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BCCNT 95/101/251

Microscopic Foundations of the Vector Meson Dominance Model via the Momentum-Space Bosonization of an Extended NJL Model

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(October, 1995)

Submitted to Physical Review D

Abstract

We use a momentum-space bosonization of a generalized Nambu–Jona-Lasinio (NJL) model to provide a microscopic foundation for the vector-meson-dominance model. (In our model the photon interacts with the constituent quarks rather than with the hadrons.) A novel feature of our model is the introduction of q^2-dependent meson decay constants, $g^\rho(q^2)$ and $g^\omega(q^2)$, as well as q^2-dependent meson-meson coupling constants, such as $g_{\rho\pi\pi}(q^2)$. We discuss the values of $g^\rho(q^2)$, $g^\omega(q^2)$ and $g_{\rho\pi\pi}(q^2)$ obtained using our generalized NJL model, considering different choices for the parameters of the model. We also provide a quark-based description of rho-omega mixing. The definition of momentum-dependent meson decay constants allows us to introduce fields for the rho and omega mesons into the analysis in an unambiguous manner, when we start with an analysis of hadronic current correlators that are expressed in terms of quark fields.

I. Introduction

Recently there has been a good deal of interest in the calculation of rho-omega mixing [1-7], since such mixing could be important in understanding charge symmetry breaking (CSB) in the nucleon-nucleon interaction. One object of interest is the matrix element $< \omega \mid H_{SB} \mid \rho >$ that provides a measure of the importance of ρ-ω mixing. Originally, that matrix element was obtained in the study of $e^+ + e^- \rightarrow \pi^+ + \pi^-$, where $q^2 \simeq m_\omega^2$. However, for nuclear physics applications, one needs to understand rho-omega mixing for $q^2 \leq 0$. Various authors have pointed out that the mixing should vanish at $q^2 = 0$ [7]. Therefore, the importance of ρ-ω mixing for understanding CSB is greatly reduced. However, there are a number of theoretical issues related to the calculation of rho-omega mixing that need to be addressed.

Recently, O'Connell, Pearce, Thomas and Williams [1] have presented a new discussion of the extraction of $< \omega \mid H_{SB} \mid \rho >$ from the experimental data for $e^+ + e^- \rightarrow \pi^+ + \pi^-$. In this connection, they reviewed two forms of the vector-meson-dominance (VMD) model, with Lagrangians denoted as $\mathcal{L}_{VMD1}$ and $\mathcal{L}_{VMD2}$. The Lagrangian for the VMD1 model is [6,8]

$$\mathcal{L}_{VMD1} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}\rho^{\mu\nu}\rho_{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu\rho^\mu - g_{\rho\pi\pi}\rho_\mu J^\mu - eA_\mu J^\mu - \frac{e}{2g^\rho}F_{\mu\nu}\rho^{\mu\nu} \quad , \quad (1.1)$$

where g^ρ is the rho decay constant, $e = \mid e \mid$, and J^μ is the hadronic current coupled to the rho. The pion component of that current is $J_\mu = i(\pi^- \partial_\mu \pi^+ - \pi^+ \partial_\mu \pi^-)$. Further, $F^{\mu\nu}$ is the electromagnetic tensor and $\rho^{\mu\nu}$ is the corresponding quantity for the rho field. As discussed in Ref. [1], one may replace $F_{\mu\nu}\rho^{\mu\nu}$ by $-2q^2 A_\mu \rho^\mu$, when the rho field is divergenceless. Thus, we see that direct photon-rho coupling vanishes at $q^2 = 0$ in the VMD1 model.

An alternate form of the VMD model is given (with $e = |e|$) by [9]

$$\mathcal{L}_{VMD2} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu\rho^\mu - g_{\rho\pi\pi}\rho_\mu J^\mu - \frac{em_\rho^2}{g^\rho}\rho_\mu A^\mu + \frac{1}{2}\left(\frac{e}{g^\rho}\right)^2 m_\rho^2 A_\mu A^\mu \quad . \tag{1.2}$$

In the limit of universality ($g_{\rho\pi\pi} = g^\rho$), these Lagrangians may be transformed into each other by a change of variables [6].

These two models yield somewhat different expressions for the pion form factor, $F_\pi(q^2)$, defined for both spacelike and timelike q^2. For the VMD1 model, we have [1]

$$F_\pi(q^2) = 1 - \frac{q^2}{g^\rho}\frac{1}{q^2 - m_\rho^2 + im_\rho\Gamma_\rho(q^2)}g_{\rho\pi\pi} \quad , \tag{1.3}$$

where $g_{\rho\pi\pi}$ is the rho-pion coupling constant, which appears in both Eqs. (1.1) and (1.2). Note that $F_\pi(q^2)$ of Eq. (1.3) satisfies the condition $F_\pi(0) = 1$. Alternatively, for the VMD2 model, we have [1]

$$F_\pi(q^2) = -\frac{m_\rho^2}{g^\rho}\frac{1}{q^2 - m_\rho^2 + im_\rho\Gamma_\rho(q^2)}g_{\rho\pi\pi} \quad . \tag{1.4}$$

These two forms are equivalent, if the universality relation, $g_{\rho\pi\pi} = g^\rho$, is valid. That may be seen in the region $q^2 < 4m_\pi^2$, where $\Gamma_\rho(q^2) = 0$. If universality is valid, we also have $F_\pi(0) = 1$, when using the expression given in Eq. (1.4).

Our work is organized in the following manner. In Section II we present the Lagrangian of our model and describe a momentum-space bosonization procedure. In Section III we consider a vector-meson-dominance model for the pion form factor. (That discussion leads naturally to the definition of momentum-dependent meson decay constants.) In Section IV we extend our considerations to a discussion of rho-omega mixing and provide an expression of the pion form factor that includes the effects of that mixing. In Section V we discuss a mixed hadronic current correlation function, $\hat{\Pi}^{\mu\nu}_{(\omega\rho)}(q)$, and show how one can define a similar correlation function using ω and ρ fields. In Section VI we present various parameter sets for our extended NJL model and also exhibit the values of various quantities that have been calculated using the model. Section VII contains some further discussion and summary of our results.

II. Momentum-Space Bosonization of an Extended NJL Model

We begin by recording the Lagrangian of our extended version of the NJL model [10],

$$\mathcal{L}(x) = \bar{q}(i\partial\!\!\!/ - m_q^0)q + \frac{G_S}{2}\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right]$$

$$- \frac{G_\rho}{2}\left[(\bar{q}\gamma^\mu\vec{\tau}q)^2 + (\bar{q}\gamma_5\gamma_\mu\vec{\tau}q)^2\right] \tag{2.1}$$

$$- \frac{G_\omega}{2}(\bar{q}\gamma^\mu q)^2 + \mathcal{L}_{conf}(x) \quad .$$

Here, $\mathcal{L}_{conf}(x)$ refers to the confining interaction we have added to the model and m_q^0 is the current quark mass matrix. (It is useful to include factors of 1/2 in the definition of the coupling constants, since these factors cancel statistical factors of 2 that appear in the evaluation of various matrix elements.)

We define the tensors [10],

$$\hat{J}^{\mu\nu}_{(\rho)}(q) = -\tilde{g}^{\mu\nu}(q)\hat{J}_{(\rho)}(q^2) \quad , \tag{2.2}$$

$$\hat{J}^{\mu\nu}_{(\omega)}(q) = -\tilde{g}^{\mu\nu}(q)\hat{J}_{(\omega)}(q^2) \quad , \tag{2.3}$$

$$\hat{K}^{\mu\nu}_{(\rho)}(q) = -\tilde{g}^{\mu\nu}(q)\hat{K}_{(\rho)}(q^2) \quad , \tag{2.4}$$

and

$$\hat{K}^{\mu\nu}_{(\omega)}(q) = -\tilde{g}^{\mu\nu}(q)\hat{K}_{(\omega)}(q^2) \quad , \tag{2.5}$$

where $\tilde{g}^{\mu\nu}(q) = g^{\mu\nu} - q^\mu q^\nu / q^2$. The diagrams giving rise to these tensors are depicted in Figs. 1a and 1b. The evaluation of those diagrams yields $-i\hat{J}^{\mu\nu}_{(\rho)}(q)$, $-i\hat{J}^{\mu\nu}_{(\omega)}(q)$, etc. The shaded area in the figures represents a vertex, $\hat{\Gamma}^\mu$, that represents the sum of a ladder of confining interactions [11]. (See Figs. 1c and 1d.) That vertex removes those cuts in the functions of Eqs. (2.2)-(2.5) that would start at $q^2 = 4m_q^2$ in the absence of a model of confinement. Thus, the functions of Eqs. (2.2)-(2.3) are real. Further, $\hat{K}^{\mu\nu}_{(\rho)}(q^2)$ and $\hat{K}^{\mu\nu}_{(\omega)}(q^2)$ only have cuts that arise when two, or three, pions go on-mass-shell. Figure 2 shows $\hat{J}_{(\rho)}(q^2)$ for both spacelike and timelike q^2. In the timelike region, $\hat{J}_{(\rho)}(q^2)$ is evaluated in Minkowski space using a cutoff of $\Lambda_3 = 0.702$ GeV for all three-momenta that appear in the integral. (See Table 1.) For spacelike q^2, confinement is not very important. We neglect confinement in that region and complete the integral in a Euclidean momentum space using a cutoff $\Lambda_E = 1.0$ GeV. (Note that, if $m_d^0 = m_u^0$, we have $\hat{J}_{(\rho)}(q^2) = \hat{J}_{(\omega)}(q^2)$.)

The functions of Eqs. (2.2)-(2.5) may be used to parameterize the quark T matrices. We sum a string of $q\bar{q}$ loop integrals and obtain

$$T_\rho(q^2) = \frac{G_\rho}{1 - G_\rho\left[\hat{J}_{(\rho)}(q^2) + \hat{K}_{(\rho)}(q^2)\right]} \tag{2.6}$$

and

$$T_\omega(q^2) = \frac{G_\omega}{1 - G_\omega\left[\hat{J}_{(\omega)}(q^2) + \hat{K}_{(\omega)}(q^2)\right]} \quad , \tag{2.7}$$

where we have suppressed reference to isospin and Dirac matrices. It will be useful to write

$$\frac{G_\rho}{1 - G_\rho\left[\hat{J}_{(\rho)}(q^2) + \hat{K}_{(\rho)}(q^2)\right]} = -\frac{g_{\rho qq}^2(q^2)}{q^2 - m_\rho^2 + im_\rho\Gamma_\rho(q^2)} \quad , \tag{2.8}$$

and a similar relation for the omega meson. The origin of Eq. (2.8) may be seen in the following discussion. In our earlier work we saw that we can use the approximations [12]

$$\hat{J}_{(\rho)}(q^2) + \mathrm{Re}\,\hat{K}_{(\rho)}(q^2) = r_1 - \frac{r_2}{q^2 - \tilde{m}_\rho^2} \quad , \tag{2.9}$$

and

$$\hat{J}_{(\omega)}(q^2) + \mathrm{Re}\,\hat{K}_{(\omega)}(q^2) = \mathrm{v}_1 - \frac{\mathrm{v}_2}{q^2 - \tilde{m}_\omega^2} \quad . \tag{2.10}$$

Here, r_1, r_2, $\tilde{m}_\rho^2$, v_1, v_2 and $\tilde{m}_\omega^2$ (with $\tilde{m}_\rho^2 = \tilde{m}_\omega^2$) are constants that are determined after an explicit calculations of the relevant functions. (See Table 1.) Inserting these expressions in Eq. (2.8), and in the corresponding expression for the omega meson, we find

$$g_{\rho qq}^2(q^2) = \frac{\tilde{m}_\rho^2 - q^2}{G_\rho^{-1} - r_1} \quad , \tag{2.11}$$

$$g_{\omega qq}^2(q^2) = \frac{\tilde{m}_\omega^2 - q^2}{G_\omega^{-1} - \mathrm{v}_1} \quad , \tag{2.12}$$

$$m_\rho^2 = \tilde{m}_\rho^2 - \frac{r_2}{G_\rho^{-1} - r_1} \quad , \tag{2.13}$$

$$m_\omega^2 = \tilde{m}_\omega^2 - \frac{\mathrm{v}_2}{G_\omega^{-1} - \mathrm{v}_1} \quad , \tag{2.14}$$

$$\Gamma_\rho(q^2) = \frac{g_{\rho qq}^2(q^2)}{m_\rho}\,\mathrm{Im}\,\hat{K}_{(\rho)}(q^2) \quad , \tag{2.15}$$

and

$$\Gamma_\omega(q^2) = \frac{g^2_{\omega qq}(q^2)}{m_\omega} \operatorname{Im} \hat{K}_{(\omega)}(q^2) \quad . \tag{2.16}$$

In Table 2, we present some values of $g_{\rho qq}(q^2)$ and $g_{\omega qq}(q^2)$ for model B that is described in Table 1.

We note that Γ_ρ and Γ_ω may be written in terms of $g_{\rho\pi\pi}$ and $g_{\omega\pi\pi}$, respectively. In general, these are q^2-dependent functions. Thus,

$$\Gamma_\rho(q^2) = \frac{q^2}{m_\rho} \frac{g^2_{\rho\pi\pi}(q^2)}{48\pi} \left[1 - \frac{4m_\pi^2}{q^2}\right]^{3/2} \theta(q^2 - 4m_\pi^2) \quad . \tag{2.17}$$

At $q^2 = m_\rho^2$, we have

$$\Gamma_\rho(m_\rho^2) = \frac{g^2_{\rho\pi\pi}}{48\pi} m_\rho \left[1 - \frac{4m_\pi^2}{m_\rho^2}\right]^{3/2} \quad , \tag{2.18}$$

where we have defined $g_{\rho\pi\pi} = g_{\rho\pi\pi}(m_\rho^2)$. If $\Gamma_\rho(m_\rho^2) = 151.2$ MeV and $m_\rho = 770$ MeV, we find that $g_{\rho\pi\pi} = 6.04$. We may now make contact with our previous work [10], where we obtained the expression

$$\Gamma_\rho(q^2) = \frac{q^2}{m_\rho} \frac{g^2_{\rho qq}(q^2)}{48\pi} g^4_{\pi qq}(m_\pi^2) n_c^2 n_f^2 \left[1 - \frac{4m_\pi^2}{q^2}\right]^{3/2} H^2(q^2)\theta(q^2 - 4m_\pi^2) \quad . \tag{2.19}$$

Here $n_c^2 = 9$ and $n_f^2 = 4$. Comparing Eqs. (2.17) and (2.19), we find

$$g^2_{\rho\pi\pi}(q^2) = 36 g^2_{\rho qq}(q^2) g^4_{\pi qq}(m_\pi^2) H^2(q^2) \quad , \tag{2.20}$$

or, for $q^2 = m_\rho^2$,

$$g_{\rho\pi\pi} = -6 g_{\rho qq}(m_\rho^2)\, g_{\pi qq}^2(m_\pi^2)\, H(m_\rho^2) \quad . \tag{2.21}$$

We have introduced a minus sign in passing from Eq. (2.20) to Eq. (2.21), since $H(q^2)$ is negative. (The precise definition of $H(q^2)$ is given in Eqs. (3.2)-(3.4).)

III. Foundation of the Vector-Meson-Dominance Model

In this section we wish to derive the expressions for electromagnetic processes that arise in our extended NJL model. In our model, the photon couples to the quarks, rather than to the hadrons. As an example of current interest, we will consider the process $e^+ + e^- \to \pi^+ + \pi^-$ that proceeds through rho-omega mixing [12]. First, let us study the direct process $\gamma \to \rho \to \pi^+ + \pi^-$. (See Fig. 3.) In Fig. 3a the dashed line is a photon of momentum q and the solid lines are quarks. We denote the value of the diagram as $(-ie/2)\left[\mathcal{F}^{\mu}_{\pi\pi}(q^2)\right]$. Then, in Fig. 3b, we show a series of quark loops. Using the definitions made in the last section, we may sum the series, including the process of Fig. 3a, to obtain (for the isovector current)

$$f^{\mu}_{V}(q) \equiv \left[\frac{-ie}{2}\right] \frac{1}{1 - G_\rho \hat{J}_{(\rho)}(q^2)} \mathcal{F}^{\mu}_{\pi\pi}(q) \quad . \tag{3.1}$$

With $n_c = 3$ and $n_f = -2$, and a statistical factor of 2, we have

$$\mathcal{F}^{\mu}_{\pi\pi}(q,\kappa) = (-1)2n_c n_f\, g^2_{\pi qq}(m^2_\pi) \int \frac{d^4k}{(2\pi)^4}\, \mathrm{Tr}\big[\hat{\Gamma}^{\mu}(q,k) \times iS(-q/2+k)\gamma_5 iS(k-\kappa)\gamma_5 iS(q/2+k)\big] \quad , \tag{3.2}$$

$$= 2\hat{\kappa}^{\mu}(q)F_{\pi\pi}(q^2) \quad . \tag{3.3}$$

(See Fig. 4.) We may write

$$F_{\pi\pi}(q^2) = (-n_f)n_c\, g^2_{\pi qq}(m^2_\pi)H(q^2) \quad , \tag{3.4}$$

thereby defining $H(q^2)$ [10]. (Note that $F_{\pi\pi}(q^2)$ and $H(q^2)$ are real and negative.) In Eqs. (3.2) and (3.3) we have made use of $\hat{\kappa}^{\mu}(q) \equiv \kappa^{\mu} - (\kappa \cdot q)q^{\mu}/q^2$ and $S(p) = [\not{p} - m_q + i\epsilon]^{-1}$. The factor of (-1) appears in Eq. (3.2) because we have a single closed fermion loop in Fig. 4.

Further, $\hat{\Gamma}^{\mu}(q,k)$ denotes the confinement vertex [11], with the property, $q_{\mu}\hat{\Gamma}^{\mu}(q,k)=0$. In order to include the coupling to the two-pion continuum, we consider diagrams such as those of Fig. 3c. These diagrams may be included, if we replace Eq. (3.1) by

$$f_{\rm V}^{\mu}(q) = \left[\frac{-ie}{2}\right] \frac{1}{1-G_{\rho}[\hat{J}_{(\rho)}(q^2)+\hat{K}_{(\rho)}(q^2)]} \mathscr{F}_{\pi\pi}^{\mu}(q,\kappa) \quad . \tag{3.5}$$

Using Eq. (2.8), we have

$$f_{\rm V}^{\mu}(q) = \left[\frac{-ie}{2}\right] \frac{1}{G_{\rho}} \frac{-g_{\rho qq}^{2}(q^2)}{q^2-m_{\rho}^2+im_{\rho}\Gamma_{\rho}(q^2)} \mathscr{F}_{\pi\pi}^{\mu}(q,\kappa) \quad , \tag{3.6}$$

which, upon use of Eq. (3.2), is

$$f_{\rm V}^{\mu}(q) = \left[\frac{-ie}{2}\right] \frac{2\hat{\kappa}^{\mu}(q)}{G_{\rho}} \frac{g_{\rho qq}^{2}(q^2)}{q^2-m_{\rho}^2+im_{\rho}\Gamma_{\rho}(q^2)} [-F_{\pi\pi}(q^2)] \quad . \tag{3.7}$$

It is useful to write the least expression as

$$f_{\rm V}^{\mu}(q) = (-ie)2\hat{\kappa}^{\mu}(q) \left[\frac{g_{\rho qq}(q^2)}{2G_{\rho}}\right] \frac{\left[-g_{\rho qq}(q^2)F_{\pi\pi}(q^2)\right]}{q^2-m_{\rho}^2+im_{\rho}\Gamma_{\rho}(q^2)} \quad . \tag{3.8}$$

From this, we see that the pion form factor is

$$F_{\pi}(q^2) = -\frac{m_{\rho}^2}{g^{\rho}(q^2)} \frac{1}{q^2-m_{\rho}^2+im_{\rho}\Gamma_{\rho}(q^2)} g_{\rho\pi\pi}(q^2) \tag{3.9}$$

where we have defined

$$g_{\rho\pi\pi}(q^2) = -g_{\rho qq}(q^2)F_{\pi\pi}(q^2) \tag{3.10}$$

and

$$\frac{m_\rho^2}{g^\rho(q^2)} = \frac{g_{\rho qq}(q^2)}{2G_\rho} \quad . \tag{3.11}$$

We may write Eq. (3.11) as

$$\frac{m_\rho^2}{g^\rho(q^2)} = \frac{1}{2} g_{\rho qq}(q^2)\left[\hat{J}_{(\rho)}(m_\rho^2) + \mathrm{Re}\hat{K}_{(\rho)}(m_\rho^2)\right] \quad , \tag{3.12}$$

since the equation

$$1 - G_\rho\left[\hat{J}_{(\rho)}(m_\rho^2) + \mathrm{Re}\hat{K}_{(\rho)}(m_\rho^2)\right] = 0 \tag{3.13}$$

determines the value of m_ρ^2 in this formalism. Thus, we see that

$$g^\rho(q^2) = \frac{2m_\rho^2}{g_{\rho qq}(q^2)\left[\hat{J}_{(\rho)}(m_\rho^2) + \hat{K}_{(\rho)}(m_\rho^2)\right]} \quad . \tag{3.14}$$

For $q^2 > 0$, $g_{\rho qq}(q^2)$ increases as q^2 is reduced in value. (See Table 2.) Therefore, $g^\rho(q^2)$ decreases with decreasing q^2.

Note that

$$F_\pi(0) = \frac{g_{\rho\pi\pi}(0)}{g^\rho(0)} \quad , \tag{3.15}$$

so that we should require $g_{\rho\pi\pi}(0)/g^\rho(0) = 1$ in a consistent formalism.

The new feature in the above equation is the q^2 dependence of $g_{\rho\pi\pi}(q^2)$ and $g^\rho(q^2)$. Near the rho pole, we find the formula of the VMD1 model,

$$F_\pi(q^2) = -\frac{m_\rho^2}{g^\rho} \frac{1}{q^2 - m_\rho^2 + i m_\rho \Gamma_\rho(m_\rho^2)} g_{\rho\pi\pi} \quad , \tag{3.16}$$

where we have put $g^\rho = g^\rho(m_\rho^2)$ and $g_{\rho\pi\pi} = g_{\rho\pi\pi}(m_\rho^2)$. We remark that $g_{\rho\pi\pi}(0)/g^\rho(0)$ could be equal to 1, even if $g_{\rho\pi\pi}(m_\rho^2)/g^\rho(m_\rho^2) = g_{\rho\pi\pi}/g^\rho \neq 1$. (For example, one has the phenomenological values $g_{\rho\pi\pi} \simeq 6.04$ and $g^\rho \simeq 5.07$, so that $g_{\rho\pi\pi}/g^\rho \simeq 1.2$.)

From Eq. (3.9) we see that, for $q^2 > 4m_\pi^2$,

$$\mathrm{Im}\, F_\pi(q^2) = \frac{m_\rho^3 \Gamma_\rho(q^2)}{\left[q^2 - m_\rho^2\right]^2 + \left[m_\rho \Gamma_\rho(q^2)\right]^2} \frac{g_{\rho\pi\pi}(q^2)}{g^\rho(q^2)} \quad . \tag{3.17}$$

Let us assume that our model provides a satisfactory result for both $\Gamma_\rho(q^2)$, and $g_{\rho\pi\pi}(q^2)/g^\rho(q^2)$, when $q^2 \simeq m_\rho^2$. Then we may obtain $F_\pi(q^2)$, for all values of q^2, from the dispersion relation

$$F_\pi(q^2) = 1 + \frac{q^2}{\pi} \int_{4m_\pi^2} ds \frac{\mathrm{Im}\, F_\pi(s)}{s(s - q^2 - i\epsilon)} \quad . \tag{3.18}$$

This dispersion relation ensures that $F_\pi(0) = 1$ and its use is, therefore, not as restrictive as imposing the condition $g_{\rho\pi\pi}(0)/g^\rho(0) = 1$.

IV. The Pion Form Factor: Rho-Omega Mixing

Equation (3.9) describes a vector-meson-dominance model of the pion form factor in the absence of ρ-ω mixing. To study the contribution of ρ-ω mixing we need to define the tensor

$$\hat{J}^{\mu\nu}_{(\rho\omega)}(q) = -\tilde{g}^{\mu\nu}(q)\,\hat{J}_{(\rho\omega)}(q^2) \tag{4.1}$$

which is shown in Fig. 5a as a (cross-hatched) quark loop. (Values of $\hat{J}_{(\rho\omega)}(q^2)$ may be found in Ref. [12].) In Fig. 5b we show the diagrams that are summed in our model. Using the various relations given in Section II, we have (for the isoscalar amplitude),

$$f^{\mu}_{S}(q) = \left(\frac{-ie}{6}\right)\frac{1}{1-G_{\omega}[\hat{J}_{(\omega)}(q^2)+\hat{K}_{(\omega)}(q^2)]}\left[\frac{\hat{J}_{(\omega\rho)}(q^2)}{i}\right]\frac{iG_{\rho}}{1-G_{(\rho)}[\hat{J}_{(\rho)}(q^2)+\hat{K}_{(\rho)}(q^2)]}\mathcal{F}^{\mu}_{\pi\pi}(q,\kappa) \tag{4.2}$$

$$= \left(\frac{-ie}{6}\right)\frac{1}{G_{\omega}}\,\frac{g^2_{\pi qq}(q^2)}{q^2-m^2_{\omega}+im_{\omega}\Gamma_{\omega}(q^2)}\,[\hat{J}_{(\omega\rho)}(q^2)]\,\frac{g^2_{\rho qq}(q^2)}{q^2-m^2_{\rho}+im_{\rho}\Gamma_{\rho}(q^2)}\,\mathcal{F}^{\mu}_{\pi\pi}(q,\kappa)\ . \tag{4.3}$$

This expression may be reorganized to yield

$$= (-ie)\,\frac{m^2_{\omega}}{g^{\omega}(q^2)}\left[\frac{2\hat{\kappa}^{\mu}(q)}{q^2-m^2_{\omega}+im_{\omega}\Gamma_{\omega}(q^2)}\right][\Theta_{(\omega\rho)}(q^2)]\left[\frac{1}{q^2-m^2_{\rho}+im_{\rho}\Gamma_{\rho}(q^2)}\right]g_{\rho\pi\pi}(q^2)\ , \tag{4.4}$$

where we have put $\mathcal{F}^{\mu}_{\pi\pi}(q,\kappa) = 2\hat{\kappa}^{\mu}(q)F_{\pi\pi}(q^2)$ and used

$$\frac{m^2_{\omega}}{g^{\omega}(q^2)} = \frac{g_{\omega qq}(q^2)}{6G_{\omega}}\ , \tag{4.5}$$

so that

$$g^{\omega}(q^2) = \frac{6m_\omega^2}{g_{\omega qq}(q^2)\left[\hat{J}_{(\omega)}(m_\omega^2) + \mathrm{Re}\,\hat{K}_{(\omega)}(m_\omega^2)\right]} \quad . \tag{4.6}$$

We have used

$$\frac{1}{G_\omega} = \left[\hat{J}_{(\omega)}(m_\omega^2) + \mathrm{Re}\,\hat{K}_{(\omega)}(m_\omega^2)\right] \quad , \tag{4.7}$$

in passing from Eq. (4.5) to (4.6) and have defined

$$\Theta_{(\omega\rho)}(q^2) = -\,g_{\omega qq}(q^2)\hat{J}_{(\omega\rho)}(q^2)\,g_{\rho qq}(q^2) \quad . \tag{4.8}$$

We can see that $\Theta_{(\omega\rho)}(m_\omega^2) = \langle \omega \,|\, H_{SB} \,|\, \rho \rangle$, with

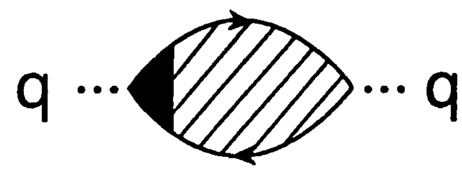
$$\langle \omega \,|\, H_{SB} \,|\, \rho \rangle = -\,g_{\omega qq}(m_\omega^2)\hat{J}_{(\omega\rho)}(m_\omega^2)\,g_{\rho qq}(m_\omega^2) \quad , \tag{4.9}$$

since the matrix element is defined at $q^2 = m_\omega^2$.

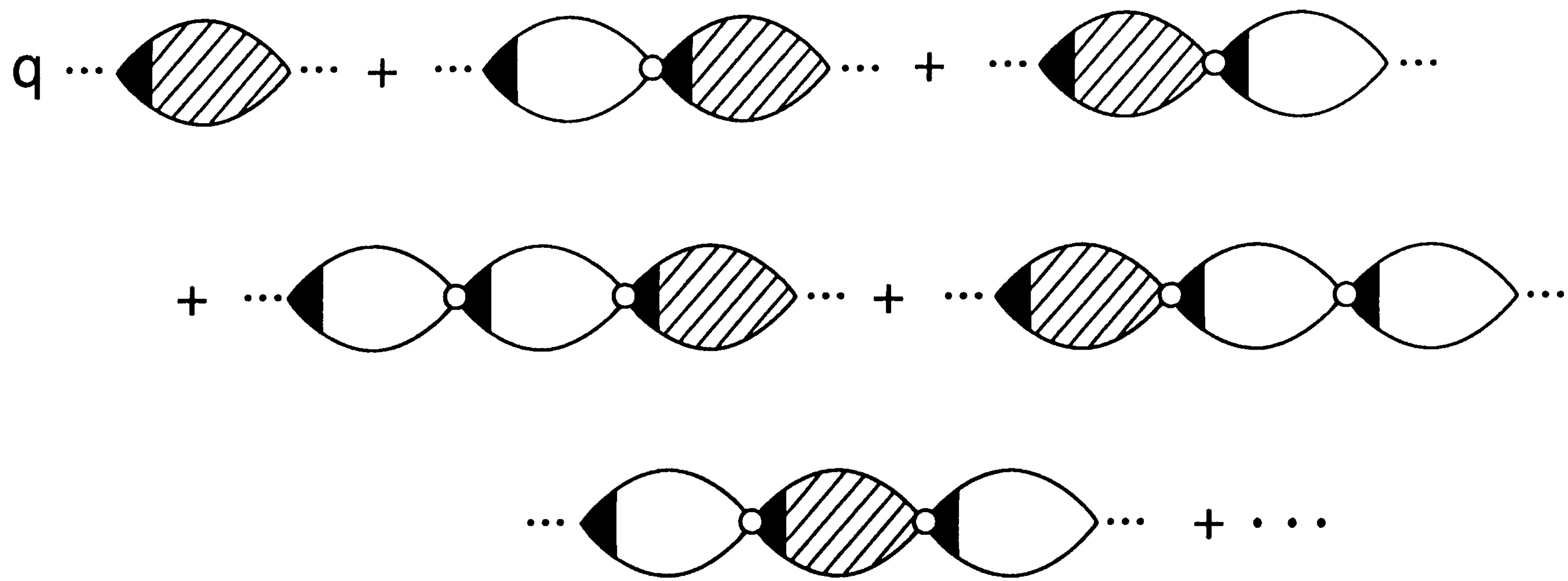
Putting these results together, we have the pion form factor

$$F_\pi(q^2) = \left\{ -\frac{m_\rho^2}{g^\rho(q^2)} - \frac{m_\omega^2}{g^\omega(q^2)} \left[\frac{1}{q^2 - m_\omega^2 + im_\omega\Gamma_\omega(q^2)}\right] \Theta_{(\omega\rho)}(q^2) \right\} \times \left[\frac{1}{q^2 - m_\rho^2 + im_\rho\Gamma_\rho(q^2)}\right] g_{\rho\pi\pi}(q^2) \quad . \tag{4.11}$$

The second term in Eq. (4.11) requires that we perform subtraction so that the term vanishes at $q^2 = 0$. Therefore, the condition that $g_{\rho\pi\pi}(0)/g^\rho(0) = 1$ remains the same as before. However, we may again use the subtracted dispersion relation of Eq. (3.18) to ensure $F_\pi(0) = 1$.



(a)



(b)

FIG. 6

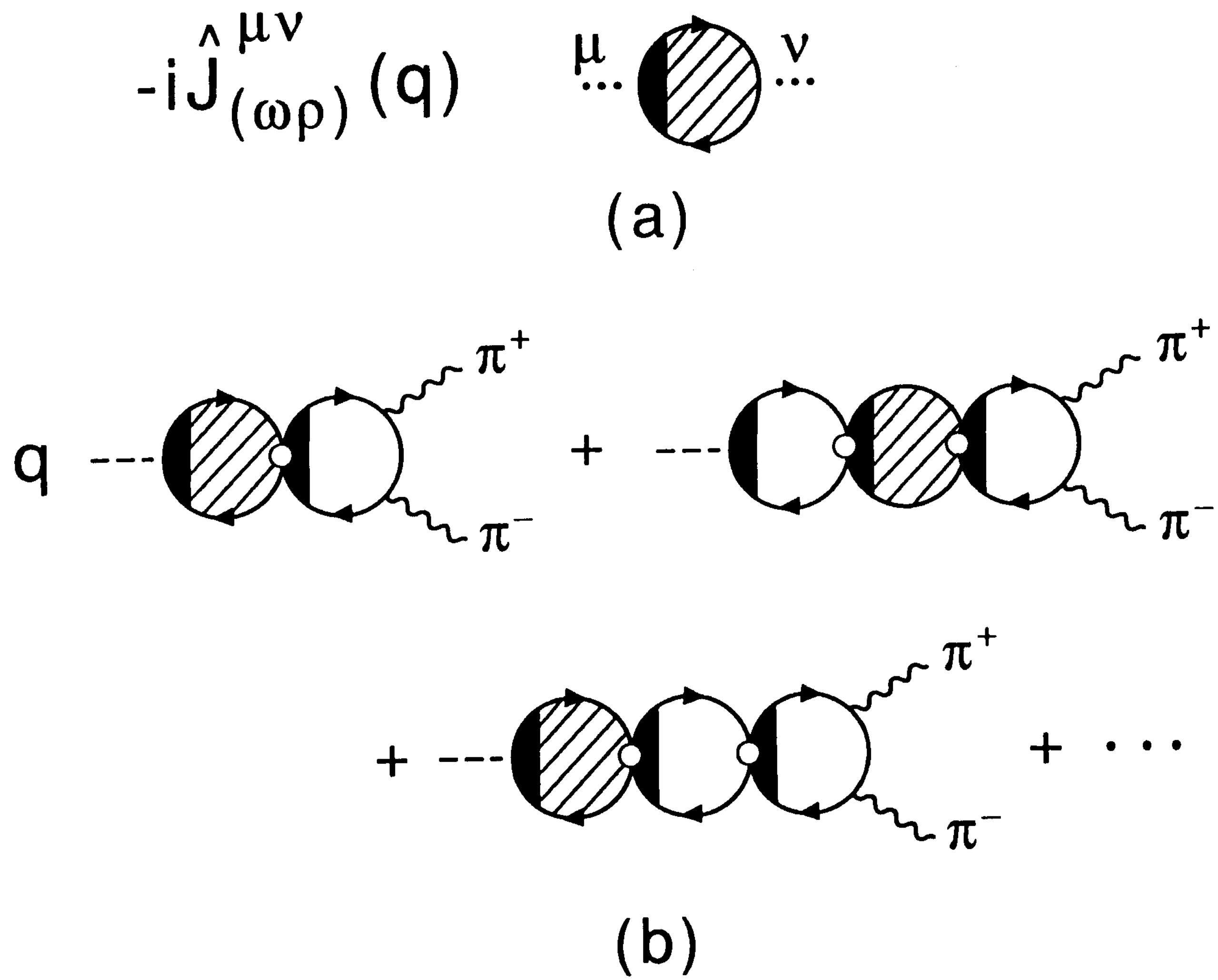


FIG. 5

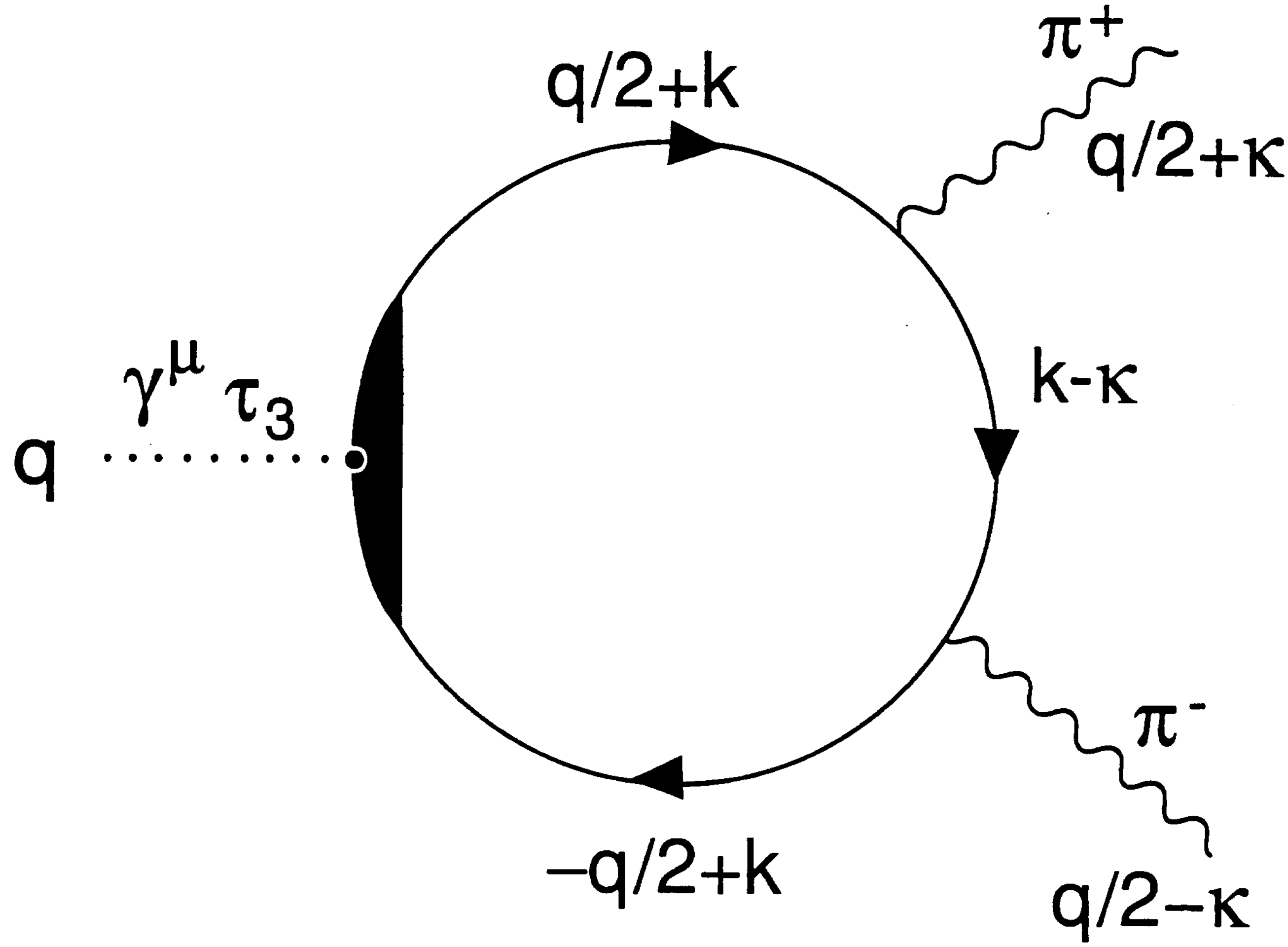


FIG. 4

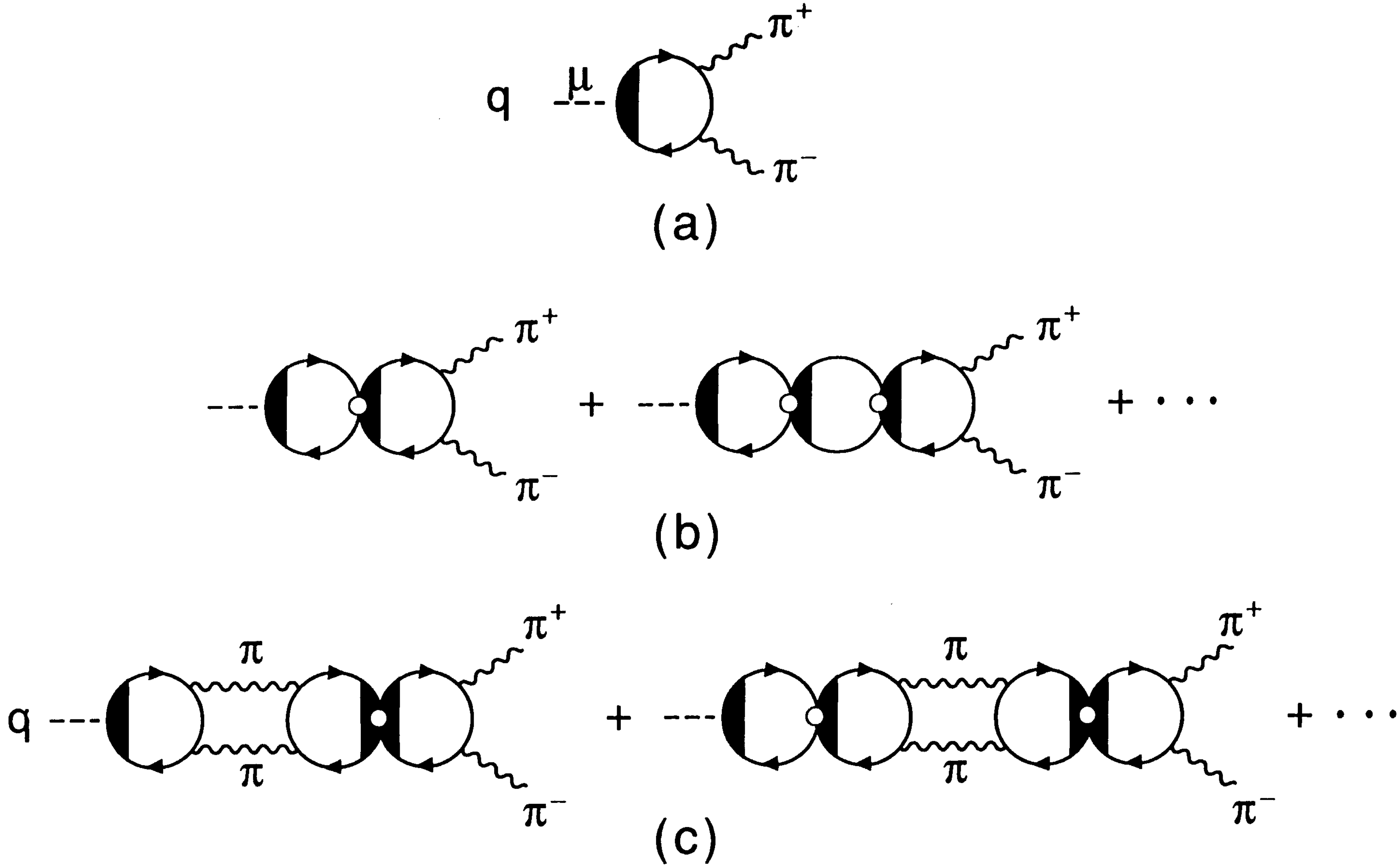


FIG. 3

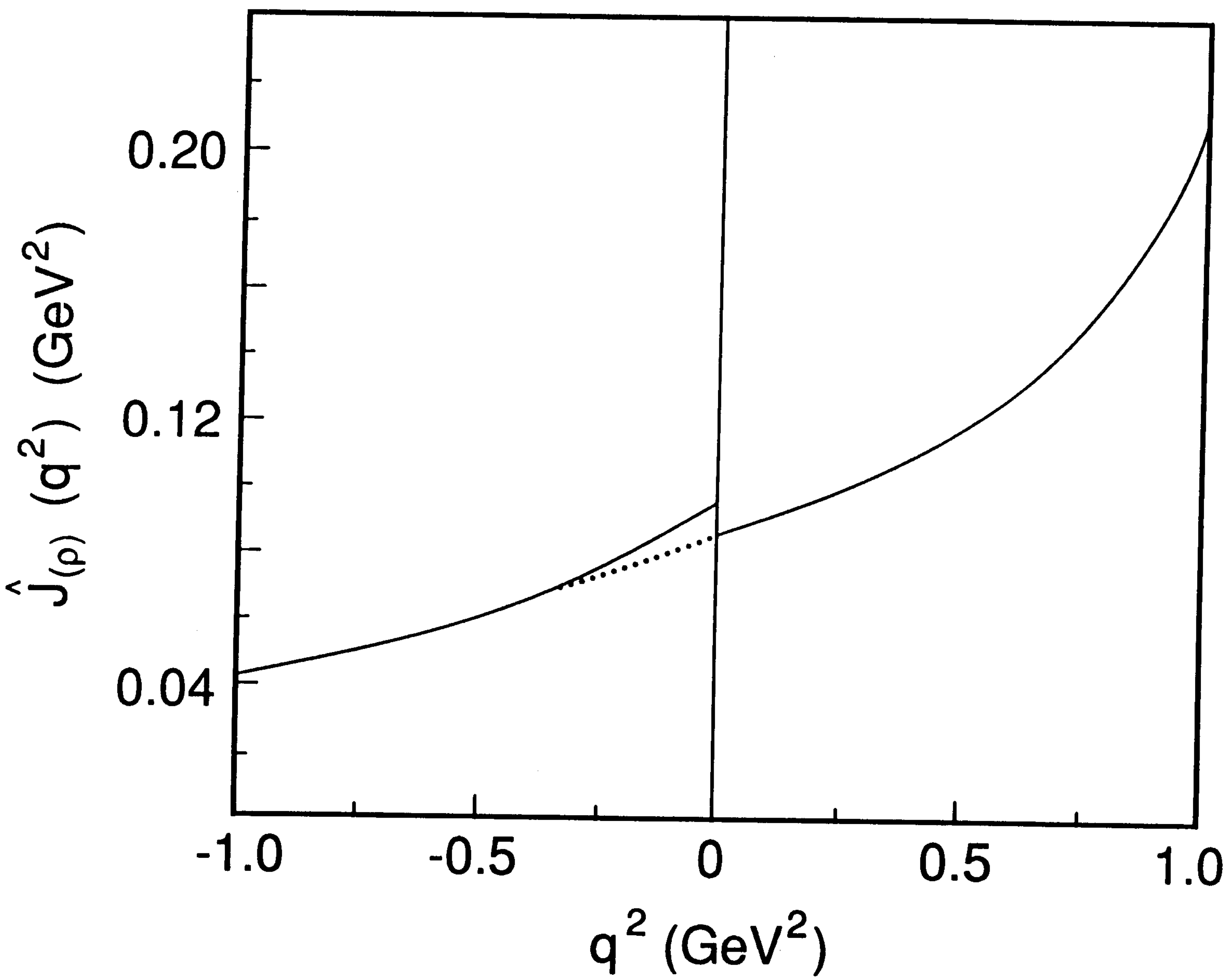


FIG. 2

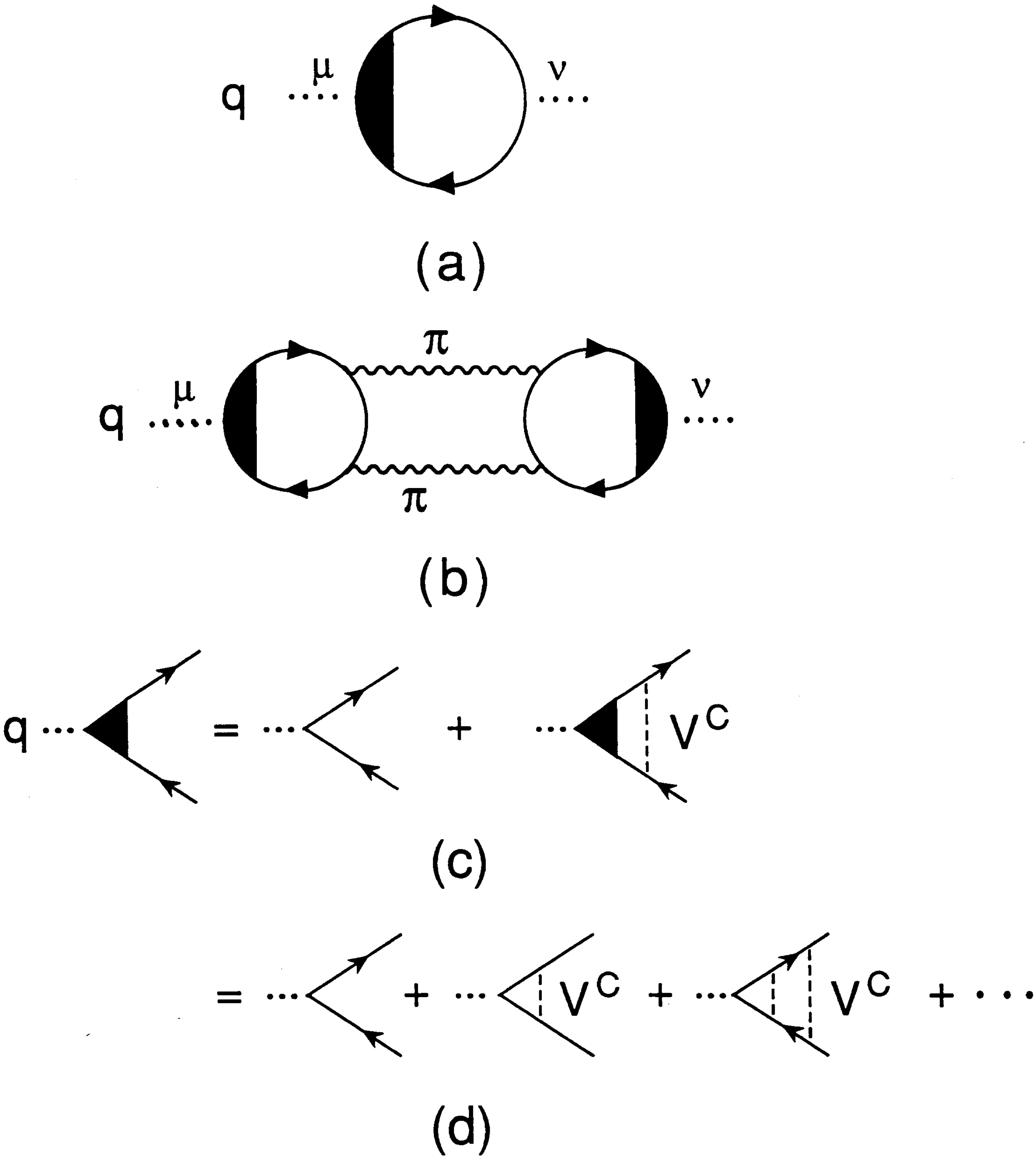


FIG. 1

b) The calculation of the mixed correlation function $\tilde{\Pi}^{\mu\nu}_{(\omega\rho)}(q) = 12\hat{\Pi}^{\mu\nu}_{(\omega\rho)}(q)$ is shown. Each diagram contains a single factor of $\hat{J}_{(\rho\omega)}(q^2)$ and a varying number of factors of $\hat{J}_{(\rho)}(q^2)$ and $\hat{J}_{(\omega)}(q^2)$, with the $\hat{J}_{(\rho)}(q^2)$ factors to the left of $\hat{J}_{(\rho\omega)}(q^2)$ and the $\hat{J}_{(\omega)}(q^2)$ factors to the right of $\hat{J}_{(\rho\omega)}(q^2)$ in the diagram. The open circles represent either iG_ρ or iG_ω, depending upon their position in the diagram. Upon bosonization one obtains Eq. (5.5). (Recall that $\hat{\Pi}_{(\omega\rho)}(q^2)$ was defined in Eq. (5.4).)

b) Additional diagrams that contribute to the amplitude $f_V^\mu(q)$. Summation of these processes yields $f_V^\mu(q)$ of Eq. (3.1). The open circles represent factors of iG_ρ and the wavy lines are pions.

c) Some additional diagrams that are included in the summation that yields $f_V^\mu(q)$ of Eq. (3.4). These diagrams introduce $\hat{K}_{(\rho)}(q^2)$ in the expression for $f_V^\mu(q)$. (See Fig. 1b.)

Fig. 4. The calculation of the diagram shown yields the function $\mathscr{F}^\mu_{\pi\pi}(q)$ of Eq. (3.2). (The isospin trace yields $n_f = -2$ and the color factor is $n_c = 3$. There is also a factor of (-1) since the figure exhibits a single closed fermion loop. (Recall that $\mathscr{F}^\mu_{\pi\pi}(q) = 2\hat{k}^\mu(q) F_{\pi\pi}(q^2)$ and that $2g_{\rho\pi\pi}(q^2) = -g_{\rho qq}(q^2) F_{\pi\pi}(q^2)$, where $F_{\pi\pi}(q^2) < 0$.)

Fig. 5. a) The figure represents the tensor $-i\hat{J}^{\mu\nu}_{(\omega\rho)}(q)$. This tensor is nonzero if $m_d^0 \neq m_u^0$.

b) Some diagrams that are summed to yield $f_S^\mu(q)$ of Eq. (4.2). The dashed lines represent photons of momentum q. The photon-quark vertex yields a factor of $(-ie/6)$. The open circles represent either factors of iG_ω or iG_ρ. [See Eq. (4.2).] Only diagrams with a single factor of $\hat{J}_{(\omega\rho)}(q^2)$ are considered since $\hat{J}_{(\omega\rho)}(q^2)$ is quite small [12].

Fig. 6. a) The diagrammatic element that serves to define $\hat{J}^{\mu\nu}_{(\rho\omega)}(q) = -\hat{g}^{\mu\nu}\hat{J}_{(\rho\omega)}(q^2)$ is shown. The shaded triangular area represents the confining vertex of Fig. 1c.

<u>Figure Captions</u>

Fig. 1. a) The diagram represents the quark-loop integral that yields $-i\hat{J}^{\mu\nu}_{(\rho)}(q)$, or $-i\hat{J}^{\mu\nu}_{(\omega)}(q)$ [10]. The shaded area represents the vertex for the confining interaction, $\hat{\Gamma}^{\mu}$ [11].

b) Calculation of this diagram yields $-i\hat{K}^{\mu\nu}_{(\rho)}(q)$ [10]. The wavy lines represent pions.

c) The inhomogeneous equation that is solved to obtain the confining vertex, $\hat{\Gamma}^{\mu}$ [11]. Here the dashed line is the confinement potential, V^C.

d) The figure shows the interation of the equation shown in c).

Fig. 2. The function $\hat{J}_{(\rho)}(q^2)$ is shown for $q^2 > 0$. The calculation is made with the parameters of model B. (See Table 1.) For $q^2 > 0$ the calculation is made in Minkowski space with a cutoff $\Lambda_3 = 0.702$ GeV. For $q^2 < 0$, we show $J_{(\rho)}(q^2)$; there, confinement is neglected and the calculation is made in a Euclidean momentum space with cutoff $\Lambda_E = 1.0$ GeV [12]. The dotted line serves to interpolate between the results of the two calculations. The difference between the dotted line and the solid line for $q^2 < 0$ provides a measure of the importance of confinement for calculations made for spacelike q^2.

Fig. 3. a) The diagram represents the function $(-ie/2)\mathcal{F}^{\mu}_{\pi\pi}(q)$. The dashed line denotes a photon of momentum q and the shaded area represents the confinement vertex, $\hat{\Gamma}^{\mu}$. (The factor $(-ie/2)$ originates in the photon-quark vertex.)

Table 2. Values of $g_{\rho qq}(q^2)$ and $g_{\omega qq}(q^2)$ are presented for model B. Here $g^2_{\rho qq}(q^2) = (\tilde{m}^2_\rho - q^2)/(G^{-1}_\rho - r_1)$ and $g^2_{\omega qq}(q^2) = (\tilde{m}^2_\omega - q^2)/(G^{-1}_\omega - v_1)$, with $G_\rho = 7.12$ GeV^{-2}, $G_\omega = 7.86$ GeV^{-2}, $\tilde{m}^2_\rho = \tilde{m}^2_\omega = 1.476$ GeV^2, $r_1 = 0.0304$ GeV^2, and $v_1 = 0.0284$ GeV^2. Note that $g_{\rho qq}(m^2_\omega) = 2.80$, $g_{\omega qq}(m^2_\omega) = 2.95$, and $g_{\rho qq}(m^2_\rho) = 2.83$.

q^2(GeV^2)	$g_{\rho qq}(q^2)$	$g_{\omega qq}(q^2)$
0.0	3.66	3.86
0.1	3.54	3.73
0.2	3.41	3.59
0.3	3.27	3.45
0.4	3.13	3.30
0.5	2.98	3.14
0.6	2.82	2.98
0.7	2.66	2.80

Table 1. Parameters and derived quantities in the extended NJL model [10,12]. Here m_q^0 is the average current quark mass, $m_q^0 = (m_d^0 + m_u^0)/2$.

Model	A [10]	B [12]	C
m_q^0	6.50 MeV	5.50 Mev	6.50 MeV
Λ_3	0.560 GeV	0.702 GeV	0.550 GeV
Λ_E	0.790 GeV	1.00 GeV	0.780 GeV
G_ρ	9.20 GeV^{-2}	7.12 GeV^{-2}	9.48 GeV^{-2}
G_ω	-----	7.86 GeV^{-2}	10.92 GeV^{-2}
G_S	15.8 GeV^{-2}	7.91 GeV^{-2}	14.53 GeV^{-2}
m_π	0.140 GeV	0.138 GeV	0.138 MeV
m_q	0.350 GeV	0.260 GeV	0.260 GeV
f_π	0.089 GeV	0.093 GeV	0.078 GeV
$<\bar{u}u>^{1/3}$	-0.230 GeV	-0.252 GeV	-0.261 GeV
m_ρ	0.770 GeV	0.770 GeV	0.770 GeV
m_ω	-----	0.783 GeV	0.783 GeV
Γ_ρ	0.160 GeV	0.144 GeV	0.146 GeV
$g_{\rho qq}(0)$	4.95	3.66	3.87
$g_{\rho qq}(m_\rho^2)$	2.40	2.83	2.56
$g_{\omega qq}(0)$	-----	3.86	3.90
$g_{\omega qq}(m_\omega^2)$	-----	2.95	2.69
$g_{\pi qq}(m_\pi^2)$	3.99	2.68	3.27
$f_\rho(m_\rho^2)$	0.160 GeV	0.256 GeV	0.175 GeV
$g_\rho(m_\rho^2)$	4.81	2.98	4.39
$f_\omega(m_\omega^2)$	-----	0.078	0.0523
$g_\omega(m_\omega^2)$	-----	9.80	14.7
$\hat{J}_{(\rho)}(0)$	0.049 GeV^2	0.0860 GeV^2	0.0510 GeV^2
$m_d^0 - m_u^0$	-----	2.73 MeV	2.81 MeV
$<\rho \mid H_{SB} \mid \omega>$	-----	4520.00 MeV^2	4520.00 MeV^2
κ	0.070 GeV^2	0.220 GeV^2	0.150 GeV^2
$g_{\rho\pi\pi}(m_\rho^2)$	6.22	5.90	5.95

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Acknowledgements

This work was supported in part by a grant from the National Science Foundation and by the PSC-CUNY Faculty Award Program of the City University of New York.

VII. Discussion

In this work we have used our extended version of the NJL model to provide a microscopic understanding of the vector-meson-dominance model. We have discussed the calculation of the pion form factor, $F_\pi(q^2)$, and have included a description of omega-rho mixing.

In a previous work [12], we have shown that $\hat{J}_{(\omega\rho)}(q^2)$ is proportional to $\Delta m = m_d^0 - m_u^0$ and that reasonable values of Δm are found when we fit the value of $< \omega \mid H_{SB} \mid \rho >$ extracted from experimental data. (See Table 1.) Our work shows how to make an unambiguous specification of the correlator of the rho and omega fields. We also show how q^2-dependent meson decay constants, $g^\omega(q^2)$ and $g^\rho(q^2)$, and q^2-dependent meson-meson coupling constants, such as $g_{\rho\pi\pi}(q^2)$, emerge naturally in this formalism.

In one of our other works, that we here designate as model B, we have used $m_q = 260$ MeV, a value that leads to a satisfactory value for the mass of an effective sigma meson, $m_\sigma = 540$ MeV. The parameters of model B are given in Table 1. Good values are obtained for m_π, f_π, $<\bar{u}u>^{1/3}$, m_ρ, m_ω and Γ_ρ. However, the values of $g_\rho = m_\rho / f_\rho$ and $g_\omega = m_\omega / f_\omega$ are only about 60% of the empirical values: $g_\rho \simeq 5.3$ and $g_\omega \simeq 15.2$. Since f_ρ and f_ω are related to the value of the square of the quark wave function at the origin, it is seen that a reduction of the confining field (by making κ smaller) will decrease both f_ρ and f_ω. That is accomplished in model A. Alternatively, working with a smaller value of m_q should reduce f_ρ. Also, we may see that f_π, f_ρ, and f_ω are sensitive to the cutoff, Λ_3 or Λ_E. For example, if the cutoff is reduced, the wave function has fewer high-momentum components. Therefore, it tends to fill a larger volume, reducing the value at the origin. That is, reducing Λ_E (or Λ_3) yields a reduction of the decay constants f_π, f_ρ, and f_ω.

In Table 1 we also present the parameters of model C, along with the predictions for various physical quantities. There we see improved values for g^ρ and g^ω relative to model B. However, that improvement is at the expense of a less good value for f_π. (We find $f_\pi = 0.078$ GeV for model C.) Therefore, we see that a more extensive parameter search may be called for. There is also the possibility that the highly simplified potential of the extended NJL model is not capable of producing very good values for f_π, g^ρ and g^ω, simultaneously.

VI. Choice of Parameters for the Extended NJL Model

If we inspect the Lagrangian of Eq. (2.1), we see the parameters m_q^0, G_S, G_ρ, and G_ω. The confining potential depends upon a parameter κ. (That potential is $V^C(r) = \kappa r e^{-\mu r}$ in coordinate space. The parameter μ is introduced to regulate the singularity of the Fourier transform of $V^C(r)$. Since μ is introduced to facilitate the numerical calculation, we do not count μ among the parameters that we vary.) In addition, we have to specify the cutoff. For Euclidean momentum-space calculations, the cutoff is Λ_E. The corresponding cutoff in Minkowski space is Λ_3. We found that $\Lambda_3 = 0.702$ GeV, if $\Lambda_E = 1.0$ GeV, for example. Thus, the parameters are m_q^0, G_S, G_ρ, G_ω, κ and Λ_E.

We may require that the analysis yield a specific value for the constituent quark mass, m_q, and the experimental value for the pion decay constant f_π ($f_\pi \simeq 93$ MeV). These requirements can be used to fix G_S and Λ_E. Then, m_q^0 may be chosen to give the correct value for m_π. Now, if we fix κ, we can determine G_ρ and G_ω so that m_ρ and m_ω are given correctly. A "good choice" for κ will also yield a satisfactory value for $\Gamma_\rho = \Gamma_\rho(m_\rho^2)$. This entire procedure does not necessarily lead to satisfactory values for g^ρ and g^ω. For example, Table 1 contains the parameters for three specific examples, which we call models A, B and C. Model A is the model considered in Ref. [10]. [We note that κ is quite small ($\kappa = 0.070$ GeV^2) for model A.] In model A, it may be seen that reasonable values are obtained for m_π, f_π, m_ρ, Γ_ρ and the quark condensate. The value of $f_\rho = 0.160$ GeV is a bit larger than the empirical value of $f_\rho \simeq 0.152$ GeV. This might seem to be a generally satisfactory result, except that the value of $m_q = 350$ MeV appears to be somewhat too large.

$$\rho^{\mu}(x) = \frac{g^{\rho}}{m_{\rho}^{2}} j_{V}^{\mu}(x) \quad , \tag{5.10}$$

to the case where the ρ and ω fields are off-mass-shell. The generalized form that appears in Eq. (5.5), for example, appears naturally in the momentum-space bosonization procedure.

In a similar fashion, we may define

$$\hat{\Pi}^{\mu\nu}_{(\rho)} = i \int d^4x e^{iq\cdot x} < 0 \,|\, T[j_V^{\mu}(x) j_V^{\nu}(0)] \,|\, 0 > \quad , \tag{5.11}$$

$$= - \tilde{g}^{\mu\nu}(q) \hat{\Pi}_{(\rho)}(q^2) \quad , \tag{5.12}$$

and

$$\hat{\Pi}^{\mu\nu}_{(\omega)} = i \int d^4x e^{iq\cdot x} < 0 \,|\, T[j_S^{\mu}(x) j_S^{\nu}(0)] \,|\, 0 > \quad , \tag{5.13}$$

$$= - \tilde{g}^{\mu\nu}(q) \hat{\Pi}_{(\omega)}(q^2) \quad , \tag{5.14}$$

Again, using the equations developed in the previous sections, we have

$$\hat{\Pi}_{(\rho)}(q^2) = \left[\frac{m_{\rho}^2}{g^{\rho}(q^2)} \right] \frac{1}{q^2 - m_{\rho}^2 + i m_{\rho} \Gamma_{\rho}(q^2)} \left[\frac{m_{\rho}^2}{g^{\rho}(q^2)} \right] \tag{5.15}$$

where the result has been organized in analogy to the expression obtained for $\Pi_{(\omega\rho)}(q^2)$ given in Eq. (5.5). We also have the relation

$$\hat{\Pi}_{(\omega)}(q^2) = \left[\frac{m_{\omega}^2}{g^{\omega}(q^2)} \right] \frac{1}{q^2 - m_{\omega}^2 + i m_{\omega} \Gamma_{\omega}(q^2)} \left[\frac{m_{\omega}^2}{g^{\omega}(q^2)} \right] \quad . \tag{5.16}$$

$$\Pi^{\mu\nu}_{(\omega\rho)}(q) = i\int d^4x e^{iq\cdot x} <0|T\big(\omega^\mu(x)\rho^\nu(0)\big)|0>$$
$$= -\tilde{g}^{\mu\nu}(q)\Pi_{(\omega\rho)}(q^2) \quad . \tag{5.6}$$

This definition is subject to some ambiguity, since the interpolating fields, $\omega^\mu(x)$ and $\rho^\mu(x)$, may undergo various transformations without changing the S matrices of the theory [13,14]. However, we may proceed to define $\Pi^{\mu\nu}_{(\omega\rho)}(q)$ in an unambiguous fashion. Let us define

$$\Pi^{\mu\nu}_{(\omega\rho)}(q) = -\tilde{g}^{\mu\nu}(q)\frac{1}{q^2-\big(m_\rho - i\Gamma_\rho(q^2)/2\big)^2}\left[-\theta(q^2)\right]$$
$$\times \frac{1}{q^2-\big(m_\omega - i\Gamma_\omega(q^2)/2\big)^2} \quad . \tag{5.7}$$

Here, $\theta(q^2)$ is the quantity defined by Maltman [13]. As we will see, $\theta(m_\rho^2) < 0$. We may now go on to exhibit the relation between $\hat{\Pi}_{(\rho\omega)_{\mu\nu}}(q)$ and $\Pi^{\mu\nu}_{(\rho\omega)}(q)$. We see that we may write

$$\hat{\Pi}_{(\omega\rho)}(q^2) = \frac{m_\omega^2}{g^\omega(q^2)}\,\frac{m_\rho^2}{g^\rho(q^2)}\,\Pi_{(\omega\rho)}(q^2) \tag{5.8}$$

with $\theta(q^2) = \Theta_{(\omega\rho)}(q^2)$. We may recall that $\Theta_{(\omega\rho)}(m_\omega^2) = <\omega\,|\,H_{SB}|\,\rho>$. The factors $m_\omega^2/g^\omega(q^2)$ and $m_\rho^2/g^\rho(q^2)$ in Eq. (5.5) serve to generalize the usual VMD relations,

$$\omega^\mu(x) = \frac{g^\omega}{m_\omega^2}\,j_S^\mu(x) \quad , \tag{5.9}$$

and

V. Hadronic Current Correlation Functions

It is useful to define the isoscalar and isovector hadronic currents,

$$j_S^\mu(x) = \frac{1}{6}\bar{q}(x)\gamma^\mu q(x) \quad , \tag{5.1}$$

and

$$j_V^\mu(x) = \frac{1}{2}\bar{q}(x)\gamma^\mu \tau_3 q(x) \quad , \tag{5.2}$$

which are combined when forming the electromagnetic current. Using these currents, we may define a mixed correlation function

$$\hat{\Pi}^{\mu\nu}_{(\omega\rho)}(q) = i\int d^4x\, e^{iq\cdot x} <0|T[j_S^\mu(x) j_V^\nu(0)]|0> \quad , \tag{5.3}$$

$$= -\tilde{g}^{\mu\nu}(q)\hat{\Pi}_{(\omega\rho)}(q^2) \quad , \tag{5.4}$$

where we recall that $\tilde{g}^{\mu\nu}(q) = g^{\mu\nu} - q^\mu q^\nu / q^2$. Using the same procedures as those used in the preceding sections, we find

$$\hat{\Pi}_{(\omega\rho)}(q^2) = \left[\frac{m_\omega^2}{g^\omega(q^2)}\right] \frac{1}{q^2 - m_\omega^2 + im_\omega\Gamma_\omega(q^2)} \left[-\Theta_{(\omega\rho)}(q^2)\right] \times \frac{1}{q^2 - m_\rho^2 + im_\rho\Gamma_\rho(q^2)} \left[\frac{m_\rho^2}{g^\rho(q^2)}\right] \quad . \tag{5.5}$$

[See Fig. 6.] This result has a simple diagrammatic representation. (Recall that $\Theta_{(\omega\rho)}(q^2) = -g_{\omega qq}(q^2)\hat{J}_{(\omega\rho)}(q^2)g_{\rho qq}(q^2)$.)

Some authors have found it useful to define a correlation function for rho and omega fields [13], We prefer to use the latter procedure since it is difficult to achieve the relation $g_{\rho\pi\pi}(0)/g^{\rho}(0) = 1$ in the NJL model.

Finally, we remark that the results of this and the previous section for $f_V^{\mu}(q)$ or $f_S^{\mu}(q)$ may be obtained by inserting at the photon-meson vertex $-iem_{\rho}^2/g^{\rho}(q^2)$ or $-iem_{\omega}^2/g^{\omega}(q^2)$. At the meson-quark vertex one introduces $-ig_{\rho qq}(q^2)$ or $-ig_{\omega qq}(q^2)$. The rho and omega propagators are $-ig^{\mu\nu}/[q^2 - m_{\rho}^2 + im_{\rho}\Gamma_{\rho}(q^2)]$ and $-ig^{\mu\nu}/[q^2 - m_{\omega}^2 + im_{\omega}\Gamma_{\omega}(q^2)]$. The diagrammatic element shown in Fig. 4 yields a factor of $\mathcal{F}^{\mu}_{\pi\pi}(q, \kappa) = 2\hat{\kappa}^{\mu}(q)F_{\pi\pi}(q^2)$ and the diagrammatic element in Fig. 5a introduces a factor of $\hat{J}^{\mu\nu}_{(\omega\rho)}(q)/i = -\tilde{g}^{\mu\nu}(q)\hat{J}_{(\omega\rho)}(q^2)/i$.