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UPDATE OF THE PLACET2 CODE FOR THE LOW-ENERGY ACCELERATION STAGES OF THE MUON COLLIDER

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Abstract

This work describes improvements made to the tracking code PLACET2 to make it possible to simulate the acceleration from 250 MeV to 63 GeV in a future muon collider. This software was selected because of its unique ability to optimally simulate recirculating linacs, which are part of the proposed layout for this initial muon acceleration stage. PLACET2 has been updated to simulate non-ultrarelativistic particles and to consider particle beams of different species, charges and masses. The main changes were introduced in the longitudinal dynamics, synchrotron radiation and wakefield descriptions. In addition, the decay of particles has been added as a new feature. The changes were benchmarked in different tests with RF-Track, a code able to simulate low energy muon beams and their decay. Finally, the lattice of the 16.6 GeV arc in the initial acceleration stage of the muon collider was simulated with both PLACET2 and RF-Track, providing another test. All the results showed excellent agreement between both codes, verifying the implementation in PLACET2.

INTRODUCTION

PLACET [1], a CERN multi-bunch tracking code developed in the 2000s to model large scale accelerators, was expanded into PLACET2 [2], providing the code with an exclusive capability to simulate recirculating machines with complex topologies. Instead of defining a lattice for each bunch, PLACET2 allows to define a single lattice, running bunches simultaneously and considering long range bunch-to-bunch effects in a realistic way. PLACET2 has been used in the design of the CLIC Drive Beam Recombination Complex (DBRC) and energy recovery linacs (ERLs) such as LHeC and PERLE [3]. Its next objective, is to simulate the recirculating linacs (RLAs) of the initial acceleration stage of the future muon collider.

The present proposal for a future muon collider [4] considers an acceleration stage, from 250 MeV to 63 GeV, based on a combination of a single-pass linac and two different RLAs [5]. The layout was proposed as an optimal architecture to use Superconducting RF (SFR) cavities, which are very profitable to perform a fast acceleration in bunches with large emittances [6]. PLACET2's ability to track bunches in complex topologies positioned it as the optimal choice for simulating the proposed layout. However, a wide update was required, as it was designed for the simulation of ultrarelativistic electrons instead of non-ultrarelativistic and decaying muons. In addition, the bunches have switched signed charges [5], so the bunch charge also had to be added as a variable.

The present work details the performed updates to PLACET2, as well as the benchmarks that allowed to verify the implementations. After this work, the code is now suitable for the simulation of the initial acceleration stage of the muon collider.

PERFORMED UPDATES

Reformulation of the Previous Expressions

In order to update the code to simulate non-ultrarelativisitc muon bunches, the following statements had to be considered:

- Regarding the relativistic factors, β can not be approximated by 1 and γ can reach small values.
- The velocities of the particles are no longer the same. In particular, the reference particle has a different velocity (v_0) from any other particle.
- The approximations which consider that the momentum or velocity are longitudinal, $p \approx p_z$, $v \approx v_z$, do not model correctly the reality. Therefore, several Taylor approximations performed in the code which neglected the normalized transversal momentum magnitudes $x' = p_x/p$ and $y' = p_y/p$, are not accurate anymore.
- To simulate bunches of different species and charges, the particle mass and charge were set as variables.

All the code expressions were extensively revised so that the above statements were taken into account. Detailing all the updates is out of the scope of this work, but some examples will be shown.

Example 1: Longitudinal Dynamics of a Drift The longitudinal coordinate of particles that travel through a drift of length L is modified as follows:

$$\Delta t = \frac{L}{v}\sqrt{1 + x'^2 + y'^2} - \frac{L}{v_0}. \quad (1)$$

However, for ultrarelativistic electrons, all the particles have the same velocity ($v = c$) and $x', y' << 1$, so Eq. (1) can be approximated as:

$$\Delta t = \frac{L}{2c}\left(x'^2 + y'^2\right). \quad (2)$$

Eq. (2) was the one implemented in PLACET2, but it does not describe correctly non-ultrarelativistic muons, so Eq.

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(1) was taken as the new expression. The longitudinal dynamics is highly affected by the non-ultrarelativistic regime and the tracking expressions were also updated for dipoles, quadrupoles, sextupoles, multipoles and rf-cavities.

Example 2: Synchrotron Radiation PLACET2 models coherent synchrotron radiation (CSR). Even though synchrotron radiation is less important for muons than for electrons, it should be still considered in the arcs. An example of an update was the CSR slippage length [7]:

$$L_s = \frac{\rho\phi^3}{24} + \frac{\rho\phi}{2\gamma^2}, \tag{3}$$

where ρ is the bending radius and ϕ the bending angle. The previous version of PLACET2 [8] neglected the second term, dependant on $1/\gamma^2$, whereas the current one uses the complete Eq. (3). In addition, the power loss caused by synchrotron radiation depends on $1/m^4$, where m is the mass of the particle, which now is a variable and not a constant.

Example 3: Long Range Transversal Wakefields In the high-intensity beams of the RLAs of the muon collider, long-range wakefields are expected to highly affect the beams. Their effect in the tracking of a particle is the following:

$$\Delta x' = \frac{\Delta p_x}{p_z} = \frac{q\Re(h)}{p_z}\frac{1}{v_z}, \tag{4}$$

where q is the charge of the particle, h is the horizontal mode that excites the bunch and $\Re(h)$ is its real part. $q\Re(h)/v_z$ represents the horizontal momentum kick imparted by the wakefield source (Δp_x). For ultrarelativistic electrons, $p_z \approx p$, $v_z \approx v = c$ and $q = e$, giving the following expression:

$$\Delta x' = \frac{e\Re(h)}{p}\frac{1}{c}. \tag{5}$$

The previous implementation used Eq.(5), whereas now it includes Eq.(4). In addition, wakefields itself are a function of the longitudinal coordinate z in meters, $W(z)$. However, PLACET2 longitudinal coordinate has units of time, (t), so its formulation for non-ultrarelativistic muons must include the relativistic β, $W(z) = W(\beta ct)$.

Muon Decay: New Feature

In order to simulate muons, their probability of decaying has to be considered. The decay has been applied to each macroparticle, such that the number of real particles inside the macroparticle is decreased with time. The decay has been modelled as follows:

$$N_f = N_0 \exp^{-\Delta t_{\mathrm{lab}}/(\gamma\tau)} \tag{6}$$

where N_f is the final and N_0 is the initial number of particles in the macroparticle, γ is the relativistic factor of the macroparticle, Δt_{lab} is the time elapsed in the laboratory reference frame and τ is the average lifetime of the selected type of particle.

RESULTS

Necessity of Non-Ultrarelativistic Dynamics

Since the muon collider initial acceleration complex starts with muons with a kinetic energy of 250 MeV, corresponding to $\beta = 0.9548$ and $\gamma = 3.366$, the previous relativistic parameters are no longer in the ultrarelativistic regime. In addition, the muon beam is characterized by a very large momentum spread and dispersion in the longitudinal coordinate, as shown in Table 1.

Table 1: Muon Beam Characteristics After the Final Cooling Currently being used for Initial Acceleration Studies [5]

Kinetic energy (Ekin)	250 MeV
Transversal normalized emittance ($\epsilon_\perp$)	22.5 µm
RMS of longitudinal coordinate (σ_z)	0.375 m
RMS relative momentum spread (σ_P/P)	0.09

The longitudinal beam dynamics of a bunch characterised with the parameters of Table 1 are highly affected by the distinction between ultrarelativistic and non-ultrarelativistic regime.

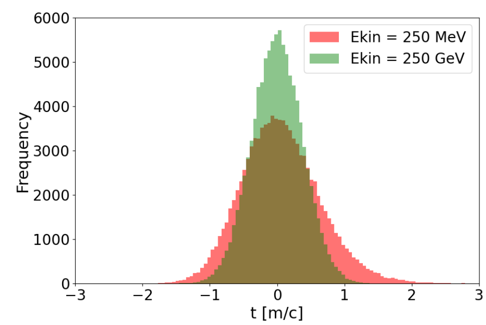

Figure 1: Comparison of the longitudinal phase space after passing a 50 m-long drift between ultrarelativistic muons (in green, with a kinetic energy of 250 GeV) and relativistic muons (in red, 250 MeV). The rest of beam parameters replicate the conditions after the final cooling of the muon collider in Table 1.

Figure 1 shows that the non-ultrarelativistic bunch is about 2 m/c wider in the longitudinal coordinate than the ultrarelativistic bunch after passing a 50 m-long drift, which is a small fraction of the length of the lattice of any of the arcs of the RLAs in the muon collider.

Benchmark 1: Simulation of the First Droplet Arc of the Second Recirculating Linac of the Muon Collider

In order to verify the performed updates, a lattice with non-ultrarelativistic muons was simulated with PLACET2 and with RF-Track [9], a tracking code developed at CERN which can handle non-ultrarelativistic particles. The chosen lattice was the 16.6 GeV first droplet arc of the second RLA of the muon collider (ARC1). This lattice is 300 m-long,

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non-linear, as it includes 135 sextupoles, and is composed by 200 quadrupoles, 100 dipoles and 428 drifts [5], providing a complete benchmark.

Phase-space Coordinates As a verification of the performed updates in the beam dynamics, the phase-space coordinates obtained after the ARC1 lattice for both codes were compared. Regarding the longitudinal beam dynamics,

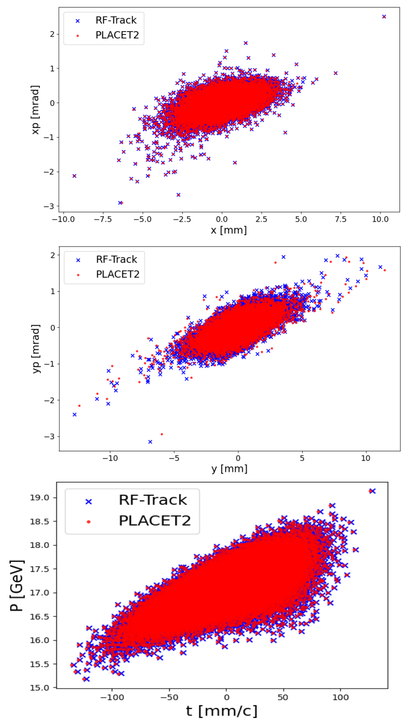

Figure 2: From top to bottom: Horizontal, vertical and longitudinal phase space coordinates after the passing ARC1 with RF-Track and PLACET2.

Fig. 2 shows an excellent agreement considering the length of the lattice: the largest discrepancy in the results is in the order of $\Delta t = 3$ mm/c for a 300 m-long lattice. Furthermore, both codes provide almost the same results in the transversal beam dynamics, with more visible discrepancies in the vertical plane. These differences are caused by second order corrections in dipole fringes, only included in RF-Track and planned upgrade for PLACET2.

Beta Functions For verifying the implementation in the transversal beam dynamics, the horizontal and vertical beta functions (β_x, β_y) along the lattice have been analyzed. Figure 3 shows excellent agreement between both codes, with very similar β functions along the 300 m of lattice, providing an excellent benchmark of the performed updates.

Benchmark 2: Muon Decay

The implementation of the muon decay could also be verified with RF-Track, which has a slightly different approach: it decays the entire macroparticle, instead of reducing the number of particles it has inside. This time, the test was

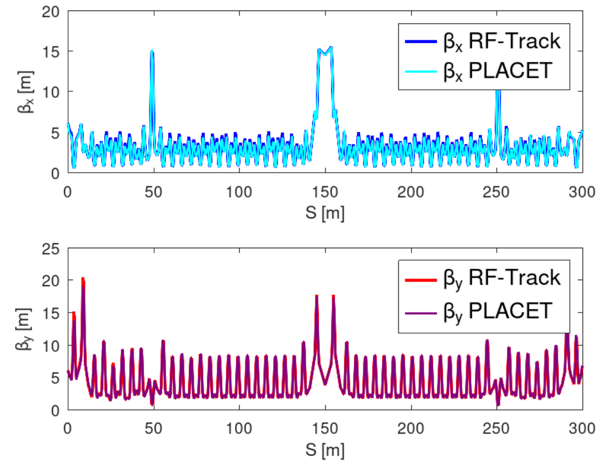

Figure 3: Horizontal (above) and vertical (below) beta functions along ARC1 with RF-Track and PLACET2.

performed through many turns of a FODO lattice. As Fig. 4 shows, the results are almost the same, with slight differences attributed to the difference in the implementations.

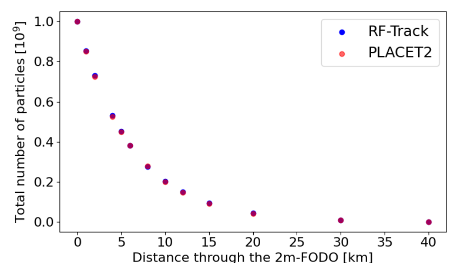

Figure 4: Remaining muons in a 1 GeV beam after several turns over a 2 m-long FODO lattice with RF-Track and PLACET2.

CONCLUSION

The updates performed to PLACET2, have enabled the code to simulate non-ultrarelativistic and decaying muons. It has been proven that this update was needed to simulate correctly the bunches in the initial acceleration stage of the muon collider. In addition, the performed updates have also been extensively benchmarked.

As future work, the beam in the initial acceleration of the muon collider is expected to have very high intensity, so intra-beam scattering, space-charge, beam-loading and beam-beam effects are very likely to impact the performance. Therefore, these effects should be included in the simulation. Backwards tracking should also be considered, as the bunches of μ^+ and μ^- travel in opposite directions in the arcs.

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