SEARCHING FOR THE BEST INITIAL BEAM PARAMETERS FOR EFFICIENT MUON IONIZATION COOLING

B. Stechauner*,[1], E. Fol, D. Schulte, CERN, Geneva, Switzerland
C. Rogers, Rutherford Appleton Laboratory, Didcot, United Kingdom
J.Schieck[1], Institute of High Energy Physics, Vienna, Austria
[1]also at TU Wien, Vienna, Austria

Abstract

Ionization cooling stands as the only cooling technique capable of efficiently reducing the phase space of a muon beam within a short time frame. The optimal cooling parameters of a muon collider aim to minimize transverse emittance while simultaneously limiting longitudinal emittance growth, resulting in optimal luminosities within the collider ring. This study shows that achieving efficient cooling performance requires selecting the best initial muon beam parameters. Because for every transvere emittance there exist an optimal beam energy for ionization cooling. We present a technique that enables the determination of these optimal initial parameters through simulations and compare them with an improved analytical scattering model.

INTRODUCTION

Muon colliders have multiple advantages over hadron and e^-e^+ colliders with a recent intensification of the interest in such machines [1, 2], including the formation of the International Muon Collider Collaboration [3]. However, technology challenges for a muon collider need to be addressed. One of the most challenging technologies is that required for ionization cooling. The muon beams are generated from pion decays and these muons occupy a large phase space. Ionization cooling is the only technical method that can reduce a muon beam's phase space on a time scale compatible with the muon lifetime. In this paper, we discuss the modified theoretical description of ionization cooling and compare it with simulations. In particular, we study the fundamental choice of initial beam energy when a muon bunch enters an ionization cooling cell. For this paper, re-acceleration of the beam is not yet taken into account.

IONIZATION COOLING

In ionization cooling the total momentum of muons is decreased by passing them through an absorber. This leads to a decrease of the muon beam's phase space. The muons are then re-accelerated subsequently using an RF cavity system, so the beam's phase space can be reduced in an energy absorbing element. Due to the lifetime of muons (2.2 µs at rest), the energy loss and re-acceleration must occur rapidly. The energy loss is achieved by muons depositing their energy within an absorbing material. One side effect is that the scattering of muons with the atomic nuclei and electrons inside the energy absorbing material, acts as a source of unwanted increase in spread of transverse momentum. The absorber is therefore placed inside a solenoid with a magnetic field strength of 30-40 T in order to suppress the spread in transverse momentum coming from multiple scattering. Low-Z materials minimize muon scattering and in particular liquid hydrogen emerges as the ideal candidate for ionization cooling. This is because the ratio between the energy deposition of muons in hydrogen and its induced spread in scattering angles per unit length is best for all materials.

Transverse Emittance Cooling

The evolution of the normalized transverse emittance $\varepsilon_{\perp,\mathrm{N}}$ in an ionization cooling cell can be expressed analytically by assuming a few approximations. Excluding any phase space correlations, D. Neuffer [4] derived the transverse emittance behavior for ionization cooling along the beam axis s by a first order differential equation

$$\frac{d\varepsilon_{\perp,\mathrm{N}}}{ds} = -\frac{\varepsilon_{\perp,\mathrm{N}}}{\beta^2 E}\left\langle\frac{\partial E}{\partial s}\right\rangle + \frac{\beta_\perp\, pc}{2\, m_\mu c^2}\frac{d\langle\vartheta^2\rangle}{ds}. \tag{1}$$

with the mass of the muon represented as $m_\mu c^2$ and its momentum as pc. The first term in Eq. (1) represents the reduction of the normalized emittance and depends on the beam energy E, the relativistic β and the muon stopping force $\langle\partial E/\partial s\rangle$, as in the Bethe-Bloch formula [5, 6]. In the final cooling channel of a muon collider design, high values of $\langle\partial E/\partial s\rangle$ are crucial, leading to a operational beam energy choice between 5 and 200 MeV. The second term in Eq. (1) is driven by the scattering variance per unit length $d\langle\vartheta^2\rangle/ds$ suppressed by the betatron function $\beta_\perp$ of the solenoid. Transverse emittance heating is minimized by focusing the beam with the solenoid to decrease the betatron oscillation $\beta_\perp \propto p/B$ within the low-Z material.

Lynch-Dahl Sattering Approximation B. Rossi and K. Greisen [7] derived the average scattering width per unit length for multiple Coulomb scatterings, based on the Rutherford cross section, by assuming several approximations. V. Highland observed experimental inconsistencies with the analytical scattering prediction for materials with $Z < 20$. He modified Rossi and Greisen's equation, incorporating fitting parameters and a logarithmic term [8]. Later, Highland's correction was further adjusted by the formula of G. Lynch and O. Dahl [9], which remains the reference formula for multiple Coulomb scattering in the particle data group book [10]. In past ionization cooling studies [11, 12],

---

* bernd.stechauner@cern.ch

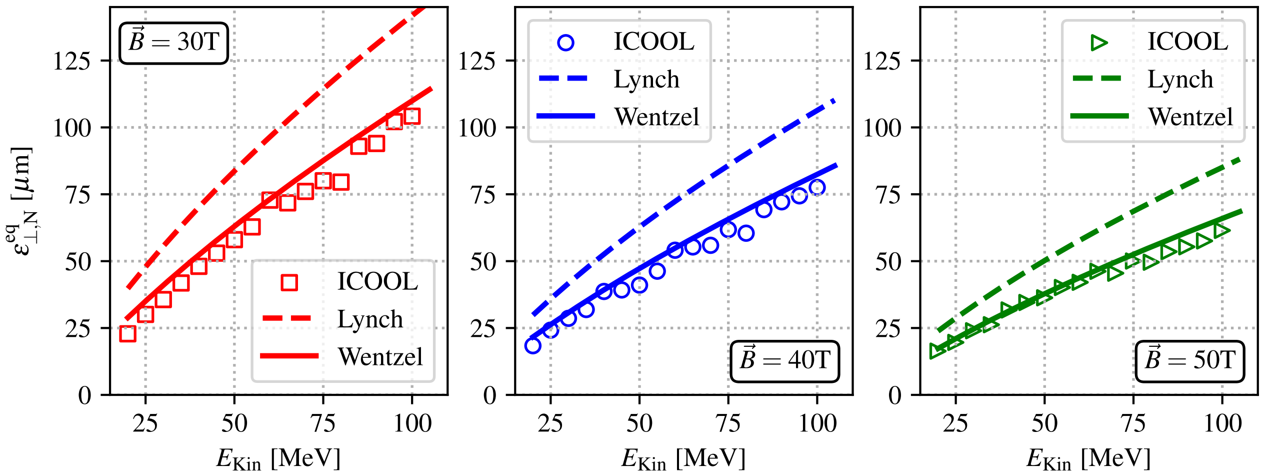

Figure 1: The equilibrium emittance of liquid hydrogen in a longitudinal static field of 30, 40 and 50 T is evaluated with ICOOL. A comparison with the analytical model shows excellent agreement when the Bethe-Wentzel scattering assumption was taken into account.

the Lynch-Dahl formula is approximated by neglecting the logarithmic term which yields the following:

$$\frac{d\langle\theta^2\rangle}{ds} = \left(\frac{13.6[\mathrm{MeV}]}{\beta pc}\right)^2 \frac{1}{L_\mathrm{R}}. \tag{2}$$

where L_R represents the radiation length. However, if one uses Eq. (2) within Eq. (1) the cooling equation leads to inconsistent results with hydrogen compared with simulations [13].

Bethe-Wentzel Scattering Angle G. Wentzel and H. Bethe proposed modifying the Rutherford cross section by introducing a minimum cut-off angle, ϑ_min, and considered the scattering from constituent electrons within the material's atoms [14, 15]. The scattering variance per unit length can be derived by separating the calculation of the nuclear and electronic deflection content [16] which results in

$$\begin{aligned}\frac{d\langle\theta^2\rangle}{ds} = \frac{k\rho}{2A}\left(\frac{Z}{\beta pc}\right)^2 \Bigg[& F\left(\frac{\vartheta_\mathrm{max}}{\vartheta_\mathrm{min}}\right) \\ & + \frac{1}{Z}F\left(\frac{\vartheta^\mathrm{e}_\mathrm{max}}{\vartheta^\mathrm{e}_\mathrm{min}}\right) - 1.19\left(1+\frac{1}{Z}\right)\Bigg].\end{aligned} \tag{3}$$

where A is the atomic mass, ρ the density of the material and $k \approx 0.157\,\mathrm{MeV^2cm^2mol^{-1}}$. The minimum and maximum nuclear scattering angles in Eq. (3) are ϑ_min and ϑ_max, while those for electron scattering are $\vartheta^\mathrm{e}_\mathrm{min}$ and $\vartheta^\mathrm{e}_\mathrm{max}$. These values are referenced in [16–18]. The function in Eq. (3) results form the integration of the modefied Rutherford cross section and is defined as

$$F(x) = \frac{1}{1+x^2} + \ln\left(1+x^2\right). \tag{4}$$

Transverse Equilibrium Emittance

In order to test the Lynch-Dahl and Bethe-Wentzel scattering models with particle tracking simulations, we consider the transverse equilibrium emittance. According to Eq. (1) for a given energy, material, and solenoid field strength, the minimum transverse emittance that can be achieved is

$$\varepsilon^\mathrm{eq}_{\perp,\mathrm{N}} = \beta^2 E \frac{d\langle\vartheta^2\rangle/ds}{\langle\partial E/\partial s\rangle}\frac{\beta_\perp pc}{2m_\mu c^2}. \tag{5}$$

We compare Eq. (5) using different models of $d\langle\vartheta^2\rangle/ds$ with results generated from ICOOL v331.1 [19]. ICOOL is a program specifically used for simulating ionization cooling and has been validated with experimental data [16]. For the simulations, we use solenoid field strengths of 30, 40 and 50 T inside a liquid hydrogen target with a density of $0.0708\,\mathrm{g\,cm^{-3}}$. For every initial beam energy, we performed a scan over multiple initial transverse beam emittances and analyzed the emittance change $\Delta\varepsilon_{\perp,\mathrm{N}}$. For the ICOOL simulations, we use the Fano-model and extracted $\varepsilon^\mathrm{eq}_{\perp,\mathrm{N}}$ by interpolating $\Delta\varepsilon_{\perp,\mathrm{N}}$. The data from ICOOL in Fig. 1 show a clear agreement with the Bethe-Wentzel model of Eq. (3), while the Lynch-Dahl scattering approximation shows an explicit overestimation of between 20 and 30%.

Longitudinal Emittance Change

The longitudinal phase space of a muon bunch in an absorber is proportional to the change in the square of its energy spread

$$\frac{d\sigma_E^2}{ds} = -2\frac{d\langle\partial E/\partial s\rangle}{dE}\sigma_E^2 + \frac{d(\Delta E^2_\mathrm{Stoch})}{ds}. \tag{6}$$

The first term in Eq. (6) depends on the slope of the Bethe-Bloch curve [20], which is negative for the final cooling. Therefore, the normalized longitudinal emittance $\varepsilon_\mathrm{L,N}$ grows as the bunch propagates through the cooling cell. The second term in Eq. (6) describes the stochastic energy fluctuation of the penetrating muons in the absorber which increases $\varepsilon_\mathrm{L,N}$

further and can be approximated as

$$\frac{d(\Delta E^2_{\mathrm{Stoch}})}{ds} = k\frac{\rho Z}{A}\gamma^2\left(1-\frac{\beta^2}{2}\right), \tag{7}$$

where γ represents the Lorentz factor. To minimize the longitudinal emittance growth, an optimized choice of the energy spread is needed.

BEST INITIAL ENERGY

In muon colliders, achieving high collision rates requires minimizing the transverse emittance, while simultaneously suppressing longitudinal emittance growth during muon cooling. This is because the luminosity scales as [3]

$$\mathcal{L} \propto \frac{1}{\varepsilon_{\perp,\mathrm{N}}\cdot\varepsilon_{\mathrm{L,N}}}. \tag{8}$$

Recipe for Finding the Optimal Parameters

For a given solenoid field strength and absorber, an optimum initial kinetic energy must be found for the beam entering the cooling cell. In beam cooling systems the parameters $\varepsilon_{\perp,\mathrm{N}}$ and $\varepsilon_{\mathrm{L,N}}$ are always constant, since they are normalized. When entering the cooling cell with a specific energy spread, we execute a scan over the initial kinetic beam energies and consider the minimum of the trade-off function $-\Delta\varepsilon_{\mathrm{L,N}}/\Delta\varepsilon_{\perp,\mathrm{N}}$ in order to achieve maximum luminosity according to Eq. (8). Particle loss is neglected, as it primarily appears in the re-acceleration phase, which is beyond the scope of this analysis. To illustrate this with an example, we start with an energy scan in liquid hydrogen for $\varepsilon_{\perp,\mathrm{N}} = 200\,\mu\mathrm{m}$, $\varepsilon_{\mathrm{L,N}} = 1\,\mathrm{mm}$, and $B = 40\,\mathrm{T}$ by solving Eq. (1) and Eq. (6) with a fourth order Runge-Kutta algorithm, in order to achieve a high accuracy. We present the results in Fig. 2 for different initial σ_E. It can be observed that the optimal initial kinetic energy, illustrated as the minimum of the curves, increases as the energy spread rises.

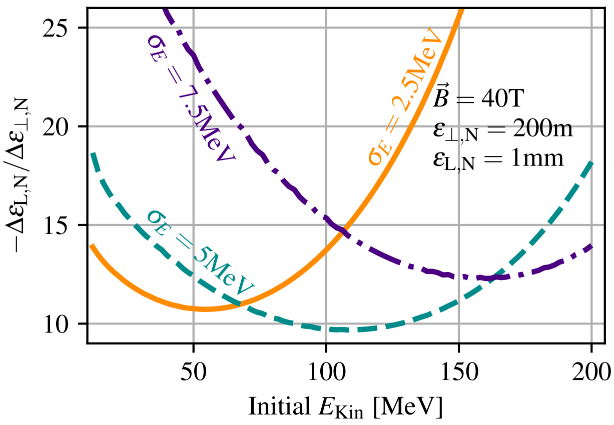

Figure 2: For given beam and machine parameters the best initial beam energy can be estimated by observing the minimum of the trade-off function $-\Delta\varepsilon_{\mathrm{L,N}}/\Delta\varepsilon_{\perp,\mathrm{N}}$.

Comparison with ICOOL Simulations

The next step involves determining the optimal starting energy for muon cooling for each initial transverse emittance. We perform a kinetic energy scan at $\varepsilon_{\mathrm{L,N}} = 1\,\mathrm{mm}$ and a relative momentum spread $\delta_{p_z} = 2\%$, and reduce the energy to 90% of its initial value. A static magnetic field of $B = 40\,\mathrm{T}$ was applied within liquid hydrogen using ICOOL, and the simulation yields the minimum trade-off function to determine the optimal energy for the initial transverse emittance. The results are illustrated in Fig. 3 and compared with the analytic model using the Bethe-Wentzel assumption. Fig. 3 shows a satisfactory agreement despite potential fluctuations in the evaluated beam parameters, since we used only 10^5 macro particles in the ICOOL simulations.

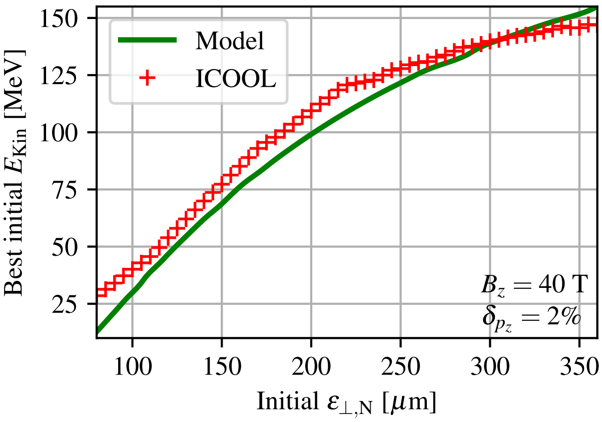

Figure 3: Best initial kinetic beam energy for ionization cooling for different initial normalized transverse emittances, for cooling with liquid hydrogen in a magnetic field of 40 T, simulated with ICOOL and calculated from the analytical model.

CONCLUSION

We searched analytically for the optimal initial muon energy for every ionizaton cooling stages. In this study, we refined Neuffer's transverse cooling equation using the Bethe-Wentzel scattering approach, showing better agreement with simulation than with the Lynch-Dahl model previously used in ionization cooling studies. An equilibrium emittance comparison of both approaches with ICOOL proofs the accuracy of Bethe-Wentzel. This enabled the determination of the optimal initial beam energy for ionization cooling, aligning well with ICOOL simulations. Our findings facilitate the selection of beam parameters for achieving highly efficient muon ionization cooling.

ACKNOWLEDGEMENT

Funded by the European Union (EU). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the EU or European Research Executive Agency (REA). Neither the EU nor the REA can be held responsible for them.

REFERENCES

[1] "US particle physicists want to build a muon collider- Europe should pitch in", *Nature*, vol.625, pp. 423-424, 2024. `doi:10.1038/d41586-024-00105-9`

[2] A. Cho, "The dream machine", *Science*, vol. 383, iss. 6690, 2024. `doi:10.1126/science.adp4452`

[3] C. Accettura, et al., "Towards a muon collider" *Eur. Phys. J. C*,vol. 83, pp. 864, 2023. `doi:10.1140/epjc/s10052-023-11889-x`

[4] D. Neuffer, "Principles and applications of muon cooling." *Proc. 3rd LAMPF II workshop*, Los Alamos, USA, Jul. 1983. `doi: 10.2172/1156195`

[5] H. Bethe, "Zur Theorie des Durchgangs schneller korpuskularstrahlen durch Materie", *Annalen der Physik*, vol. 397, 1930. `doi:10.1002/andp.19303970303`

[6] F. Bloch, "Zur Bremsung rasch bewegter Teilchen beim Durchgang durch Materie", *Annalen der Physik*, vol. 408, 1933. `doi:10.1002/andp.19334080303`

[7] B. Rossi, K. Greisen, "Cosmic-Ray theory", *Rev. Mod. Phys.*, vol. 13, pp. 240-309, 1941. `doi:10.1103/RevModPhys.13.240`

[8] V.L. Highland, "Some practical remarks on multiple scattering", *Nucl. Instru. Meth.*, vol. 129, pp. 497-499, 1975. `doi:10.1016/0029-554X(75)90743-0`

[9] G.R. Lynch, O.I. Dahl, "Approximations to multiple Coulomb scattering", *Nucl. Instrum. Meth. B*, vol. 58, pp. 6-10, 1991. `doi:10.1016/0168-583X(91)95671-Y`

[10] R.L. Workman *et al.*, "The Review of Particle Physics", *Prog. Theor. Exp. Phys.*, Vol. 2022, pp. 549-564, Aug. 2022, `doi:10.1093/ptep/ptac097`

[11] D. Stratakis *et al.*, "Tapered channel for six-dimensional muon cooling towards micron-scale emittances", *Phys. Rev. ST Accel. Beams*, vol.16, no. 091001, Sep. 2013. `doi:10.1103/physrevstab.16.091001`

[12] H.K. Sayed *et al.*, "High field – low energy muon ionization cooling channel", *Phys. Rev. ST Accel. Beams*, vol. 18, no. 091001, Sep. 2015. `doi:10.1103/physrevstab.18.091001`

[13] B. Stechauner *et al.*, "Comparison of tracking codes for beam-matter interactions", in *Proc. IPAC'23*,Venice, Italy, May 2023, paper MOPL165, pp. 921-924.

[14] H.A. Bethe, "Molière's theory of multiple scattering", *Phys. Rev.*, Vol. 89, pp. 1256, 1953. `doi: 10.1103/PhysRev.89.1256`

[15] G. Wentzel, "Zwei Bemerkungen über die Zerstreuung korpuskularer Strahlen als Beugungserscheinung", *Z. Physik*, Vol. 40, pp. 590–593, 1926. `doi:10.1007/BF01390457`

[16] T. Carlisle, "Step IV of the Muon Ionization Cooling Experiment (MICE) and the multiple scattering of muons", Ph.D. thesis, University of Oxford, United Kingdom, 2013.

[17] J.D. Jackson, "Classical Electrodynamics", New York, NY, USA, Wiley, 1998.

[18] B. Stechauner *et al.*, "Introducing a semi-Gaussian mixture model for simulating multiple Coulomb scattering in RF-Track", presented at IPAC'24, Nashville, USA, MAY 2024, paper WEPR31, this conference.

[19] R. C. Fernow, "ICOOL: a simulation code for ionization cooling of muon beams", in *Proc. PAC'99*, New York, NY, USA, Mar. 1999, paper THP31, pp. 3020-3022. `doi: 10.1109/PAC.1999.792132.`

[20] D. Neuffer, "Introduction to muon cooling", *Nucl. Inst. and Meth. in Phys. Research A*, vol. 532, pp. 26–31, 2004. `doi: 10.1016/j.nima.2004.06.051`