by $v(r)$, Eq. (12), and electrons hitting the wall can produce secondary electrons, if $r/b = (N_b r_0/b)\sqrt{2mc^2/E^{\eta}_{\rm min}}$. This condition is easy to satisfy: $r/b = 0.7$ for the PEP-II parameters at 3A current.

As mentioned above, the motion of an electron may be quite complicated, an electron can be kicked several times by bunches before it reaches a wall. At the high N_b corresponding to the 3 A current for PEP-II, an electron experiences on average three kicks before it reaches the wall. At lower currents, the situation is even more complex: the number of kicks increases, see Fig. 3, an electron can come back to the wall where it started, and, in general, motion becomes quite stochastic, see Fig. 4. Neglecting the effect of the space charge, it is easy to show that electrons at large distances from the beam are indeed unstable. The coordinate and impulse of an electron are changed from bunch-to-bunch as

$$x \to \hat{x} = x + \frac{s_b}{mc}\hat{p}_x; \quad p_x \to \hat{p}_x = p_x - \frac{2N_b e^2 x}{c(x^2+\sigma_t^2)} . \tag{25}$$

Here, σ_t is the transverse rms bunch size. The electron is stable if the trace $|SpM| < 2$ where M is the matrix transforming Δx, Δp_x to $\hat{\Delta} x$, $\hat{\Delta} p_x$. Calculations give

$$\frac{1}{2}SpM = 1 + \frac{N_b r_e s_b}{2}\frac{x^2-\sigma_t^2}{(x^2+\sigma_t^2)^2} , \tag{26}$$

which is larger than one for $|x| > |\sigma_t|$. Hence, the motion is random, and electron trajectories tend to cover all phase space at large distances.

It seems that if multipactoring occurs, fast electrons may multiply to large densities and become dangerous for beam stability. The process stops, however, due to the finite energy spread E_0 of the secondary electrons. This results from both the initial distribution in energy of the secondary electrons (see Figs. 1, 2), and from the difference in the kick gained from a bunch with a finite bunch length σ_l.

As a result of the initial spread in velocity Δv, the primary group of electrons is extended by $\Delta x = \Delta v s_b/c$ at the time when the interaction with the next bunch occurs. Interaction with the bunch transforms Δx into additional spread in velocities

and, as result, in the difference in time Δt when electrons hit the wall. The effective yield of secondary electrons is $\eta_{\rm eff} = \eta\Delta t/(\sigma/c)$. Estimate gives

$$\frac{\eta_{eff}}{\eta} = \frac{N_b r_0 \sigma_l}{2bs_b}\sqrt{\frac{2mc^2}{E_0}} . \tag{27}$$

For PEP-II parameters, the ratio is of the order of 10^{-2} at 3 A current, and even smaller for smaller N_b. This result agrees with what can be expected from a stochastic behavior of electron trajectories at low currents. Hence, the resonance multipactoring does not lead to a large electron density. Effect of the secondary electrons is considered below. The space-charge effect changes the dynamics of the background electrons and has to be taken into account in the estimate of the equilibrium density.

Fig. 5 shows the variation of the electron density with time for the 1-D problem. The result was obtained by tracking photoelectrons taking into account the space-charge effect, interaction with the beam, and the secondary electron emission at the walls.

The equilibrium density of the background electrons

Let us estimate the steady-state density of the background electrons. The density depends in the self-consistent way on the average space charge of electrons and on the yield of the secondary electrons from the walls.

The problem is quite complicate and consideration is restricted to the 1-D problem for particles within the range $0 < x < 2b$, $|y| < h_g/2$ with the density $n(x, v, t)$ being independent on the longitudinal and vertical y coordinates.

The dimensionless variables are used

$$z = \frac{x}{2b}, \quad u = \frac{s_b v}{2bc}, \quad \tau = \frac{ct}{s_b} . \tag{28}$$

The equations of motion in this units are

$$\frac{dz}{d\tau} = u, \quad \frac{du}{d\tau} = F_t(z,\tau), \quad 0 < z < 1, \tag{29}$$

where the total force F_t includes periodic kicks from the beam with the period $\tau = 1$

and the space-charge force $F(z)$, given by the Poisson equation

$$F_t = \beta F(z,\tau) - \frac{\alpha}{2z-1}\delta_1(\tau), \quad \alpha = \frac{N_b r_0 s_b}{b^2}, \quad \beta = 4\pi r_0 s_b^2 \tag{30}$$

$$F_t = -\frac{\partial U(z)}{\partial z}, \quad \frac{\partial F(z)}{\partial z} = n(z), \tag{31}$$

$$F(z) = \int_0^z z' dz' n(z') - \int_z^1 dz'(1-z')n(z'). \tag{32}$$

Here $n(z,\tau) = \int dun(u,z,\tau)$ is the density with the usual dimension cm^{-3} defined in such a way that $n(x,t) = n(z,\tau)$ for corresponding z and x. The total number of particles is $n_{\mathrm{tot}} = \int n(x)dx = 2b\int n(z)dz$. We use the Vlasov equation:

$$\frac{\partial n(z,u,\tau)}{\partial \tau} + u\frac{\partial n(z,u,\tau)}{\partial z} + F_t\frac{\partial n(z,u,\tau)}{\partial u} =$$

$$[Q_0\delta(\tau)+\eta_0 J_+^\eta]\Phi(-u)\delta(1-z) - un(u,1)\delta(1-z)\Theta(u) + [\eta_0 J_-^\eta \Phi(u) + un(u,0)\Theta(-u)]\delta(z). \tag{33}$$

where we neglected collisions between the electrons, but included the interaction of electrons with the beam and the average effect of the space charge. The source terms in the RHS include the term proportional to the flux dN_e/ds of photoelectrons per unit length of the beam pipe per bunch:

$$Q_0 = \frac{1}{2bh_g}(\frac{dN_e}{ds}). \tag{34}$$

Other terms describe particles loss at the wall and the secondary emission of the electrons. For Al, the maximum yield $\eta_0 = 2.6$ corresponds to the energy 400 eV of the incident electron, rolling off to $\eta = 1$ at $E_{\min} = 50$ eV and $E_{\max} = 2.6$ keV. Eq.

(33) uses notation

$$J_+^\eta = \int_{u_{\min}}^{u_{\max}} udun(u,z); \quad J_-^\eta = -\int_{-u_{\max}}^{-u_{\min}} udun(u,z) \tag{35}$$

for the number of incident electrons able to produce the secondary electrons, where

$$u_{\min} = (\frac{s_b}{b})\sqrt{\frac{E_{\min}}{2mc^2}}. \tag{36}$$

The distribution of the secondary electrons is described here by the function $\Phi(u)$, which is non zero only for $u > 0$, and normalized to one, $\int_0^\infty du\Phi(u) = 1$. The distribution may be characterized by momentums

$$u_0 = \int_0^\infty udu\Phi(u), \quad \hat{u}_0^2 = \int_0^\infty u^2 du\Phi(u). \tag{37}$$

$\Phi(u)$ has a sharp maximum at the energy of few eV. We will neglect another peak with the energy close to the energy of the incident electron with a small yield.

The equilibrium density can be obtained by deriving a set of equations for the momentums of $n(u,z,\tau)$. Let us define momentums

$$J^+(z) = \int_0^\infty udun(u,z), \quad J^-(z) = -\int_{-\infty}^0 udun(u,z),$$

$$n(z)T^+(z) = \int_0^\infty u^2 dun(u,z), \quad n(z)T^-(z) = \int_{-\infty}^0 u^2 dun(u,z). \tag{38}$$

Both $J^\pm > 0$ and the total current $J(z) = n(z)v(z)$ is $J(z) = J^+ - J^-$. Similarly, $T(z) = T^+(z) + T^-(z)$. To close the system of equations, we assume that the third momentums can be approximated as

$$\int_0^\infty u^3 dun(u,z) \simeq J^+(z)T^+(z), \quad \int_{-\infty}^0 u^3 dun(u,z) = -J^-(z)T^-(z). \tag{39}$$

Integrating Eq. (2-5) over du, udu, and u^2du gives the system of equations

$$\frac{\partial n(z,\tau)}{\partial \tau} + \frac{\partial J(z,\tau)}{\partial z} = 0, \tag{40}$$

$$\frac{\partial J}{\partial \tau} + \frac{\partial n(z,\tau)T}{\partial z} - F_t n = 0, \tag{41}$$

$$\frac{\partial n(z,\tau)T}{\partial \tau} + \frac{\partial J(z,\tau)T}{\partial z} - 2F_t J = 0, \tag{42}$$

and the boundary conditions:

$$J^-(1) = [Q_0\delta(\tau) + \eta_0 J^\eta_+], \quad J^+(0) = \eta_0 J^\eta_- . \tag{43}$$

$$T^-(1)n(1) = u_0[Q_0\delta(\tau) + \eta_0 J^\eta_+], \quad T^+(0)n(0) = u_0\eta_0 J^\eta_-, \tag{44}$$

$$T(1)J(1) = -\hat{u}_0^2[Q_0\delta(\tau) + \eta_0 J^\eta_+] + T^+(1)J^+(1),$$

$$T(0)J(0) = \hat{u}_0^2\eta_0 J^\eta_- - J^-(0)T^-(0). \tag{45}$$

Averaging in time over the period $\tau = 1$ changes this equation only by replacing $\delta(\tau) \to 1$ and taking away terms with derivatives over τ.

After averaging, Eq. (40) gives $J(z) = J_0 = \text{const}$. Eq. (42) then gives the energy conservation

$$\frac{T(z)}{2} + U(z) = \text{const} = \frac{T(0)}{2} = \frac{T(1)}{2} \tag{46}$$

where $U(z)$ is defined by Eq. (34) with the conditions $U(0) = U(1) = 0$. Eqs. (41), (42) give together

$$n^2T = \text{const} = A^2 = n(0)^2T(0) = n(1)^2T(1) \tag{47},$$

and the remaining equation defines the density

$$\frac{\partial}{\partial z}\frac{A^2}{n} = F_t(z)n(z). \tag{48}$$

It follows from Eqs. (46), (47) that $T(0) = T(1)$, $n(0) = n(1)$.

Define $J^\pm$ and $T^\pm$ from Eqs. (45). This gives

$$T(0) = \hat{u}_0^2 + \frac{u_0}{n(0)}[\eta_0 J^\eta_- - J_0], \quad T(1) = \hat{u}_0^2 + \frac{u_0}{n(1)}[Q_0 + \eta_0 J^\eta_+ + J_0]. \tag{49}$$

Condition $T(0) = T(1)$ defines J_0:

$$2J_0 = -[Q_0 + \eta_0(J^\eta_+ + J^\eta_-)]. \tag{50}$$

Hence,

$$T(1) = \hat{u}_0^2 + \frac{u_0}{2n(1)}[Q_0 + \eta_0(J^\eta_+ + J^\eta_-)]. \tag{51}$$

By definitions,

$$T(1) = \frac{1}{n(1)}\int u^2 dun(1,u) = (\overline{u})^2 + \overline{(u-\overline{u})^2}. \tag{52}$$

Identify $\overline{(u-\overline{u})^2} = \hat{u}_0^2$, $\overline{u} = u_0$, then, the boundary conditions are

$$n(0) = n(1), \qquad 2n(1)u_0 = Q_0 + \eta_0(J^\eta_+ + J^\eta_-). \tag{53}$$

We now need to define the current J^η of high energy particles going toward the wall. This cannot be done from the steady-state equations: particles in the steady state move in an effective potential, and they return to the wall with exactly the same energy they leave the wall. To define J^{η}, we go to the basic equation of motion (31). A kick from a bunch has to increase the energy of a particle to the the range from $E_{\rm min}$ to $E_{\rm max}$. This means that a particle has to be at the distances from a bunch in the range

$$u_{\rm min} < \frac{\alpha}{(2z-1)} < u_{\rm max} \tag{54}$$

where $u_{\rm min}$, $u_{\rm max}$ are defined by Eq. (36). If $u_{\rm min}$ is large compared to the velocity u of a particle before kick, then

$$J^{\eta} = \int dz' n(z', \tau) \tag{55}$$

where the integral is taken over the range Eq. (54). The minimum distance from the beam here is $\Delta z = \alpha/(2u_{\rm max})$ and has to be replaced by the dimensionless transverse rms bunch size $\sigma_x/2b$ if $\Delta z < \sigma_x/2b$.

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Taking another derivative over z in Eq. (48), we get

$$\frac{\partial}{\partial z}\frac{1}{n}\frac{\partial}{\partial z}\frac{1}{n} = \frac{\beta}{A^2}n + \frac{2\alpha}{A^2}\frac{1}{(2z-1)^2}. \tag{56}$$

The first term in the RHS describes the space-charge effect. It is small compared to the second term describing the average field of the bunch for the densities $n < 2\alpha/\beta$, or $n < 5. \times 10^6$ cm^{-3} for PEP-II at 3 A current.

Neglecting the space-charge term, we get the solution

$$n(z) = \frac{Q_0}{u_0}[1 + \frac{\alpha}{2u_0^2}\ln|\frac{1}{2z-1}|]^{-1/2} \tag{57}$$

where, for simplicity we put $\hat{u}_0^2 = u_0^2$.

The density Eq. (57) is of the order of $n \simeq Q_0/2u_0$, or $n = 4.0 \times 10^5$ for PEP-II at 3 A current. That is larger than $n_{\rm max}$ of the initial density of a strip, but decreases logarithmically at the beam position $z = 1/2$ due to the interaction with the beam.

At very large densities, the RHS of Eq. (56) is dominated by the space-charge term. Neglecting the second term, we get the solution similar to the problem of the space-charge dominated current from a cathode:

$$n(z) = (\frac{2}{9\beta})^{1/3}|\frac{A}{z-1/2}|^{2/3}. \tag{58}$$

This solution should be matched with the solution Eq. (57), which is always correct in the close vicinity to the bunch.

Result of the numeric integration of Eq. (56) are shown in Fig. 6. The equilibrium density is maximum at the wall and is much larger than the density of the photo-electrons, $n \simeq 5. \times 10^7$ cm^{-3}.

The equation was solved numerically for the variable $y(z) = n(1)/n(0)$. The solution depends on two parameters α, u_0, for different values of the parameter $\hat{\beta} = \beta n(1)$. After the solution of the equation with the boundary conditions $y(0) = y(1) = 1$ was found by the shooting method, the integrals $\hat{J}^{\eta} = J^{\eta}/n(1)$ can be defined. Then, the boundary condition Eq. (53) defines the solution $n(z) = \hat{\beta}/[\beta y(z)]$, which corresponds to

$$\eta_0(\hat{\beta}) = \frac{u_0}{\hat{J}^{\eta}}[1 - \frac{\beta Q_0}{2u_0\hat{\beta}}]. \tag{59}$$

The quasi-equilibrium solution for given Q_0 exists only for certain range of $\eta_0 > 0$ as it can be seen from Eq. (59). Outside the range, the solution is unstable what corresponds to the avalanche of the secondary electrons.

The self-consistent regime is described here approximately using averaging in time. If η_0 is two small, the bunch-to-bunch modulation of the density become large and all electrons can go to the wall within before another bunch arrives. The minimal η_0 can be obtained from the first relation Eq. (55) estimating the current to the wall J^+ as $J^+ = \int_{z_{\rm max}}^1 dzn(z)$.

Effect of Ions

The inelastic collisions in the residual gas with pressure p have a typical cross-section $\sigma_c \simeq 2$ Mbarn and, at normal temperature, generate electrons with the rate $dN_e/ds = 0.06\,(p/\text{torr})$ cm^{-1} per positron. The photoeffect on the residual gas gives rate comparable with the rate of inelastic collisions. For a pressure $p = 5$ ntor (the density of the residual gas at normal temperature $n_g = 1.5 \times 10^8$ cm^{-3}) this rate is 2×10^4 smaller than the rate of photoelectron production.

Let us consider therefore the rate of ionization of the residual gas by the photo- and background electrons. The potential problem here is the induced de-gassing from the wall, which may produce pressure instability.

The photoelectrons hitting the walls do not cause the problem. They are produced with the rate

$$\frac{d^2N_e}{dsdt} = (\frac{dN_e}{ds})\frac{I_b}{eN_b} \tag{60}$$

where I_b is the average beam current. This rate is very high,of the order of 10^{14} cm^{-1}sec^{-1}, and the electrons can produce neutrals hitting the walls. However, the yield of neutrals per electron is smaller than that for ions. More than that, this effect is independent of pressure and, hence, does not lead to the run-away increase of the pressure of the residual gas, although more pumping may be required. Similarly, in the case of the formation of the equilibrium plasma, the flux of ions to the wall is equal to the flux of the photoelectrons in the beam pipe. This flux is large, but again does not depend on the residual gas density n_g.

Photoelectrons can also ionize residual atoms, which, hitting the wall, produce more neutrals. This process increases effective yield of the neutrals produced by ions at the walls and reduces the threshold of the pressure build-up. The cross section of the ionization of the residual gas is of the order of $\sigma_i \simeq 10^{-16}$ cm^2 at low energy. The rate of ion production per electron moving from wall-to-wall is then

$$\frac{d^2N_{ie}}{dsdt} = 2bn_g\sigma_i(\frac{dN_e}{ds})\frac{I_b}{eN_b}. \tag{61}$$

Comparing this yield with the rate of ions produced in the inelastic collisions of the

beam with the residual gas

$$\frac{d^2N_i}{dsdt} = n_g\sigma_c\frac{I_b}{e}, \tag{62}$$

we see that the rate in Eq. (61) is small, only few percent of the rate in Eq. (62), even with the relatively low σ_c.

The rate of ions produced by the background electrons with density n_e may be higher. An electron produces an ion within the ionisation length $l = (n_g\sigma_i)^{-1}$, and if the average velocity of the electrons is v, they produce

$$\frac{d^2N_i}{dsdt} = n_g n_e \sigma_i 4abv \tag{63}$$

ions per unit length of the beam pipe with the dimensions $a \times b$. This rate is $d^2N_i/dsdt = 2.0 \times 10^7$ cm^{-1}sec^{-1} for $n_e = 10^5$ cm^{-3}, $v/c = 10^{-2}$, and $b = 2a = 5$ cm. This has to be compared with the rate from Eq. (63) $d^2N_i/dsdt = 6.0 \times 10^9$ cm^{-1}sec^{-1}.

Thus, the rate of the ion production by the electrons is small compared to the rate of ion production by the beam.

Finally, ions may, in their turn, affect the density of the background electrons. The ionization of the residual gas may produce more than enough ions to make a neutral plasma, which would change the space charge effect and the dynamics of the background electrons. The Debye length at the plasma density $n = 10^7$ cm^{-3} and the temperature of the order of $E_{\min} = 50$ eV is of the order of 1.6 cm. Plasma with these parameters may affect the condition of the equilibrium. However, the ionization rate is relatively low: the cross section $\sigma \simeq 10^{-16}$ cm^2 corresponds at this density to ionization length 3. $\times 10^7$ cm or to the ionization time 20 msec at the electron velocity $v/c = 5 \times 10^{-3}$. This time is much larger than the revolution period 7 μsec, but the ion density can increase up to 10^4 cm^{-3}. The 100 m gap in the bunch train will clear electrons, and their space charge drags ions to the wall. However, the gap in the PEP-II positron ring is partially filled, and ions can survive and be accumulated to make the low-density neutral plasma.

The plasma oscillations at such density have the wavelength $2\pi/\sqrt{4\pi r_0 n}$ of the order of 100 m, too large to affect the beam.

Conclusion

The production rate and dynamics of the photoelectrons are studied for the PEP-II parameters. The growth rate of the transverse instability driven by the primary photoelectrons is of the order of 0.7 msec for the PEP-II parameters. This is comparable with the rate of instability driven by ions in the HER: the large flux of the primary photoelectrons is compensated by the low density and small number of interacting bunches.

The multipactoring at resonance currents cannot produce large electron density due to the final energy spread caused by the finite bunch length and the intrinsic energy spread of the secondary electrons.

Production of the secondary electrons may lead to large average densities. This effect is studied in the 1-D model, taking into account the space-charge effect, interaction with the beam, and production of the photoelectrons.

The ions can be produced in electron collisions with the residual gas with density of the order of the electron density. They may not be cleared out by a partially filled gap. While going to the wall, the ions can increase the rate of production of neutral atoms and increase pressure of the residual gas.

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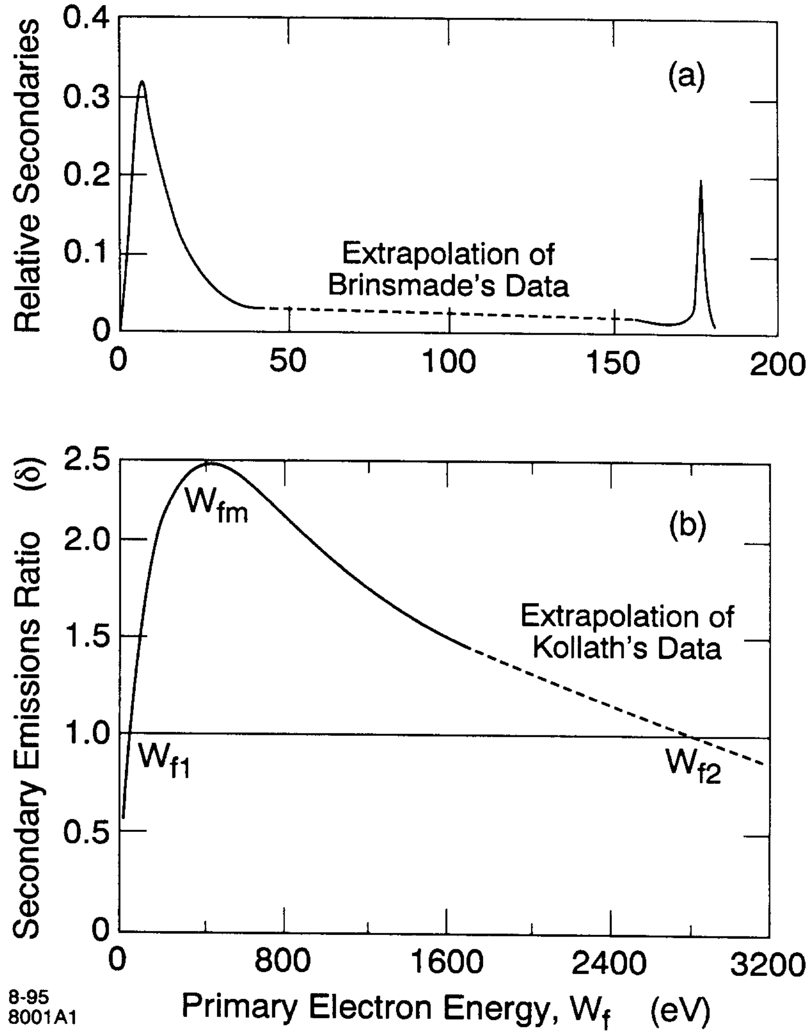

Figure 1

The yield of the secondary electrons: dependence on the energy of the incident electron (below, from R. Kollath, Phys. Z. 38, p. 202, 1937), and the energy spectrum of the secondary electrons (above, from J. R. Brinsmade, Phys. Rev. 30, p.494, 1927).

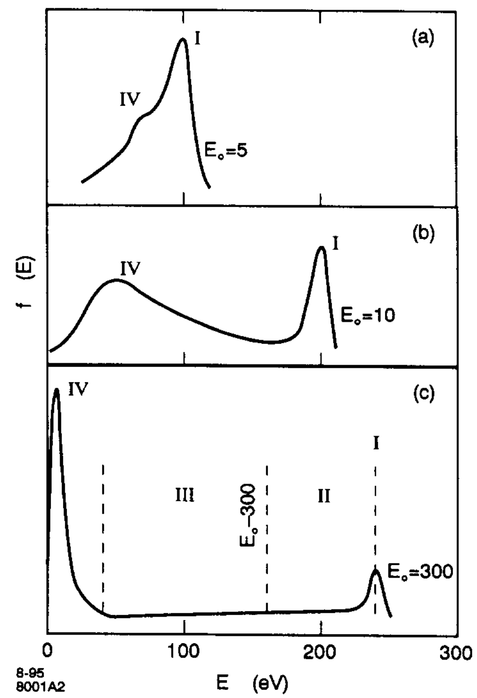

Figure 2

Dependence of the yield of the secondary electrons on initial energy for different materials, LBL Engineering Manual.

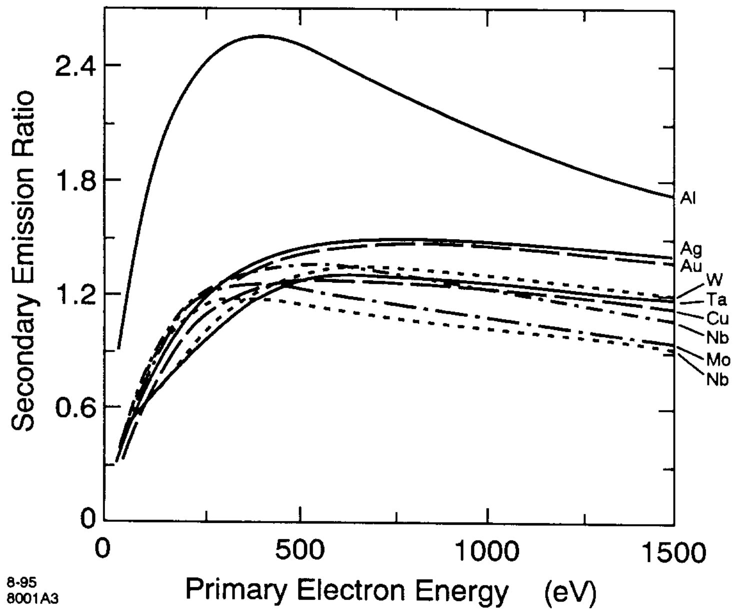

Figure 3

Dependence of number of kicks an electron receives traveling from wall to wall on current.

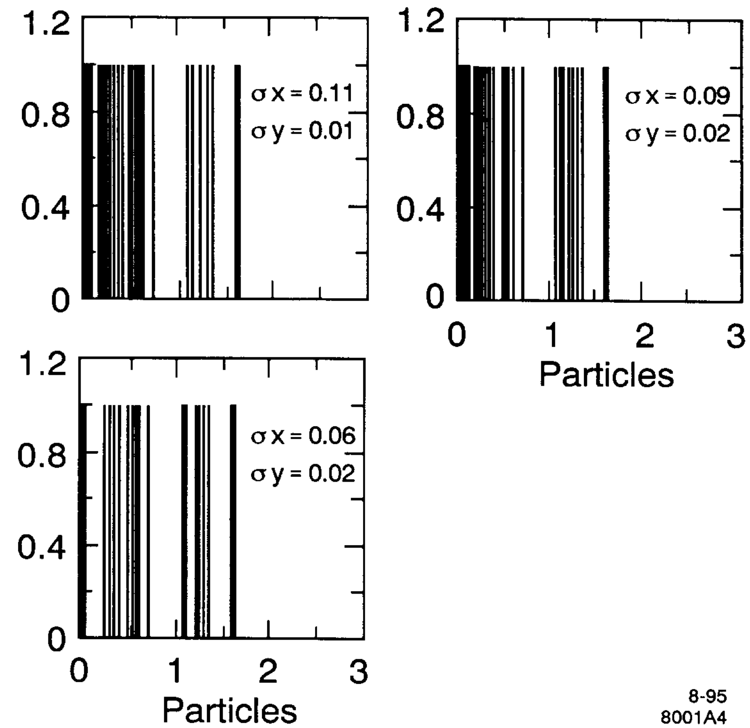

Figure 4

Examples of the trajectories in the phase plane for three different location around the ring (i.e. for different transverse rms σ_t) at $I =$ 0.015 A.

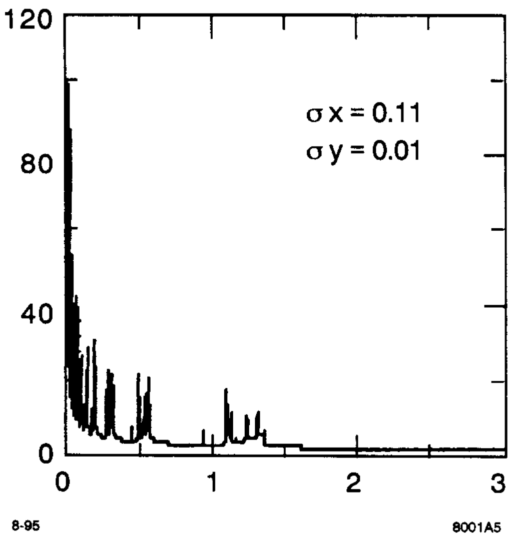

Figure 5

Dependence of the average density on time in the 1-D model including both effects of the photoelectrons and of the secondary emission. $dN_e/ds = 6.8 \times 10^5$, $\eta_0 = 2.6$. Time in units s_b/c.

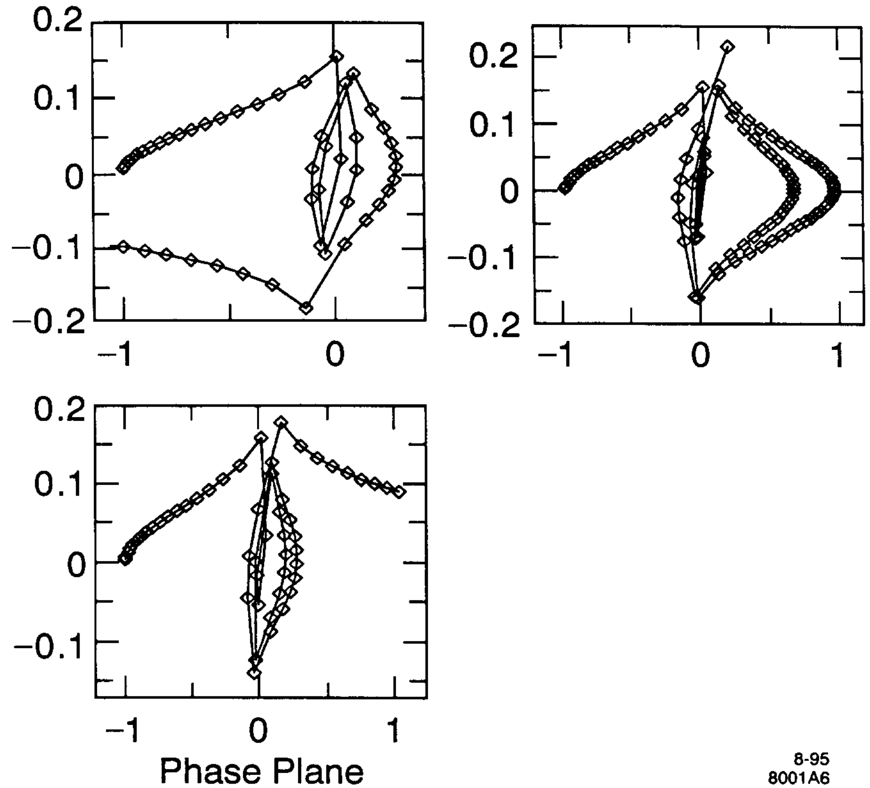

Figure 6

Density profile for the same case as in Fig. 5.