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NUMERICAL METHODS FOR EMITTANCE COMPUTATION FROM LUMINOSITY

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Abstract

The beam transverse emittances play a critical role for the performance of high-energy colliders. Various measurement techniques are thus employed to measure them. In particular, the so-called luminosity emittance scans are used to evaluate the convoluted beam emittances. This method usually assumes different emittances in the two planes but identical emittances in the two beams. In this paper, we propose an approach to relax this assumption. We present a new measurement protocol and discuss its potential and limits. We also analyse the impact of statistical measurement errors of the luminosity value.

INTRODUCTION

The transverse beam emittances directly affect the luminosity of a collider and its overall performance. In the Large Hadron Collider (LHC) they are typically measured by means of the Wire Scanners (WS) and the Beam Synchrotron Radiation Telescope (BSRT) [1]. The WS cannot be used systematically during the operation due to its limitation to low beam intensity [2]. The BSRT can be used for continuous monitoring, but comes with a relative error of about 10/20% [3].

In colliders, one could leverage on the diagnostic provided by the detectors themselves, performing the so-called luminosity emittance scans [4, 5] (or Van der Meer scans [6]). By controlling the transverse separation of the beam at the Interaction Points (IP) with a good knowledge of the machine parameters and of the luminosity dependence on those parameters, one can invert the problem. This inversion can be done analytically, introducing convenient hypotheses. Assuming that the transverse emittances in the two transverse planes (horizontal and vertical) are different but are identical between the two beams (convoluted emittance hypothesis), and that the beam profiles are Gaussian and ignoring the dispersion contribution, it is possible to obtain the dependence of the luminosity on the horizontal offset δ_x between the beams at the IP:

$$\mathscr{L}(\epsilon_x; \beta_x|\delta_x) = \mathscr{L}_0\, e^{-\frac{\delta_x^2}{4\epsilon_x\beta_x}}, \tag{1}$$

where $\mathscr{L}_0$ is the luminosity for $\delta_x = 0$ and ϵ_x is the convoluted horizontal emittance. Knowing β_x, it is possible to compute the ϵ_x by measuring $\mathscr{L}/\mathscr{L}_0$ for different values of δ_x. This approach can be trivially extended to compute the emittance also in the vertical plane.

Our objective is to generalize the inversion of Eq. (1) solving numerically the problem assuming the emittances of the two beams and for the two planes are fully independent ($\epsilon_{x1}, \epsilon_{x2}, \epsilon_{y1}, \epsilon_{y2}$). In doing so, we could not find a generalization of the inversion of Eq. (1) in closed form, but we propose a numerical inversion of the luminosity integral [7]:

$$\mathscr{L} = \mathscr{L}(\epsilon_{x1}, \epsilon_{x2}, \epsilon_{y1}, \epsilon_{y2}; \beta_{x1}, \beta_{x2}, \dots|\delta_x, \delta_y, \delta\theta_x, \delta\theta_y), \tag{2}$$

where $\delta\theta_{x,y}$ is the variation of the crossing angle.

NUMERICAL INVERSION METHOD

Obtaining the four emittances from the luminosity by inverting Eq. (2) is clearly an underdetermined problem. On the other hand, one can apply several times small variations to the variables in Eq. (2) (δ_x, δ_y,…) and measure the luminosity $\mathscr{L}/\mathscr{L}_0$ producing artificially a well-posed system, which can be inverted numerically. This method is yet to be tested experimentally in the LHC. We propose a numerical procedure to test our method, where we compute the $\mathscr{L}/\mathscr{L}_0$ values using the numerical integral of the luminosity model [7] and the collider parameters (β_x, β_y,…), assuming some values for the four emittances. Subsequently, we can formulate a system of equations, the inversion of which retrieves the initially assumed emittances. In practice, we produce a non-linear system of equation that is solved using SciPy's Nonlinear Least Squares method (LS) [8]. Since the inversion is purely numerical, we can include in the luminosity integral more complex effects, like hourglass effect [7], geometrical reduction factor [9], dispersion, crabbing. [10]. In practice, the variations of the initial parameters are chosen to perturb $\mathscr{L}$ by at the most 1%, allowing the scan to be, in principle, almost transparent for the luminosity production. As an example, for the typical LHC parameters [11] at $\beta^* = 30$ cm, this 1% is attained with a variation of $\delta_y \sim 1.98 \times 10^{-6}$ cm and $\delta\theta_y \sim 3.27$ µrad.

RESULTS AND DISCUSSION

In the previous section, we presented what we refer to as the measurement protocol. Unfortunately, this is not enough to compute the true beam emittances in all circumstances, since the inversion is ill-conditioned.

This section will briefly detail how we improved the robustness of the inversion also by taking into account the statistical measurement error on the luminosity value. The

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main tool we used to guide us in the tuning of the numerical inversion is the LS penalty function plot. We gradually increased the difficulty of the problem, progressing through sequential steps. For additional details, see [12].

No Statistical Measurement Error on the Luminosity

In this subsection, we show the results of the inversion of the general Eq. (2), neglecting statistical errors in the luminosity determination. It is possible to show, that assuming full symmetry in the machine parameters, the problem is ill-posed. In fact, from the general luminosity formula of [7], assuming that beta parameters ($\beta^*_{\{x1,x2,y1,y2\}}$,$\alpha_{\{x1,x2,y1,y2\}}$) are equal for both beams and planes, it is possible to obtain this dependence on the luminosity

$$\mathscr{L}(\epsilon_{x1},\epsilon_{x2},\epsilon_{y1},\epsilon_{y2};\dots) = \mathscr{L}((\epsilon_{x1}+\epsilon_{x2}),(\epsilon_{y1}+\epsilon_{y2});\dots). \quad (3)$$

More details about this dependence can be found in Ref. [11].

It is evident that due to this symmetry, the LS can estimate the two sum emittances ($(\epsilon_{x1}+\epsilon_{x2})$,$(\epsilon_{y1}+\epsilon_{y2})$), but it is unable to distinguish between the emittances in the same plane for the different beams. In Fig. 1 it is possible to visualize the last statement even in a simplified case ($\epsilon_1 = \epsilon_{x1} = \epsilon_{y1}, \epsilon_2 = \epsilon_{x2} = \epsilon_{y2}$), where the red point in the center of the image is the first guess of the optimizer. During several iterations, the solver, from the current guess, picks two points along the two orthogonal axes, and from the evaluation of the penalty functions in these three points obtains the tangent plane, and it proceeds towards its deepest descent direction obtaining a new guess, and so on. Changing the nominal value of β^* for one of the two beams in each plane, adding a small value (1×10^{-4} cm, smaller than its measurement error), it is possible to see in Fig. 2 how accurate the emittance reconstruction becomes. For convenience, we will refer to the estimated emittance from the LS as $\hat{\epsilon}$. Note that all the histograms that will be shown in the following are the result of 1600 random choices of emittances in a square of 20% of the usual value (2.3×10^{-6}).

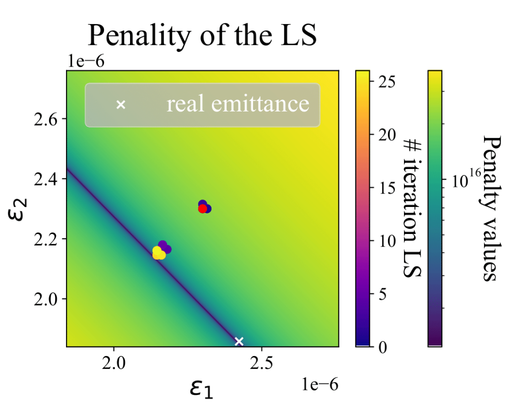

Figure 1: Plots of the penalty function when $\epsilon_1 = \epsilon_{x1} = \epsilon_{y1}, \epsilon_2 = \epsilon_{x2} = \epsilon_{y2}$, with configuration from [11], represented in the second bar, while the first bar represents the colour of the different iterations of the optimizer.

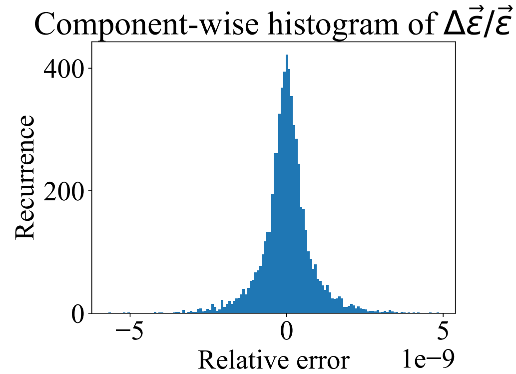

Figure 2: Histogram of the relative error for all the random choices of emittances component-wise ($[(\epsilon_{x1} - \hat{\epsilon_{x1}})/\epsilon_{x1},\dots]$) without considering the luminosity measurement error, where ($\epsilon_{x1} \neq \epsilon_{x2} \neq \epsilon_{y1} \neq \epsilon_{y2}$).

Statistical Measurement Error on the Luminosity

In the analytical approach of Eq. (1) and in our numerical one, there are two hidden assumptions: we neglected the statistical or systematic errors in the luminosity and in the machine parameter values.

The purpose of this subsection is to extend the preceding analysis by examining how the results are affected by adding a statistical error to the luminosity values used in the system. We model this error like a Gaussian, with 0 mean and standard deviation of 0.1% $\mathscr{L}_0$.

Using the same measurement protocol described in the previous case, it is possible to show that the LS is particularly affected by this random error, leading to an increased relative error of the estimation, even in a simpler case of convoluted emittance ($\epsilon_x = \epsilon_{x1} = \epsilon_{x2}, \epsilon_y = \epsilon_{y1} = \epsilon_{y2}$), as can be seen in Fig. 3. As a result, alternative strategies are proposed. Even in this case, the examination of the penalty function behaviour can guide us. Figure 4a depicts the penalty function of the LS in the (ϵ_x,ϵ_y) case, in the nominal configuration in [11]. A vertical line near the center of the plot is observed, delineating a zone characterized by small values of the penalty function, containing a global minimum. The orientation of that region is due to the crossing angle plane. In fact, the line represents the vulnerable aspect of our inversion approach: indeed adding a random error on the luminosity value, the LS can invert the ϵ_x but it is unable to invert the ϵ_y.

To condition this problem, we can use the information of the other detectors of the LHC, in which the crossing plane is rotated by 90 degrees. In particular, in ATLAS the crossing is in the vertical plane, whereas in CMS it is in the horizontal. This process yields a composite penalty function comprised of two distinct functions, each characterized by orthogonal "minima". The intersection point of these lines will stabilize the inversion, as illustrated in Fig. 4b.

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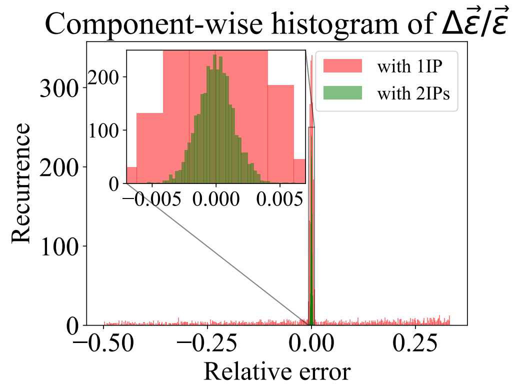
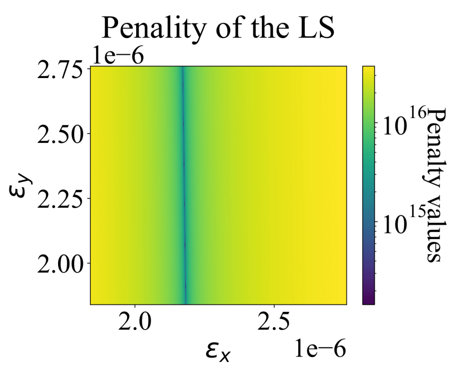
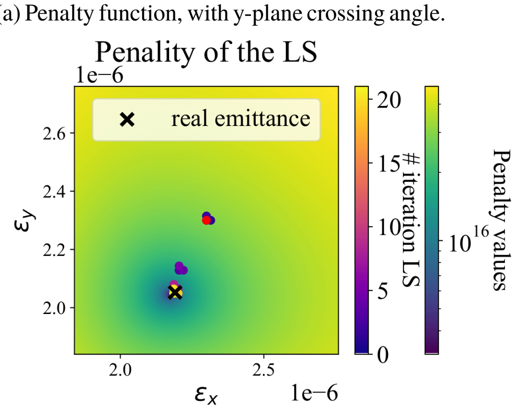


Figure 3: Histogram of the relative error component-wise($\Delta\vec{\epsilon}/\vec{\epsilon} = [(\epsilon_x - \hat{\epsilon_x})/\epsilon_x, \dots]$) without considering the luminosity measurement error, ($\epsilon_x = \epsilon_{x1} = \epsilon_{x2}, \epsilon_y = \epsilon_{y1} = \epsilon_{y2}$). In red the error using the luminosity from one detector, in green the error using the luminosity from two detectors with different transverse crossing plane.



(a) Penalty function, with y-plane crossing angle.



(b) Penalty function with 2 crossing angle planes.

Figure 4: Plots of the penalty functions when $\epsilon_x = \epsilon_{x1} = \epsilon_{x2}, \epsilon_y = \epsilon_{y1} = \epsilon_{y2}$, in different choices of the configuration system.

The same strategy can also be employed in the most difficult example, where all four emittances differ. In this situation, a larger number of variables is required, namely during the scan one has to vary also the β^*. For example, in order to invert the problem, we need also 2 different β^* configuration for each detector. In particular, we need two different β^* between the two beams (between the planes the β^* can be equal). Table 1 illustrates an example of different β^* configurations to condition the inversion.

Table 1: Table Representing the Different System Configurations for Each Detector

1st detector	$\Delta\theta_x \neq 0$, $\beta_1^* = \beta_2^*$	$\Delta\theta_x \neq 0$, $\beta_1^* = 1.02\beta_2^*$
2nd detector	$\Delta\theta_y \neq 0$, $\beta_1^* = \beta_2^*$	$\Delta\theta_y \neq 0$, $\beta_1^* = 1.02\beta_2^*$

The shift for β^* is in the order of 2%, so similar to its measurement error, but it is in principle more complex to be performed if compared to the orbit related variables. The histogram in Fig. 5 shows the accuracy of this, relatively involved, measurement protocol. The success of this strategy is apparent in the precision of the reconstructed emittances (standard deviation of $\sim 6 \times 10^{-3}$).

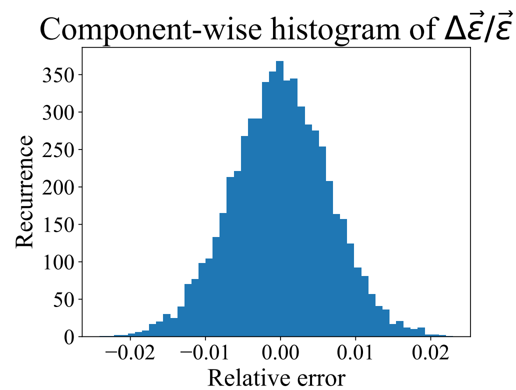


Figure 5: Histogram of the relative error component-wise($\Delta\vec{\epsilon}/\vec{\epsilon} = [(\epsilon_{x1} - \hat{\epsilon_{x1}})/\epsilon_{x1}, \dots]$) considering the luminosity measurement error, where ($\epsilon_{x1} \neq \epsilon_{x2} \neq \epsilon_{y1} \neq \epsilon_{y2}$).

CONCLUSIONS

In this paper, we proposed a generalization of the luminosity scan measurement protocol with the aim of relaxing the convoluted emittance assumption. It is based on a fully numerical approach to analyze the problem. The efficacy and constraints of this methodology have been investigated, and several alternative strategies have been suggested to mitigate the inherent limitations of the ill-posed problem.

Subsequent research endeavours could entail expanding the error analysis to encompass both the measured parameters and the underlying model. Additionally, experimental validation of this approach on operational runs within the LHC could provide valuable insights into its real-world applicability.

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