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wilson

A package for renormalization group running in the SMEFT with Sterile Neutrinos

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Abstract: Sterile neutrinos are well-motivated beyond the Standard Model (BSM) particles. The Standard Model Effective Field Theory (SMEFT) augmented with these new fields is known as the νSMEFT. We present the first code for solving the renormalization group equations (RGEs) of the νSMEFT in an automated way. For this purpose, we have implemented the νSMEFT as a new effective field theory (EFT) in the Wilson coefficient exchange format `WCxf`. Furthermore, we included anomalous dimensions depending on the gauge couplings and Yukawas in the python package `wilson`[1]. This novel version of `wilson` allows a consistent inclusion of νSMEFT renormalization group (RG) running effects above the electroweak (EW) scale in phenomenological studies involving sterile neutrinos. Moreover, this new release allows us to study EW, strong, and Yukawa running effects separately within the SMEFT.

[1] https://wilson-eft.github.io/

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1 Introduction

Right-handed neutrinos are a natural extension of the Standard Model (SM) of particle physics. Since the observation of neutrino oscillations, neutrinos are known to have non-zero masses. The mass hierarchy remains however yet to be determined [1], exhibiting either normal ordering, i.e. $m_1 < m_2 < m_3$ or inverse ordering ($m_3 < m_1 < m_2$). In order for the neutrinos to become mass states, a simple solution is to add right-handed neutrinos to the SM. These can acquire a Majorana mass or, in combination with the coupling to the Higgs a Dirac mass. Allowing for even higher-dimensional operators involving right-handed neutrinos and the SM field content one arrives at the so-called νSMEFT, which is the Standard Model Effective Field Theory [2] (for reviews, see [3, 4]), augmented by right-handed neutrinos.

The νSMEFT has been studied extensively in the past few years [5–12]. However, concerning loop corrections, the νSMEFT is not as mature as SMEFT. Recently the renormalization of the νSMEFT has been completed at the 1-loop level [13–17]. The one-loop anomalous dimensions of the νSMEFT depending upon both the gauge and Yukawa couplings have been computed in these references. Such effects can be important for the running between the new physics (NP) scale Λ and the electroweak (EW) scale. Indeed these turn out to be crucial for phenomenological studies in the νSMEFT [13, 16, 18]. However, as of now, there is no public code that provides numerical solutions to the RG running within the νSMEFT for the complete set of operators.

In this work, we present a major upgrade of `wilson` [19], a Python package for the running and matching of Wilson coefficients (WCs) above and below the EW scale. Provided with the numerical values of the WCs at a high NP scale, the original `wilson` package is capable of performing the RG evolution within the SMEFT [20–22], matching onto the weak effective theory (WET) at the EW scale [23–25], as well as to perform the full QCD/QED RG evolution below the EW scale down to hadronic scales relevant for low-energy precision tests [26, 27]. In this upgraded version, we have included the functionality of solving the renormalization group equations (RGEs) of the νSMEFT in an automated way. Moreover, the subtle non-standard RG running effects due to back-rotation [28] can now be included in νSMEFT.

The article is organized as follows. In Section 2, we introduce the νSMEFT, including its Lagrangian together with the $SU(3)^6$ flavour rotations of the νSMEFT operators. In Section 3, we discuss the RG running in the νSMEFT. In Section 4, the implementation of the νSMEFT in `wilson` is discussed, along with additional upgrades. In Section 5, we provide details of the checks performed on the output of our code, and in Section 6 we give a summary and future prospects.

2 νSMEFT

In this section, we discuss our conventions regarding the νSMEFT and choice of flavour basis for νSMEFT operators involving fermions.

2.1 Lagrangian and Operator Basis

In addition to the SM fields, the νSMEFT contains sterile neutrinos. We denote the corresponding fields by n_p, where p is the generation index and we assume $p \in \{1, 2, 3\}$.

The νSMEFT Lagrangian is given by

$$\mathcal{L}_{\nu\text{SMEFT}} \supset i\bar{n}\partial\!\!\!/ n + (-\frac{1}{2}m_\nu(n^T C n) + h.c.) + \mathcal{L}_{\text{Yukawa}} + (\sum_i \mathcal{C}_i \mathcal{O}_i + h.c.)\,, \tag{2.1}$$

where the first term is the kinetic term for the sterile neutrinos. If neutrinos are assumed to be of Majorana nature, one can also add a Majorana mass term m_ν. The presence of such a term does not explicitly enter most of the anomalous dimension matrices (ADMs). However, one exception is the mixing of dipole operators into the Weinberg operator [16]. In the current implementation, this piece of the ADM is not included. In the Majorana mass term C stands for the charge-conjugate operator. Furthermore, $\mathcal{C}_i$ are the WCs of the higher dimension (≥ 5) operators. We have omitted the $\mathcal{L}_{\text{SM}}$ part containing the usual dimension-four SM terms. Note that our actual implementation of the νSMEFT in `wilson` follows `WCxf` conventions [29], in which the complex conjugated part for the higher dimensional operators is added only for the non-hermitian operators. In this way, only one out of two operators related through hermitian conjugation is considered in the basis.

A subset of $\mathcal{L}_{\text{SM}}$, the Yukawa terms plus a new Dirac mass term for the neutrinos are given by

$$\mathcal{L}_{\text{Yukawa}} = -[\phi^{\dagger j}\bar{d}Y_d q_j + \tilde{\phi}^{\dagger j}\bar{u}Y_u q_j + \phi^{\dagger j}\bar{e}Y_e \ell_j + \tilde{\phi}^{\dagger j}\bar{n}Y_n \ell_j + \text{h.c.}]\,, \tag{2.2}$$

where ϕ is the Higgs doublet and its conjugate field is $\tilde{\phi}^j = \epsilon^{jk}\phi_k^*$. Note that the neutrino Yukawa matrix Y_n is a NP parameter, unlike the other dimension-four terms in $\mathcal{L}_{\text{SM}}$.

Ignoring the different flavour permutations, there are in total 16 ($\Delta B = 0, \Delta L = 0$) new dimension-six operators as compared to the SMEFT. We assume that the right-handed neutrinos have a lepton number $L = 1$. In Table 1a, we show these new operators in the Warsaw basis convention [30], where $prst$ are the flavour indices. In addition, we also show two $B-$ and $L-$ violating operators in Table 1b that were first discussed in [31] and agree with the findings in [32]. Further, two new dimension-five operators are given in Table 1c.

$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$		$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$	
$\mathcal{O}_{nd}$	$(\bar{n}_p\gamma_\mu n_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qn}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{n}_s\gamma^\mu n_t)$	$\mathcal{O}_{\ell n\ell e}$	$(\bar{\ell}_p^j n_r)\epsilon_{jk}(\bar{\ell}_s^k e_t)$
$\mathcal{O}_{nu}$	$(\bar{n}_p\gamma_\mu n_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{\ell n}$	$(\bar{\ell}_p\gamma_\mu \ell_r)(\bar{n}_s\gamma^\mu n_t)$	$\mathcal{O}^{(1)}_{\ell nqd}$	$(\bar{\ell}_p^j n_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
$\mathcal{O}_{ne}$	$(\bar{n}_p\gamma_\mu n_r)(\bar{e}_s\gamma^\mu e_t)$			$\mathcal{O}^{(3)}_{\ell nqd}$	$(\bar{\ell}_p^j \sigma_{\mu\nu} n_r)\epsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} d_t)$
$\mathcal{O}_{nn}$	$(\bar{n}_p\gamma_\mu n_r)(\bar{n}_s\gamma^\mu n_t)$			$\mathcal{O}_{\ell nuq}$	$(\bar{\ell}_p^j n_r)(\bar{u}_s q_t^j)$
$\mathcal{O}_{nedu}$	$(\bar{n}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu u_t)$				
$\psi^2\phi^3$		$\psi^2\phi^2 D$		$\psi^2 X\phi$	
$\mathcal{O}_{n\phi}$	$(\phi^\dagger\phi)(\bar{l}_p n_r\tilde{\phi})$	$\mathcal{O}_{\phi n}$	$i(\phi^\dagger \overleftrightarrow{D}_\mu\phi)(\bar{n}_p\gamma^\mu n_r)$	$\mathcal{O}_{nW}$	$(\bar{\ell}_p\sigma^{\mu\nu}n_r)\tau^I\tilde{\phi}W^I_{\mu\nu}$
		$\mathcal{O}_{\phi ne}$	$i(\tilde{\phi}^\dagger D_\mu\phi)(\bar{n}_p\gamma^\mu e_r)$	$\mathcal{O}_{nB}$	$(\bar{\ell}_p\sigma^{\mu\nu}n_r)\tilde{\phi}B_{\mu\nu}$

(a) $\mathcal{B}$- and L- conserving dimension-six operators in the νSMEFT in the Warsaw basis notation.

$\Delta\mathcal{B} = \Delta L = 1$ + h.c.	
$\mathcal{O}_{qqdn}$	$\epsilon^{\alpha\beta\gamma}\epsilon_{ij}(q_p^{i\alpha\,T}Cq_r^{j\beta})(d_s^{\gamma\,T}Cn_t)$
$\mathcal{O}_{uddn}$	$\epsilon^{\alpha\beta\gamma}(u_p^{\alpha\,T}Cd_r^{\beta})(d_s^{\gamma\,T}Cn_t)$

(b) $\mathcal{B}$- and L- violating dimension-six operators in the νSMEFT.

$\psi^2 X$ and $\psi^2\phi^2$	
$\mathcal{O}_{nnB}$	$(n_p^T C\sigma^{\mu\nu}n_r)B_{\mu\nu}$
$\mathcal{O}_{nn\phi\phi}$	$(n_p^T Cn_r)\phi^\dagger\phi$

(c) L- violating dimension-five operators in the νSMEFT.

Table 1: νSMEFT operator basis up to dimension-six.

The remaining Warsaw basis operators which are also part of the νSMEFT can be found in [30]. The complete set of νSMEFT operators in `WCxf` conventions with explicit flavour indices will be published on the `WCxf` webpage.

2.2 Choice of Flavour Basis

Extending the SM with three RH neutrino fields n_p (calling the resulting model to be νSM), the maximal flavour symmetry group possessed by the νSM is then

$$\mathrm{SU(3)_q} \otimes \mathrm{SU(3)_u} \otimes \mathrm{SU(3)_d} \otimes \mathrm{SU(3)}_\ell \otimes \mathrm{SU(3)_e} \otimes \mathrm{SU(3)_n}. \tag{2.3}$$

Two-fermion	Four-fermion
$C^{n\phi} = U^\dagger_{e_L} C'^{n\phi} U_{n_R}$	$(C^{nd})_{f_1f_2f_3f_4} = (U_{n_R})_{g_2f_2}(U_{d_R})_{g_4f_4}(U_{n_R})^*_{g_1f_1}(U_{d_R})^*_{g_3f_3}(C'^{nd})_{g_1g_2g_3g_4}$
$C^{\phi n} = U^\dagger_{n_R} C'^{\phi n} U_{n_R}$	$(C^{nu})_{f_1f_2f_3f_4} = (U_{n_R})_{g_2f_2}(U_{u_R})_{g_4f_4}(U_{n_R})^*_{g_1f_1}(U_{u_R})^*_{g_3f_3}(C'^{nu})_{g_1g_2g_3g_4}$
$C^{\phi ne} = U^\dagger_{n_R} C'^{\phi ne} U_{e_R}$	$(C^{ne})_{f_1f_2f_3f_4} = (U_{n_R})_{g_2f_2}(U_{e_R})_{g_4f_4}(U_{n_R})^*_{g_1f_1}(U_{e_R})^*_{g_3f_3}(C'^{ne})_{g_1g_2g_3g_4}$
$C^{nW} = U^\dagger_{e_L} C'^{nW} U_{n_R}$	$(C^{nn})_{f_1f_2f_3f_4} = (U_{n_R})_{g_2f_2}(U_{n_R})_{g_4f_4}(U_{n_R})^*_{g_1f_1}(U_{n_R})^*_{g_3f_3}(C'^{nn})_{g_1g_2g_3g_4}$
$C^{nB} = U^\dagger_{e_L} C'^{nB} U_{n_R}$	$(C^{nedu})_{f_1f_2f_3f_4} = (U_{e_R})_{g_2f_2}(U_{u_R})_{g_4f_4}(U_{n_R})^*_{g_1f_1}(U_{d_R})^*_{g_3f_3}(C'^{nedu})_{g_1g_2g_3g_4}$
$C^{nnB} = U^T_{n_R} C'^{nnB} U_{n_R}$	$(C^{qn})_{f_1f_2f_3f_4} = (U_{d_L})_{g_2f_2}(U_{n_R})_{g_4f_4}(U_{d_L})^*_{g_1f_1}(U_{n_R})^*_{g_3f_3}(C'^{qn})_{g_1g_2g_3g_4}$
$C^{nn\phi\phi} = U^T_{n_R} C'^{nn\phi\phi} U_{n_R}$	$(C^{\ell n})_{f_1f_2f_3f_4} = (U_{e_L})_{g_2f_2}(U_{n_R})_{g_4f_4}(U_{e_L})^*_{g_1f_1}(U_{n_R})^*_{g_3f_3}(C'^{\ell n})_{g_1g_2g_3g_4}$
	$(C^{\ell n\ell e})_{f_1f_2f_3f_4} = (U_{n_R})_{g_2f_2}(U_{e_R})_{g_4f_4}(U_{e_L})^*_{g_1f_1}(U_{e_L})^*_{g_3f_3}(C'^{\ell n\ell e})_{g_1g_2g_3g_4}$
	$(C^{\ell nqd(1)})_{f_1f_2f_3f_4} = (U_{n_R})_{g_2f_2}(U_{d_R})_{g_4f_4}(U_{e_L})^*_{g_1f_1}(U_{d_L})^*_{g_3f_3}(C'^{\ell nqd(1)})_{g_1g_2g_3g_4}$
	$(C^{\ell nqd(3)})_{f_1f_2f_3f_4} = (U_{n_R})_{g_2f_2}(U_{d_R})_{g_4f_4}(U_{e_L})^*_{g_1f_1}(U_{d_L})^*_{g_3f_3}(C'^{\ell nqd(3)})_{g_1g_2g_3g_4}$
	$(C^{\ell nuq})_{f_1f_2f_3f_4} = (U_{n_R})_{g_2f_2}(U_{d_L})_{g_4f_4}(U_{e_L})^*_{g_1f_1}(U_{u_R})^*_{g_3f_3}(C'^{\ell nuq})_{g_1g_2g_3g_4}$
	$(C^{qqdn})_{f_1f_2f_3f_4} = (U_{d_L})_{g_2f_2}(U_{n_R})_{g_4f_4}(U_{d_L})^*_{g_1f_1}(U_{d_R})^*_{g_3f_3}(C'^{qqdn})_{g_1g_2g_3g_4}$
	$(C^{uddn})_{f_1f_2f_3f_4} = (U_{d_R})_{g_2f_2}(U_{n_R})_{g_4f_4}(U_{u_R})^*_{g_1f_1}(U_{d_R})^*_{g_3f_3}(C'^{uddn})_{g_1g_2g_3g_4}$

Table 2: Definitions (preserving $SU(3)^6$) for the νSMEFT WCs of operators involving fermions in the down-basis (i.e. obtained by setting $U_q = U_{d_L},\ U_\ell = U_{e_L},\ U_u = U_{u_R},\ U_d = U_{d_R},\ U_e = U_{e_R}$ and $U_n = U_{n_R}$).

The flavour symmetry transformations can be defined through:

$$\begin{aligned} q &\to U_q q\,, & \ell &\to U_\ell \ell\,, \\ u &\to U_u u\,, & d &\to U_d d\,, \\ e &\to U_e e\,, & n &\to U_n n\,, \end{aligned} \tag{2.4}$$

with U_ψ to be unitary matrices. The $\mathrm{SU(3)_n}$ flavour symmetry can be broken down to $\mathrm{O(3)_n}$ by the Majorana mass term for the neutrinos (see (2.1)). The remaining symmetry of νSM is broken by the Yukawas terms [33]. In the absence of Majorana mass term, νSMEFT also possess the full flavour symmetry (2.3) up to redefinitions of the WCs. Thus the flavour basis of νSMEFT is not unique.

Two convenient choices of bases are defined by assigning specific values to U_q and U_ℓ:

$$U_q = U_{d_L}\,, \qquad U_\ell = U_{e_L} \qquad \text{(down-basis)}\,, \tag{2.5}$$

$$U_q = U_{u_L}\,, \qquad U_\ell = U_{\nu_L} \qquad \text{(up-basis)}. \tag{2.6}$$

In the down-basis (up-basis), the down-type (up-type) Yukawa matrices take a diagonal form. In Table. 2, we show the redefinitions of the νSMEFT WCs in the down-basis convention. The corresponding redefinitions for the pure SMEFT operators can be found in Ref. [34]. As a result in these two bases, the only unknown parameters are the WCs, the SM parameters, and $\hat{Y}_n$. In the current implementation of the νSMEFT in `wilson` we have adopted the down-basis convention assuming no Majorana mass term.

3 Evolution in the νSMEFT

The RG running in the νSMEFT from the NP scale to the EW scale is controlled by:

$$\dot{C}_i(\mu) = 16\pi^2 \mu \frac{d}{d\mu} C_i(\mu) = \hat{\gamma}_{ij}(g_1, g_2, g_3, \hat{Y}_\psi) C_j(\mu)\,, \tag{3.1}$$

where μ is the renormalization scale, and g_i and $\hat{Y}_\psi$ are the gauge couplings and Yukawa matrices for the quarks, leptons, and sterile neutrinos. The latter gives rise to a neutrino Dirac mass term after EW symmetry breaking (EWSB). We implemented the full gauge and Yukawa dependence of the ADM $\hat{\gamma}_{ij}$. For the code implementation, the explicit expressions for $\hat{\gamma}_{ij}$ are taken from [14] (gauge-coupling dependence) and [15] (Yukawa dependence)[1]. The running of the Baryon number violating operators was taken from [31]. For the purpose of implementation in `wilson`, to match the `WCxf` convention, we have used the conjugate of Yukawas (denoted by $\hat{Y}_\psi$) as compared to Eq. (2.2) (corresponding to the original convention of Ref. [15]). The structure of the ADMs within the νSMEFT due to gauge couplings exhibits a block structure for the SMEFT and νSMEFT specific operators, meaning the corresponding operators mix only among themselves. However, the Yukawa-dependent ADMs (or the ADMs depending upon the combination of Yukawa and gauge couplings) also introduce mixing between pure νSMEFT and pure SMEFT operators.

For the purpose of RG evolution within the νSMEFT, all dimension-four parameters are required at the input scale. In contrast to the SMEFT case, where all dimension-four parameters at the NP scale can be determined from their corresponding SM values at the EW scale, the neutrino Yukawa couplings $\hat{Y}_n$ in the νSMEFT

[1] While this work was in preparation, in a recent study [17] the missing Yukawa terms in the ADMs have been computed. These will be included in the future update of the `wilson` package.

belong to the unknown NP parameter category, which must be inputted along with the νSMEFT WCs to solve the above RGEs. At dimension-six level $\hat{Y}_n$ also receives corrections from the $C_{n\phi}$ WC. Also, one can add a dimension-four Majorana mass term, in addition to the dimension-five part originating within SMEFT from the Weinberg operator. But such terms do not affect the νSMEFT ADMs directly [12, 14, 16], in most cases.

4 νSMEFT implementation in `wilson`

The inclusion of νSMEFT evolution in `wilson` has the following components:

1. Addition of the νSMEFT as a new EFT in the `WCxf` format.
2. Addition of a basis file for the νSMEFT in the `WCxf` format. In `wilson` we continue to call this the `Warsaw` basis, as this basis has been inspired by the corresponding SMEFT Warsaw basis.
3. Addition of the νSMEFT ADMs in `wilson`.

Apart from that, we have made important changes to the `wilson` package. The major one is replacing the `SMEFT` class with the `EFTEvolve` class. While the former was dedicated to the SMEFT evolution, the latter is designed to be able to perform the RG evolution within all three EFTs: SMEFT, νSMEFT, and WET. In the current version, we do not utilize it for the WET evolution, which is kept as it was. In the forthcoming versions, we plan to use `EFTEvolve` for the WET as well. The main difference between the two classes is that the `__init__` method of the `EFTEvolve` class also requires the *beta-function* in form of a dictionary, in addition to the `WCxf` instance representing a parameter point in the EFT space. At the user level, the evolution within the νSMEFT can be performed using a few lines of code:

```
from wilson import Wilson
mywilson = Wilson({'nd_1111': 1e-6, 'lnle_1111':1e-6},
                   scale=1e3, eft='nuSMEFT', basis='Warsaw')
mywilson.match_run(91, 'nuSMEFT', 'Warsaw')
```

where the values of input dimension-six WCs need to be specified in units of GeV^{-2}.

Unlike in the SMEFT, the dimension-four parameter $\hat{Y}_n$ has to be provided as an input parameter at the input scale. This can be done by the command `set_option`, with the option `yukawa_scale_in`, which allows the user to input any Yukawa matrix ($\hat{Y}_\psi,\ \psi = u, d, e, n$). Its usage is demonstrated below:

```python
from wilson import Wilson
import numpy
mywilson = Wilson({'nd_1111': 1e-6, 'lnle_1111':1e-6},
                        scale=1e3, eft='nuSMEFT', basis='Warsaw')
mywilson.set_option('yukawa_scale_in', {'Gn': numpy.eye(3), 'Gu':
    numpy.zeros((3,3))})
```

The Yukawa matrices (represented by Gψ within the code) are given in a Python dictionary, with the key being the name and its values passed on as a `numpy` array. In this example, as an illustration, we input only two Yukawa matrices, where we set $\hat{Y}_n$ as a 3×3 matrix with diagonal entries equal to 1, and $\hat{Y}_u$ is a 3×3 null matrix. If not specified, the SM Yukawas $\hat{Y}_u, Y_d, Y_e$ are internally determined within `wilson`, while $\hat{Y}_n$ is set to zero.

Similarly, another option named `gauge_higgs_scale_in` for the command `set_option` allows the user to enter gauge couplings g_1, g_2 and g_3 (denoted by `gp, g` and `gs`, respectively in the program), and Higgs parameters m_H^2 and λ (denoted by `m2` and `Lambda`, respectively in the program) in the form of a dictionary. Its usage is demonstrated in the following code snippet:

```python
from wilson import Wilson
import numpy
mywilson = Wilson({'nd_1111': 1e-6, 'lnle_1111':1e-6},
                        scale=1e3, eft='nuSMEFT', basis='Warsaw')
mywilson.set_option('gauge_higgs_scale_in', {'g': 0, 'gp': 1, 'gs':
    0.5, 'Lambda': 100, 'm2': 100})
```

The values of m_H^2 and Λ have to be given in the units of GeV^2 and GeV, respectively.

5 Comparison and cross-checks

To test the proper functioning of the new code, we made several checks by running internal test functions, as well as comparing the output with known results. Two major checks of the output were performed for the case of the SMEFT and the νSMEFT, as only the corresponding part of the code was upgraded (the WET EFT remains unmodified).

5.1 SMEFT evolution

To test the proper functioning of our upgraded code for the SMEFT, we compare the results from our new version of the code with the previous version. For this purpose dictionaries of all 1635 SMEFT Wilson coefficients in the Warsaw basis (including all the non-redundant combinations of the indices) with randomly generated input values at $\Lambda = 1$ TeV were generated. The running is then performed to the EW scale using the new and the old versions of the implementation. As expected, the output matched precisely, confirming the stable working of the new code for the SMEFT case.

5.2 νSMEFT evolution

To verify the results of the novel implementation for the νSMEFT, we have reproduced the results of the article [14]. For this test, we set the indices $prst = 1111$ and list the 16×16 νSMEFT WCs in the basis

$$\vec{\mathcal{C}} = \{\mathcal{C}_{nd},\, \mathcal{C}_{nu},\, \mathcal{C}_{ne},\, \mathcal{C}_{qn},\, \mathcal{C}_{\ell n},\, \mathcal{C}_{\phi n},\, \mathcal{C}_{n\phi},\, \mathcal{C}_{nW},\, \mathcal{C}_{nB},\, C^{(1)}_{\ell nqd},\, C^{(3)}_{\ell nqd},\, \mathcal{C}_{nedu},\, \mathcal{C}_{\ell n\ell e},\, \mathcal{C}_{\ell nuq},\, \mathcal{C}_{\phi ne},\, \mathcal{C}_{nn}\}\,. \tag{5.1}$$

The 16 WCs at the EW scale and at $\Lambda = 1$ TeV are then related by

$$\frac{\delta\mathcal{C}(M_Z)}{10^{-3}} = \left(\begin{array}{cccccccccccccccc}
-0.89 & 1.77 & -0.89 & 0.89 & -0.89 & 0.44 & & & & & & & & & & \\
1.77 & -3.54 & 1.77 & -1.77 & 1.77 & -0.89 & & & & & & & & & & \\
-2.66 & 5.32 & -2.66 & 2.66 & -2.66 & 1.33 & & & & & & & & & & \\
0.44 & -0.89 & 0.44 & -0.44 & 0.44 & -0.22 & & & & & & & & & & \\
-1.33 & 2.66 & -1.33 & 1.33 & -1.33 & 0.66 & & & & & & & & & & \\
1.33 & -2.66 & 1.33 & -1.33 & 1.33 & -0.66 & & & & & & & & & & \\
 & & & & & & 46.26 & 39.39 & -8.9 & & & & & & & \\
 & & & & & & 0 & 7.17 & 5.27 & & & & & & & \\
 & & & & & & 0 & 15.81 & -1.19 & & & & & & & \\
 & & & & & & & & & 136.72 & -107.42 & & & & & \\
 & & & & & & & & & -2.24 & -25.01 & & & & & \\
 & & & & & & & & & & & 7.97 & & & & \\
 & & & & & & & & & & & & -0.31 & & & \\
 & & & & & & & & & & & & & 138.71 & & \\
 & & & & & & & & & & & & & & 5.98 & \\
 & & & & & & & & & & & & & & & 0
\end{array}\right)\mathcal{C}(\Lambda)\,, \tag{5.2}$$

The results shown in this matrix agree well with those given in [14]. The running effects in the 6×6 and 3×3 blocks are small because only EW gauge couplings contribute. The mixing in the 2×2 block is large as it is governed by QCD running.

6 Summary

The lack of observations of new particles at the LHC indicates the scale of NP to be well above the EW scale. Potential BSM effects can then be encoded in terms of Wilson coefficients of the SMEFT in a general manner. However, the field content of the SMEFT is restricted to SM particles, which makes it unfit for certain NP models containing light sterile neutrino states. Adding such fields to the SMEFT results in new higher-dimensional operators starting at the dimension-five and dimension-six levels. Those operators can mix within themselves as well as with the standard SMEFT operators at the one-loop level. While anomalous dimensions for such mixing terms are known, including their effects in the physical observables requires solving the corresponding RGEs. For this purpose, a careful consideration of properly evaluating the SM parameters at the NP scale is a must, but often ignored in phenomenological studies, where typically only first leading log solutions are considered.

In this work, we have presented an upgrade of the `wilson` package, which is the first public code that includes the full numerical evolution of the νSMEFT parameters including subtle effects such as back-rotation of flavour bases. For the purpose of RG running within the νSMEFT, the neutrino Yukawa matrix has to be provided along with the Wilson coefficients while all other dimension-four parameters are internally evaluated within `wilson`, unless provided by the user. This implementation allows to include RG effects in studies related to light sterile neutrino particles and their correlations with other sectors such as flavour physics. In the future, we plan to include the WET augmented with sterile neutrinos in `wilson`, together with the corresponding matching conditions from the νSMEFT and a proper treatment of neutrino masses and mixing.

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References

[1] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler, J. a. P. Pinheiro, and T. Schwetz, *NuFit-6.0: Updated global analysis of three-flavor neutrino oscillations*, arXiv:2410.05380.

[2] W. Buchmuller and D. Wyler, *Effective Lagrangian Analysis of New Interactions and Flavor Conservation*, *Nucl. Phys. B* **268** (1986) 621–653.

[3] I. Brivio and M. Trott, *The Standard Model as an Effective Field Theory*, *Phys. Rept.* **793** (2019) 1–98, [arXiv:1706.08945].

[4] G. Isidori, F. Wilsch, and D. Wyler, *The standard model effective field theory at work*, *Rev. Mod. Phys.* **96** (2024), no. 1 015006, [arXiv:2303.16922].

[5] F. del Aguila, S. Bar-Shalom, A. Soni, and J. Wudka, *Heavy Majorana Neutrinos in the Effective Lagrangian Description: Application to Hadron Colliders*, *Phys. Lett. B* **670** (2009) 399–402, [arXiv:0806.0876].

[6] A. Aparici, K. Kim, A. Santamaria, and J. Wudka, *Right-handed neutrino magnetic moments*, *Phys. Rev. D* **80** (2009) 013010, [arXiv:0904.3244].

[7] S. Bhattacharya and J. Wudka, *Dimension-seven operators in the standard model with right handed neutrinos*, *Phys. Rev. D* **94** (2016), no. 5 055022, [arXiv:1505.05264]. [Erratum: Phys.Rev.D 95, 039904 (2017)].

[8] Y. Liao and X.-D. Ma, *Operators up to Dimension Seven in Standard Model Effective Field Theory Extended with Sterile Neutrinos*, *Phys. Rev. D* **96** (2017), no. 1 015012, [arXiv:1612.04527].

[9] I. Bischer and W. Rodejohann, *General neutrino interactions from an effective field theory perspective*, *Nucl. Phys. B* **947** (2019) 114746, [arXiv:1905.08699].

[10] J. Alcaide, S. Banerjee, M. Chala, and A. Titov, *Probes of the Standard Model effective field theory extended with a right-handed neutrino*, *JHEP* **08** (2019) 031, [arXiv:1905.11375].

[11] T. Han, J. Liao, H. Liu, and D. Marfatia, *Right-handed Dirac and Majorana neutrinos at Belle II*, *JHEP* **04** (2023) 013, [arXiv:2207.07029]. [Erratum: JHEP 09, 016 (2023)].

[12] A. Datta, H. Liu, and D. Marfatia, *$B^-\to D(*)\ell X^-$ decays in effective field theory with massive right-handed neutrinos*, *Phys. Rev. D* **106** (2022), no. 1 L011702, [arXiv:2204.01818].

[13] M. Chala and A. Titov, *One-loop running of dimension-six Higgs-neutrino operators and implications of a large neutrino dipole moment*, *JHEP* **09** (2020) 188, [arXiv:2006.14596].

[14] A. Datta, J. Kumar, H. Liu, and D. Marfatia, *Anomalous dimensions from gauge couplings in SMEFT with right-handed neutrinos*, *JHEP* **02** (2021) 015, [arXiv:2010.12109].

[15] A. Datta, J. Kumar, H. Liu, and D. Marfatia, *Anomalous dimensions from Yukawa couplings in SMNEFT: four-fermion operators*, *JHEP* **05** (2021) 037, [arXiv:2103.04441].

[16] K. Fuyuto, J. Kumar, E. Mereghetti, S. Sandner, and C. Sun, *Sterile neutrino dark matter within the νSMEFT*, *JHEP* **09** (2024) 042, [arXiv:2405.00119].

[17] M. Ardu and X. Marcano, *Completing the one-loop νSMEFT renormalization group evolution*, *JHEP* **10** (2024) 212, [arXiv:2407.16751].

[18] V. Cirigliano, W. Dekens, J. de Vries, K. Fuyuto, E. Mereghetti, and R. Ruiz, *Leptonic anomalous magnetic moments in ν SMEFT*, *JHEP* **08** (2021) 103, [arXiv:2105.11462].

[19] J. Aebischer, J. Kumar, and D. M. Straub, *Wilson: a Python package for the running and matching of Wilson coefficients above and below the electroweak scale*, *Eur. Phys. J. C* **78** (2018), no. 12 1026, [arXiv:1804.05033].

[20] E. E. Jenkins, A. V. Manohar, and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators I: Formalism and lambda Dependence*, *JHEP* **10** (2013) 087, [arXiv:1308.2627].

[21] E. E. Jenkins, A. V. Manohar, and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators II: Yukawa Dependence*, *JHEP* **01** (2014) 035, [arXiv:1310.4838].

[22] R. Alonso, E. E. Jenkins, A. V. Manohar, and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators III: Gauge Coupling Dependence and Phenomenology*, *JHEP* **04** (2014) 159, [arXiv:1312.2014].

[23] J. Aebischer, A. Crivellin, M. Fael, and C. Greub, *Matching of gauge invariant dimension-six operators for $b \to s$ and $b \to c$ transitions*, *JHEP* **05** (2016) 037, [arXiv:1512.02830].

[24] E. E. Jenkins, A. V. Manohar, and P. Stoffer, *Low-Energy Effective Field Theory below the Electroweak Scale: Operators and Matching*, *JHEP* **03** (2018) 016, [arXiv:1709.04486]. [Erratum: JHEP 12, 043 (2023)].

[25] W. Dekens and P. Stoffer, *Low-energy effective field theory below the electroweak scale: matching at one loop*, *JHEP* **10** (2019) 197, [arXiv:1908.05295]. [Erratum: JHEP 11, 148 (2022)].

[26] J. Aebischer, M. Fael, C. Greub, and J. Virto, *B physics Beyond the Standard Model at One Loop: Complete Renormalization Group Evolution below the Electroweak Scale*, *JHEP* **09** (2017) 158, [arXiv:1704.06639].

[27] E. E. Jenkins, A. V. Manohar, and P. Stoffer, *Low-Energy Effective Field Theory below the Electroweak Scale: Anomalous Dimensions*, *JHEP* **01** (2018) 084, [arXiv:1711.05270]. [Erratum: JHEP 12, 042 (2023)].

[28] J. Aebischer and J. Kumar, *Flavour violating effects of Yukawa running in SMEFT*, *JHEP* **09** (2020) 187, [arXiv:2005.12283].

[29] J. Aebischer et al., *WCxf: an exchange format for Wilson coefficients beyond the Standard Model*, *Comput. Phys. Commun.* **232** (2018) 71–83, [arXiv:1712.05298].

[30] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, *Dimension-Six Terms in the Standard Model Lagrangian*, *JHEP* **10** (2010) 085, [arXiv:1008.4884].

[31] R. Alonso, H.-M. Chang, E. E. Jenkins, A. V. Manohar, and B. Shotwell, *Renormalization group evolution of dimension-six baryon number violating operators*, *Phys. Lett. B* **734** (2014) 302–307, [arXiv:1405.0486].

[32] J. Aebischer, W. Altmannshofer, E. E. Jenkins, and A. V. Manohar, *Dark matter effective field theory and an application to vector dark matter*, *JHEP* **06** (2022) 086, [arXiv:2202.06968].

[33] V. Cirigliano, B. Grinstein, G. Isidori, and M. B. Wise, *Minimal flavor violation in the lepton sector*, *Nucl. Phys. B* **728** (2005) 121–134, [hep-ph/0507001].

[34] A. Dedes, W. Materkowska, M. Paraskevas, J. Rosiek, and K. Suxho, *Feynman rules for the Standard Model Effective Field Theory in R_ξ -gauges*, *JHEP* **06** (2017) 143, [arXiv:1704.03888].