# Theoretical predictions for $B \to X_s\gamma$ and $B \to K^*\gamma$ *)

**M. Ciuchini**[a,b], **E. Franco**[b], **G. Martinelli**[b,c], **L. Reina**[d] **and L. Silvestrini**[e]
[a] INFN, Sezione Sanità, V.le Regina Elena 299, 00161 Roma, Italy.
[b] Dip. di Fisica, Università degli Studi di Roma "La Sapienza" and INFN, Sezione di Roma, P.le A. Moro 2, 00185 Rome, Italy.
[c] TH Division, CERN, CH-1211 Geneva 23, Switzerland.
[d] Service de Physique Théorique, Université Libre de Bruxelles, Boulevard du Triomphe, CP 225 B-1050 Brussels, Belgium.
[e] Dip. di Fisica, Univ. di Roma 'Tor Vergata'and INFN, Sezione di Roma II, Via della Ricerca Scientifica 1, I-00173 Rome, Italy.

**Abstract**

We present upgraded theoretical predictions for inclusive and esclusive radiative decays of B mesons in the Standard Model. Our results include those next-to-leading order corrections which have been already computed. Our best estimates in the Standard Model are $BR(B \to K^*\gamma) = (4.3 \pm 0.9^{+1.4}_{-1.0}) \times 10^{-5}$, $BR(B \to X_s\gamma) = (1.9 \pm 0.2 \pm 0.5) \times 10^{-4}$, $\Gamma(B \to K^*\gamma)/\Gamma(B \to X_s\gamma) = 0.23 \pm 0.09$.

* Talk given by G. Martinelli

# Introduction

Radiative decays of $B$ mesons represent very important tests of the weak interactions and of the role of effective flavour-changing neutral currents. Among these decays, $b \to s\gamma$ and $b \to sg$ are theoretically clean and sensitive to physics beyond the Standard Model, e.g. charged scalar Higgs models and/or SUSY models[1]–[2]. The experimental measurement of the exclusive branching fraction $BR(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ by the CLEOII collaboration [3] and the imminent measurement of the inclusive rate, on which an upper limit $BR(B \to X_s\gamma) < 5.4 \times 10^{-4}$ (95% C.L.) [4] already exists, offer the opportunity to compare experimental results and theoretical predictions for these quantities.

From the theoretical point of view, the prediction of the rates consists as usual of two steps. On the one hand, it is necessary to compute the renormalization of the coefficients of the effective Hamiltonian to take into account the effects of strong interactions at short distances, i.e. for scales $m_b \leq \mu \leq M_W$. It turns out that renormalization effects at the leading order (LO) have important consequences, since they almost double the amplitude obtained without their inclusion [5, 6]. Unfortunately, the full set of next-to-leading corrections to these decays, which are necessary for a consistent use of $\Lambda_{QCD}$, of the renormalization scale, and for more accurate predictions, are not available yet. On the other hand, it is necessary to compute the hadronic matrix elements of the operators appearing in the effective Hamiltonian. For inclusive decays, in the framework of the Heavy Quark Effective Theory (HQET), it is possible to predict the rate using the parton model, with computable corrections that are expected to be of order $1/m_b^2$ [7]–[11]. For exclusive decays, one has to know the relevant hadronic form factors, obtained from a non-perturbative estimate. For $B \to K^*\gamma$ there is only one form factor, which we will denote in the following by $F_1(0)$. The exclusive channel had a bad reputation because different predictions of the rate varied by orders of magnitude. In the recent past, however, lattice QCD [12, 13] and QCD sum rules [14]–[16] have procured more reliable results and the theoretical uncertainties have been substantially reduced. This makes the exclusive decays more interesting as tests of the Standard Model.

We present an upgraded analysis of the inclusive and exclusive $b \to s\gamma$ decay rates, which takes into account several improvements made recently:

1) The coefficient of the magnetic (chromo-magnetic) operator, $C_7$ is now established. In refs. [17, 18], the long-standing problem of the regularization dependence of the LO coefficients of these operators was solved; the results were found to be different from all previous calculations [19]. Those of refs. [17, 18] were subsequently

confirmed in ref. [20].

2) The next-to-leading order corrections to the anomalous dimension matrix are partially known [21, 22]. We will make use of this information to try to evaluate the effect of the next-to-leading terms and include it in the error on the final predictions. We also compare results obtained in the HV and NDR regularization schemes, since a spurious regularization dependence is introduced by the incomplete NLO terms. The NLO corrections that we can already include diminish $C_7$ in both the HV and NDR schemes. It turns out that in HV the dependence on the renormalization scale is substantially reduced. A similar effect is also found in NDR, where nevertheless a sizeable dependence is still present. We will also make use of the partial calculation of the $O(\alpha_s)$ corrections to the inclusive rate, which has been computed in ref. [27].

3) We will study both the inclusive and exclusive channels, the latter being ignored in most of more recent analyses. We will make use of lattice and QCD sum rules predictions for the relevant form factor $F_1(0)$.

4) By varying within their errors the experimental and theoretical quantities, we obtain a distribution of values for the theoretical predictions, from which we estimate the theoretical uncertainty.

Our predictions contain several differences with respect to the recent analysis of ref. [23]:

i) the estimates we present are also for the exclusive $B \to K^*\gamma$ rate;

ii) according to refs. [28, 29], we have used the running mass, and not the pole mass, for the evaluation of the inclusive rate;

iii) $O(1/m_b^2)$ terms have been taken into account in the present study;

iv) NLO corrections to the coefficient function [27] and to the anomalous dimension matrix [21, 22] have been included by us in evaluation of the values and theoretical uncertainties for the rates.

Of all these differences, the most important is the last one, since the known NLO corrections diminish the strong $\mu$ dependence and reduce the values of the rates. Apart from this, the bulk of our results is substantially in agreement with those of ref. [23].

# Basic Formulae

The effective Hamiltonian responsible for $b \to s\gamma$ decays can be written as [5, 6], [17]–[20]:

$$H^{eff} = -V_{tb}V_{ts}^{*}\frac{G_F}{\sqrt{2}}C_7^{eff}(\mu)O_7(\mu), \tag{1}$$

where:

$$O_7 = \frac{Q_d e}{16\pi^2}m_b\left(\bar{s}\sigma^{\mu\nu}b\right)_R F_{\mu\nu} \tag{2}$$

and $\mu$ is the renormalization scale of the relevant operators. From the effective Hamiltonian one can derive the decay rates. In the following, we report the main formulae that will be used in the numerical evaluation of the inclusive and exclusive branching fractions.

**Inclusive** $B \to X_s\gamma$:

$$BR(B \to X_s\gamma) = \left[\frac{\Gamma(B \to X_s\gamma)}{\Gamma(B \to Xl\nu_l)}\right]^{th} \times BR^{exp}(B \to Xl\nu_l), \tag{3}$$

where :

$$\left[\frac{\Gamma(B \to X_s\gamma)}{\Gamma(B \to Xl\nu_l)}\right]^{th} = \frac{|V_{ts}^{*}V_{tb}|^2}{|V_{cb}|^2}\frac{\alpha_e}{6\pi g(m_c/m_b)} \times F \times |C_7^{eff}(\mu)|^2 \tag{4}$$

with the phase-space factor $g(z)$ given by:

$$g(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4\ln(z) \tag{5}$$

and

$$F = \frac{K(m_t/M_W,\mu)}{\Omega(m_c/m_b,\mu)}, \tag{6}$$

In eq. (6) the quantity $\Omega(z)$ contains the $O(\alpha_s)$ QCD corrections to the semileptonic decay rate [24, 25]. Within a good approximation it is given by [26]:

$$\Omega(z,\mu) \simeq 1 - \frac{2\alpha_s(\mu)}{3\pi}\left[\left(\pi^2 - \frac{31}{4}\right)(1-z)^2 + \frac{3}{2}\right]; \tag{7}$$

the factor $K(m_t/M_w,\mu)$ contains the $O(\alpha_s)$ NLO corrections to the $B \to X_s\gamma$ rate, due to real and virtual gluon emission [27]. The calculation of ref. [27] does not contain the full set of $O(\alpha_s)$ corrections, which would require a two-loop calculation of the coefficient function and a three loop calculation of the anomalous dimension.

In ref. [27], the authors only included some terms, which one may argue to be the most important ones. We have used the results of ref. [27] in our analysis, in the same spirit in which we considered the NLO anomalous dimension, which is only partially known.

The scale $\mu$, which appears in some of the previous formulae, is there for two reasons. In the numerator of eq. (4), $\mu$ denotes the renormalization scale of the effective $b \to s\gamma$ Hamiltonian. It also represents the scale at which we decided to compute the expansion parameter $\alpha_s$ for the QCD corrections to the semileptonic decay rate. For simplicity we have taken the same value of $\mu$ in both cases. According to refs. [28, 29], in the framework of the HQET, to avoid problems with renormalon (non-perturbative) effects, we have used as expansion parameter the running mass $m_b = m_b(\mu)$. For this reason, since the $b \to s\gamma$ operators are evaluated at a generic $\mu$, we have omitted the factor $(m_b(\mu)/m_b(\text{pole}))^2$ which was included in $F$ by the authors of ref. [23], see eq. (6).

**Exclusive $B \to K^*\gamma$:**

$$\begin{aligned} BR(B \to K^*\gamma) &= \left[\frac{\Gamma(B \to K^*\gamma)}{\Gamma(B \to X_s\gamma)}\right]^{th} \times \left[\frac{\Gamma(B \to X_s\gamma)}{\Gamma(B \to Xl\nu_l)}\right]^{th} \\ &\times \; BR^{exp}(B \to Xl\nu_l), \end{aligned} \tag{8}$$

where

$$\left[\frac{\Gamma(B \to K^*\gamma)}{\Gamma(B \to X_s\gamma)}\right]^{th} = \left(\frac{M_b}{m_b}\right)^3 \left(1 - \frac{M_{K^*}^2}{M_B^2}\right)^3 \times \frac{1}{1 + (\lambda_1 - 9\lambda_2)/(2m_b^2)} \times |F_1(0)|^2 \tag{9}$$

In the HQET formalism, the parameters $\lambda_1$ and $\lambda_2$ describe the leading non-perturbative corrections (at order $O(1/m_b^2)$) to the parton model predictions for the inclusive rate [11]. They are related to the kinetic energy of the $b$ quark (inside the $B$ meson) and to the $B$–$B^*$ mass splitting. The $1/m_b^2$ corrections cancel in $\Gamma(B \to X_s\gamma)/\Gamma(B \to Xl\nu_l)$ but not in the ratio (9); $F_1(0)$ is the relevant form factor defined by:

$$\langle K^*(p',\eta)|\bar{s}\sigma_{\mu\nu}(1+\gamma_5)q^\nu b|B(p)\rangle = 2i\epsilon_{\mu\nu\rho\sigma}\eta^{*\nu}p^\rho p'^\sigma F_1(q^2) \tag{10}$$

$$+ 2\left[\eta^*_\mu(M_B^2 - M_{K^*}^2) - (\eta^*\cdot q)(p+p')_\mu\right] G_2(q^2) \tag{11}$$

with $G_2(0) = F_1(0)/2$.

| Parameters | Values |
| --- | --- |
| $\lvert V_{ts}^* V_{tb}\rvert^2/\lvert V_{cb}\rvert^2$ | $0.95 \pm 0.04$ |
| $m_c/m_b$ | $0.316 \pm 0.013$ |
| $m_t$(GeV) | $174 \pm 16$ |
| $\lambda_1$(GeV$^2$) | $-0.15 \pm 0.15$ |
| $\lambda_2$(GeV$^2$) | $0.12 \pm 0.01$ |
| $m_b(\mu = m_b)$(GeV) | $4.65 \pm 0.15$ |
| $F_1(0)$ | $0.35 \pm 0.05$ |
| $BR^{exp}(B \to X l \nu_l)$ | $0.107 \pm 0.005$ |
| $\Lambda_{QCD}^{n_f=5}$(GeV) | $0.240 \pm 0.090$ |
| $\mu$ | $m_b/2$–$2m_b$ |

Table 1: *Values of the different parameters used to predict the inclusive and exclusive radiative b decay rates.*

## Predictions and Uncertainties for the Decay Rates

We give in table 1 the range of variation of all the quantities appearing in eqs. (3)–(9), which have been used to obtain our results.

Using the central values of table 1, we show in fig. 1 the $\mu$ dependence of $C_7^{eff}(\mu)$ at the LO and NLO. For LO we mean that we have taken the anomalous dimension matrix at the LO. For the total rate, we put to zero all the $O(\alpha_s)$ terms that appear in eqs. (3)–(9), including those relative to the semileptonic rate. In this case one has also to use $K(m_t/M_W, \mu) = 1$. In the NLO case we turn on all the known NLO corrections, including the $(6 \times 6)$ anomalous-dimension matrix computed in refs. [21, 22]. In this case we have varied $K(m_t/M_W, \mu)$ according to the results of ref. [27], i.e. $0.79 \le K(m_t/M_W, \mu) \le 0.86$ for $m_b/2 \le \mu \le 2m_b$. We notice that the $\mu$ dependence is reduced both in the HV and in the NDR cases. If we call $R = (C_7^{eff}(\mu)|_{\mu=m_b/2}/C_7^{eff}(\mu)|_{\mu=2m_b})^2$, we get $R^{LO} \sim 1.72$, $R^{NDR} \sim 1.54$ and $R^{HV} \sim 1.25$ [1]. One would prefer *HV* because of the reduced $\mu$ dependence, as shown fig. 1. In the absence of a complete calculation, however, we cannot decide which of the results, NDR or HV, is closer to the real NLO result. For this reason, for all the quantities reported below, we will combine the results obtained with NDR

[1]This means that the coefficient itself varies only of $\sim 10-15\%$ in a range of scales as large as $\mu \sim 2-9$ GeV.

| | $BR(B \to X_s\gamma) \times 10^4$ | | |
|---|---|---|---|
| $\mu$ (GeV) | LO | NLO$_{HV}$ | NLO$_{NDR}$ |
| $m_b/2$ | $3.81 \pm 0.47$ | $1.92 \pm 0.19$ | $2.77 \pm 0.32$ |
| $m_b$ | $2.93 \pm 0.33$ | $1.71 \pm 0.18$ | $2.25 \pm 0.25$ |
| $2m_b$ | $2.30 \pm 0.26$ | $1.56 \pm 0.17$ | $1.91 \pm 0.21$ |
| | $BR(B \to K^*\gamma) \times 10^5$ | | |
| $\mu$(GeV) | LO | NLO$_{HV}$ | NLO$_{NDR}$ |
| $m_b/2$ | $6.9 \pm 1.5$ | $4.4 \pm 0.8$ | $6.4 \pm 1.3$ |
| $m_b$ | $5.3 \pm 1.1$ | $3.8 \pm 0.8$ | $5.0 \pm 1.0$ |
| $2m_b$ | $4.2 \pm 0.9$ | $3.3 \pm 0.7$ | $4.1 \pm 0.8$ |

Table 2: *Theoretical predictions for exclusive and inclusive radiative B decays.*

and HV with $m_b/2 \leq \mu \leq 2m_b$, and include the differences in the final estimate of the error. As central value we will take the average between the NDR and HV result. Regarding all the other quantities, we allow them to vary within the ranges reported in table 1, with Gaussian distributions for the experimental parameters and flat distributions for the theoretical ones. In a given regularization scheme and for a fixed value of $\mu$, this procedure generates a pseudo-Gaussian distribution of values, from which we deduce the error. For more details, see ref. [30].

We want to add some comments on the value and error of $F_1(0)$ which has been used in the exclusive case. Recent lattice and QCD sum rules calculations of this quantity give: $F_1(0) = 0.20 \pm 0.02 \pm 0.06$ [12], $F_1(0) = 0.30^{+10}_{-8}$ [13] and $F_1(0) = 0.35 \pm 0.05$ [14], $F_1(0) = 0.32 \pm 0.05$ [15], $F_1(0) = 0.310 \pm 0.013 \pm 0.033 \pm 0.060$ [16]. For this reason we have chosen to use $F_1(0) = 0.35 \pm 0.05$, which covers most of the theoretical predictions (see table 1). From the numbers given in table 2 we quote $BR(B \to K^*\gamma) = (4.3 \pm 0.9^{+1.4}_{-1.0}) \times 10^{-5}$, $BR(B \to X_s\gamma) = (1.9 \pm 0.2 \pm 0.5) \times 10^{-4}$ and $\Gamma(B \to K^*\gamma)/\Gamma(B \to X_s\gamma) = 0.23 \pm 0.09$. The first error comes from the width of the pseudo-Gaussian distribution of the theoretical values, the second includes the $\mu$ dependence and regularization dependence of the results.

# References

[1] S. Glashow and S. Weinberg, Phys. Rev. D15 (1977) 1958; L.F. Abbott, P. Sikivie and M.B. Wise, Phys. Rev. D21 (1980) 1393.

[2] S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, Nucl. Phys. B353 (1991) 591; R. Barbieri and G.F. Giudice, Phys. Lett. B309 (1993) 86; F. Borzumati, preprint DESY 93-090 (August 1993) (hep-ph/9310212); S. Bertolini and F. Vissani, SISSA 40/94/EP (March 1994).

[3] R. Ammar et al. (CLEO Collaboration), Phys. Rev. Lett. 71 (1993) 674.

[4] E. Thorndike (CLEO Collaboration), in *Proc. of the American Physical Society Meeting*, Washington (DC), April 1993; T. Browder, K. Honscheid and S. Playfer, CLNS 93/1261, UH-511-778-93, OHSTPY-HEP-E-93-018, HEPSY 93-10 (1993).

[5] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Phys. Rev. 18 (1978) 2583.

[6] B. Grinstein, R. Springer and M.B. Wise, Phys. Lett. B202 (1988) 138; Nucl. Phys B339 (1990) 269.

[7] M. Voloshin and M. Shifman, Yad. Fiz. 41 (1985) 187 [Sov. Nucl. Phys. 41 (1985) 120]; ZhETF 91 (1986) 1180 [Sov. Phys.-JETP 64 (1986) 698.

[8] J. Chai, H. Georgi and B. Grinstein, Phys. Lett. B247 (1990) 399.

[9] I. Bigi, N. Uraltsev and A. Vainshstein, Phys. Lett. B293 (1992) 430; (E) B297 (1993) 477.

[10] B. Blok and M. Shifman, Nucl. Phys. B399 (1993) 441 and 459.

[11] A.F. Falk, M. Luke and M.J. Savage, SLAC-PUB-6317, UCSD-PTH-93-23, JHU-TIPAC-930020,UTPT-93-19, CMU-HEP-93-12, DOE-ER-40682-37.

[12] C. Bernard, P. Hsieh and A. Soni, Nucl. Phys. (Proc. Suppl.) B26 (1992) 347 (note that there is a factor 2 missing in eq. (4) of this paper); WASH-U-HEP-93-35.

[13] K.C. Bowler et al. (CN-UKQCD Collaboration), SHEP 93-94-01 (November 1993).

[14] P. Colangelo, C.A. Dominguez, G. Nardulli and N. Paver, Phys. Lett. B317 (1993) 183.

[15] A. Ali, V.M. Braun and H. Simma, CERN-TH.7118/93, MPI-Ph/93-97, DESY 93-193.

[16] S. Narison, CERN-TH-7166/94.

[17] M. Ciuchini, E. Franco, G. Martinelli, L. Reina and L. Silvestrini, Phys. Lett. B316 (1993) 127.

[18] M. Ciuchini, E. Franco, L. Reina and L. Silvestrini, ROME prep. 973/93, INFN-ISS 93/6, ULB-TH 15/93 (November 1993), to appear in Nucl. Phys. B.

[19] R. Grigjanis, P.J. O'Donnel, M. Sutherland and H. Navelet, Phys. Lett. B213 (1988) 355; Phys. Lett. B223 (1989) 239; Phys. Lett. B237 (1990) 252; G. Cella, G. Curci, G. Ricciardi and A. Viceré, Phys. Lett. B248 (1990) 181; M Misiak, Phys. Lett. B269 (1991) 161, B321 (1994) 193; Nucl. Phys. B393 (1993) 23; K. Adel and Y.P. Yao, Mod. Phys. Lett. A8 (1993) 1679; UM-TH 93-20, IP-ASTP-29-93 (1993).

[20] G. Cella, G. Curci, G. Ricciardi and A. Viceré, Phys. Lett. B325 (1994) 227.

[21] A.J. Buras, M. Jamin, M.E. Lautenbacher and P.H. Weisz, Nucl. Phys. B370 (1992) 69, Addendum, ibid. Nucl Phys. B375 (1992) 501; A.J. Buras, M. Jamin and M.E. Lautenbacher, Nucl. Phys. B400 (1993) 37 and B400 (1993) 75.

[22] M. Ciuchini, E. Franco, G. Martinelli and L. Reina, Phys. Lett. B301 (1993) 263; Nucl. Phys. B415 (1994) 403.

[23] A.J. Buras, M. Misiak, M. Münz and S. Pokorski, MPI-Ph/93-77, TUM-T31-50/93 (November 1993).

[24] N. Cabibbo, G. Corbò and L. Maiani, Nucl. Phys. B155 (1979) 93.

[25] M. Jezabek and J.M. Kühn, Nucl. Phys. B320 (1989) 20 and B314 (1989) 1.

[26] R.E. Behrends, R.J. Finkelstein and A. Sirlin, Phys. Rev. 101 (1956) 866; T. Kinoshita and A. Sirlin, Phys. Rev. 113 (1959) 1652.;
N. Cabibbo and L. Maiani, Phys. Lett. B79 (1978) 109.

[27] A. Ali and C. Greub, Z. Phys. C60 (1993) 433.

[28] M. Beneke and V.M. Braun, MPI-PhT/94-9, UM-TH-94-4 (February 1994).

[29] I. Bigi et al., CERN-TH-7171/94, TPI-MINN-94/4-T, UMN-TH-1239/94, UND-HEP-94-BIG03 (February 1994).

[30] M. Ciuchini, E. Franco, G. Martinelli and L. Reina, Rome preprint no.1024 (June 1994).

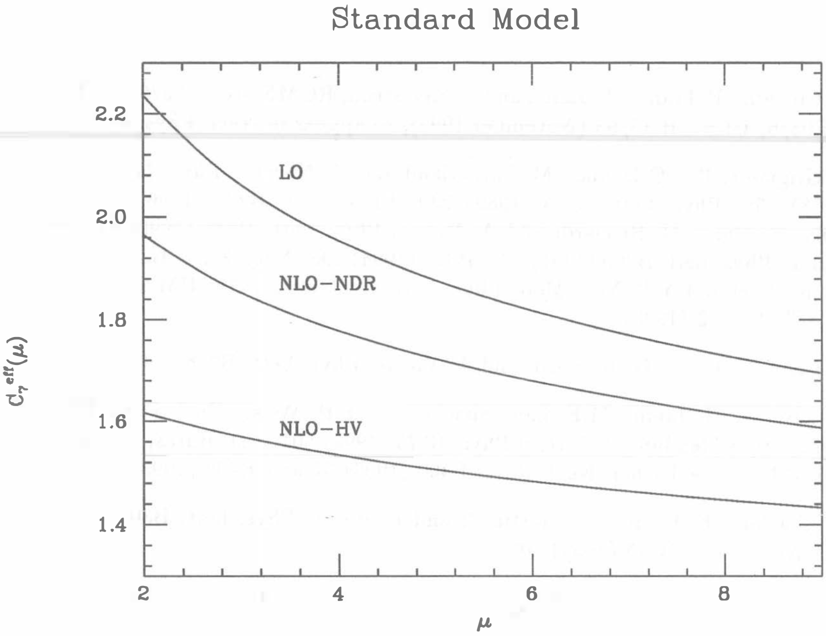


Figure 1: $C_7^{eff}(\mu)$ *as a function of* $\mu$ *in the LO, NLO-HV and NLO-NDR cases. For the definition of NLO see the text.*