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Chapter 28

Non-linear Optics Measurements and Corrections

E. H. Maclean[a], F. Carlier[a], J. W. Dilly[a], M. Giovannozzi[a], N. Karastathis[b],
T. H. B. Persson[a] and R. Tomás[a]

[a]*CERN, BE Department, Genève 23, CH-1211, Switzerland*

[b]*Former CERN member*

Nonlinear optics errors in low-β^* insertions pose a serious challenge to successful operation of the HL-LHC. LHC experience however has demonstrated that the previously assumed correction strategy, based upon ideal compensation of selected nonlinear resonances, as determined from magnetic measurements, suffers from several limitations. A beam-based correction approach yielded a positive operational impact in the LHC, and dedicated machine studies have helped establish new methods for nonlinear optics corrections in HL-LHC.

1. Motivation for Correction

Nonlinear errors in low-β^* Insertion Regions (IRs) can dramatically perturb the beam-dynamics (where β^* denotes the Courant-Snyder β function at the experimental Interaction Points, IP). At small β^* the errors in such insertions are expected to be the dominant source of nonlinear optics perturbations in both the LHC and HL-LHC. Traditionally concern in relation to nonlinear errors in the low-β IRs has focused on loss of dynamic aperture (DA, the boundary in phase space below which particle motion remains bounded for a given number of turns). DA results in beam-losses and lifetime reduction,[1] and depends on the nonlinearities present in the machine. For example, Figure 1 (left) shows simulated HL-LHC DA after 10^6 turns in the operational configuration (with beam-beam and Landau octupoles expected at end of levelling with $\beta^* = 0.15$ m), with and without normal dodecapole corrections applied in IR1 and IR5. A clear deterioration of DA is seen in the absence of nonlinear correction. Multiple studies predict that correction of nonlinear errors in experimental IRs is necessary to maintain a stable extent of phase space sufficient for productive operation in HL-LHC.

Additionally, measurement of linear optics and nonlinear observables at at peak energies in HL-LHC will rely heavily on excitation of driven betatron oscillations with an AC-dipole. Machine studies in the LHC demonstrated the DA of such forced oscillations can be dramatically smaller than for free betatron oscillations.[2] This poses a serious challenge to HL-LHC commissioning, since a good DA will not only be required during luminosity production, sufficient forced-DA will also be necessary in order to perform optics measurements.

DA is also not the only challenge. Uncorrected IR-nonlinear errors perturb linear optics via feed-down from IR orbit-bumps. Figure 1 (right, red) shows the peak-$\frac{\Delta\beta}{\beta}$ ('*peak beta-beating*', which characterises relative linear optics errors), generated in simulations of HL-LHC at end-of-squeeze due to feed-down from uncorrected nonlinear errors in the triplets and separation dipoles. Histograms over 60 instances of the errors (representative of ex-

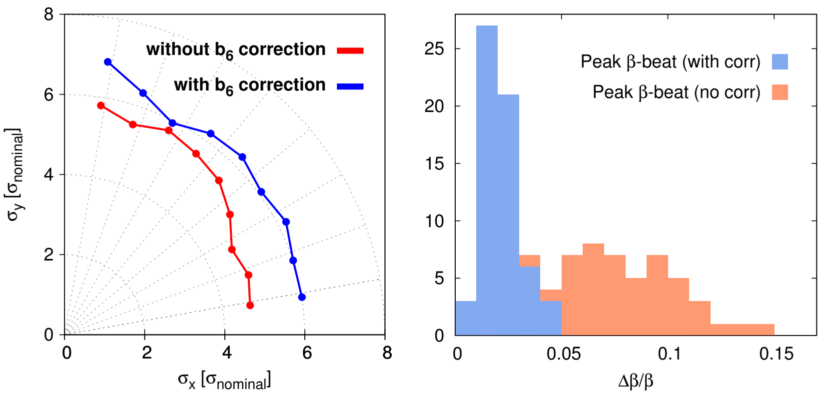


Fig. 1. Left: simulated DA after 10^6 turns during HL-LHC luminosity production, including beam-beam, with/without IR-dodecapole correction. Right: histogram over 60-seeds of simulated peak-$\Delta\beta/\beta$ in HL-LHC, without beam-beam, due to feed-down from IR orbit-bumps at 0.15 m, without (red) and with (blue) correction of all available nonlinear multipoles in the low-β^* IRs.

pected tolerances and uncertainties) are shown. In the most extreme cases, uncorrected nonlinear errors in the IRs generate a peak-$\frac{\Delta\beta}{\beta}$ which approaches machine protection limits ($\frac{\Delta\beta}{\beta} \leq 20\,\%$[3]), while even more moderate cases significantly impinge on the $\frac{\Delta\beta}{\beta}$-margin available to accommodate residuals from linear optics commissioning (typically of the order of 7 % in the LHC before any orbit-bumps in the IRs, and hence any feed-down from nonlinearities, are introduced). The potential luminosity imbalance due to uncorrected feed-down can also become unacceptable, with ATLAS/CMS β^* imbalances showing comparable distributions as the peak beta-beat. The role of nonlinear errors in perturbing linear optics must therefore be considered in HL-LHC commissioning strategy.

Normal-octupole errors in the low-β^* IRs, as well as skew-octupoles and feed-down to linear coupling, can substantially distort the tune-footprint, leading to loss of Landau damping. Uncorrected normal octupole errors in the LHC have already been observed to have an impact on the instability threshold.[4] Uncorrected normal octupole errors in the HL-LHC at end-of-squeeze could generate tune footprint distortion up to 4 times larger than those obtained in LHC.[5] Control of collective instabilities therefore provides additional motivation for correction of nonlinear errors in the HL-LHC.

2. Motivation for Beam-based Measurement and Correction

The baseline correction strategy[6] for nonlinear errors in HL-LHC IRs assumes the possibility of calculating ideal corrections for a wide range of nonlinear resonances based on magnetic measurements during construction, with the principle objective of optimizing dynamic aperture of free betatron oscillations. Even in this ideal case, it should be expected that beam-based measurement will still be necessary in order to validate corrections and assess residual errors: for example, quantifying residual detuning from the IRs to inform Landau damping strategy.

LHC experience however, has highlighted the limitations of the baseline approach and suggests a beam-based approach to correction may be a necessary complement to any magnetic measurements. In the LHC several discrepancies were observed between corrections based on the magnetic model and those required to minimize corresponding beam-based observables. For example,

Figure 2 shows a discrepancy between amplitude detuning expected from magnetic measurements (shown in grey, where 60 instances of the magnetic model are represented, corresponding to uncertainties in the measured errors) and that measured with beam (red). The resulting disparity in required corrections is shown in Figure 3 (center), which compares model- and beam-based settings of octupole correctors in IR1 and 5. A global discrepancy is seen at the level of 30 % in amplitude detuning. Such discrepancies between the magnetic model and real accelerator could lead to sub-optimal performance.

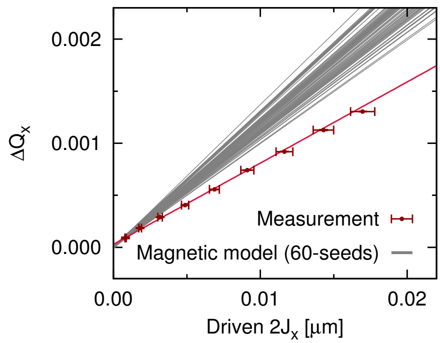


Fig. 2. Measured detuning ($\beta^* = 0.4$ m) in LHC compared to expectations from magnetic model.

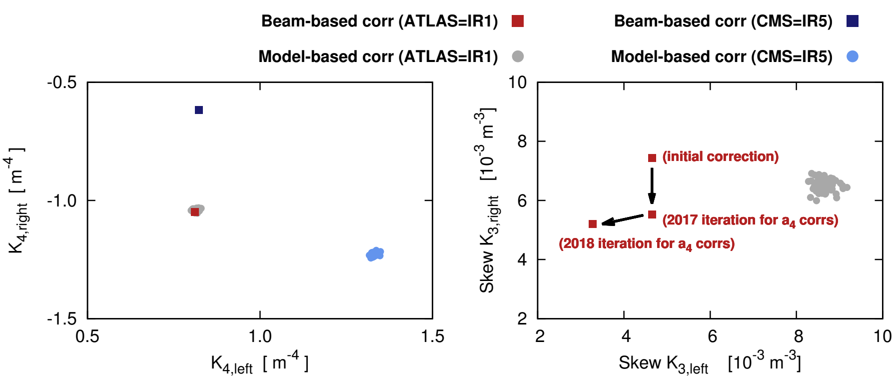


Fig. 3. Left: Beam-based normal octupole corrections in LHC IR1/5 compared to corrections from magnetic model. Right: Iterations of (beam-based) skew-sextupole corrections in LHC IR1 due to changes in skew-octupole corrector powering, compared to corrections from magnetic model.

The reason for this discrepancy is unclear. Regardless of the source however, such discrepancies motivate application of beam-based methodology to help define or improve nonlinear corrections.

Additionally, during LHC studies and commissioning it was observed that alignment errors of the high-order correctors spoiled compensation of lower-order nonlinear errors.[8,9] For example, Figure 3 (right) shows repeated iterations of skew-sextupole corrections which had to be implemented in the LHC as a consequence of changes in skew-octupole corrector powering in 2017 and 2018. The required iterations could be identified with an anomalous 1 mm misalignment or orbit offset of the skew-octupole corrector on the right side of IR1, which introduced additional sextupole errors through feed-down once powered. Such geometric errors are not currently accounted for in the model-based correction strategy. Similarly, additional complications to a model-based correction strategy arise from the large longitudinal variation of β functions over the triplet lengths. For high-order errors (such as dodecapole sources) longitudinal variation of the error distribution within the triplet are therefore capable of causing significant changes to required corrections,[12] which are also not accounted for in the existing LHC model-based correction strategy. Such complications may be relevant to HL-LHC commissioning and motivate further development of both model- and beam-based strategies.

3. Nonlinear Optics Commissioning Experience at the LHC

A beam-based approach to nonlinear correction was adopted for LHC commissioning since 2017 (pre-2017 no IR-nonlinear corrections were performed). LHC optics commissioning strategy emphasised the interrelated nature of linear and nonlinear corrections, with several iterations of interleaved linear and nonlinear optics corrections performed. Detailed reviews of the strategy and outcome are provided.[8–10]

Inclusion of beam-based nonlinear optics corrections into LHC commissioning strategy yielded a number of operational benefits. Of particular note, correction of nonlinear errors improved the performance of online tune measurement. This is visible in Figure 4 (left) which shows substantial reduction to noise in the tune measurement (red) as octupole correction (blue) is applied. Without this improved performance of tune instrumentation the ability to commission the linear optics in the IRs via K-modulation would be significantly hindered, highlighting the importance of adopting an iterative approach between the linear and nonlinear optics corrections. Correction of feed-down from sextupole errors in the ATLAS and CMS insertions also significantly improved optics-related luminosity imbalance between the experiments, while better control of feed-down to linear coupling from IR-nonlinearities and better control of tune-footprint during the β^*-squeeze have been correlated with an improved performance of Landau damping since 2017.[11] Finally in dedicated machine studies at $\beta^* = 0.14$ m application of nonlinear corrections was observed to improve beam-lifetime during optics measurements, as seen in Figure 4 (right) which shows the change in fractional intensity for the two minutes immediately prior (red), and following (blue), application of nonlinear corrections.

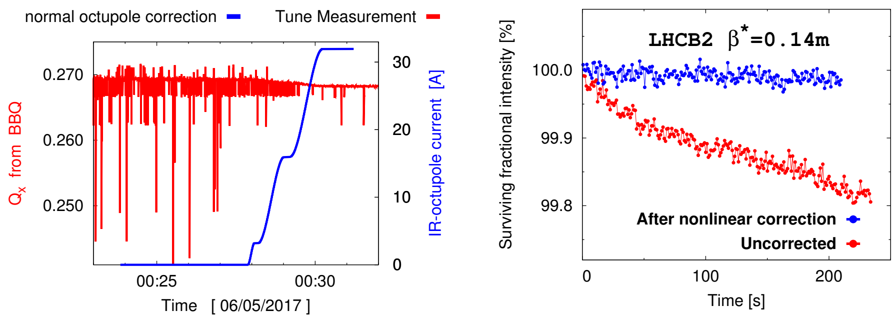


Fig. 4. Left: online measurement of LHC tune during application of octupole corrections in ATLAS and CMS insertions. Right: change in fractional intensity over 2 minutes, before and after application of nonlinear corrections in ATLAS and CMS insertions at $\beta^* = 0.14$ m during dedicated machine tests.

4. Beam-based Measurement Techniques Used at the LHC

To facilitate IR-nonlinear correction in LHC and HL-LHC, beam-based techniques applicable to slow-cycling hadron synchrotrons were developed. Detailed reviews of the measurement techniques employed and tested at the LHC can be found in.[8,9,12,13]

Some success had previously been obtained for IR-nonlinear corrections at RHIC via minimization of feed-down to tune for various orbit bumps applied across an IR.[14] Observation of feed-down also proved effective in the LHC.[8,9,12] Linear and quadratic feed-down to tune was studied for various orbit bumps in the H and V planes across each low-β IR. While the use of several custom assymetric orbit bumps was explored in dedicated tests,[26] in practice studies of feed-down to tune for LHC commissioning primarily utilized the nominal crossing-angle orbit bumps, allowing correctors in IR1 and 5 to be powered in order to minimize tune shifts as a function of the operational bump.[8,9] Figure 5 illustrates this, showing tune-shift vs IR5 crossing-angle before (red) and after (blue) sextupole correction. Where studies at RHIC focused on feed-down to tune, for LHC commissioning this was extended to also consider linear and quadratic feed-down to the f_{1001} linear coupling resonance driving term as a function of the crossing-angle orbit bumps. Optimizing tune and coupling stability vs crossing-angle was particularly relevant for crossing-angle luminosity levelling, where changes to these properties can detrimentally influence lifetime and instabilities. A primary concern during such scans is orbit leakage from the IR-bumps distorting the measurement.[26] Precise control of closed-orbit leakage will be a necessary prerequisite to successful nonlinear optics correction in HL-LHC.

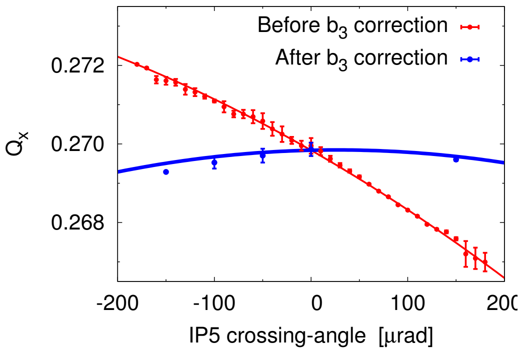


Fig. 5. Tune-shift with CMS crossing-angle, before (red) and after (blue) sextupole correction.

A further key observable developed for nonlinear optics measurements in the LHC is amplitude-detuning via AC-dipole excitation (detuning measurements via single kicks are not possible at top energy due to machine protection concerns and the beam-destructive nature of the single-kicks). Such measurements required both theoretical and experimental developments,[15] but are now a routine component of LHC optics commissioning and were used in the LHC to help define normal octupole corrections in the ATLAS and CMS IRs. An example of such a detuning measurement, used to define normal octupole corrections, is shown in Figure 2. Given the importance of high-order corrections in the HL-LHC, use of the AC-dipole was also developed for measurement of second-order detuning ($\partial^2 Q/\partial J^2$) and feed-down to first-order detuning from orbit bumps over the IRs ($\partial^2 Q/\partial J \partial \theta$) in dedicated machine tests. Both observables appear viable for study of normal/skew decapole and normal dodecapole errors at top energy in the HL-LHC.[12,13] Examples of measurement of second-order detuning and feed-down to first-order detuning are shown in Figure 6 (left/right, respectively).

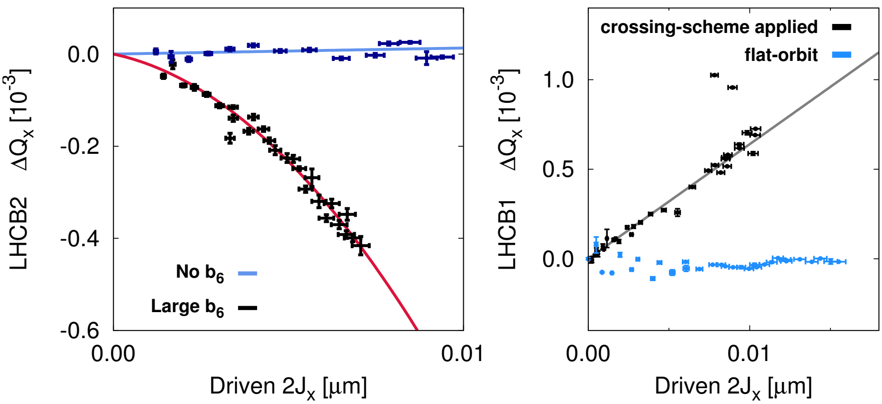


Fig. 6. Left: detuning at flat-orbit (an orbit with all IR orbit-bumps removed) with well-corrected nonlinearities (blue) and enhanced dodecapoles (red) causing a quadratic change of tune with action. Right: detuning at flat-orbit with well-corrected octupolar errors (blue), and with the IR5 crossing-angle orbit bump applied (black), causing feed-down from decapoles and dodecapoles to generate linear detuning with action.

Resonance strengths can be directly characterized by Resonance Driving Terms (RDTs). Free and forced RDTs can be measured with a single kick[21] and AC-dipoles[22] respectively. Minimization of RDTs is already used extensively in the LHC for linear coupling correction. Numerous studies of RDT measurement via AC-dipole excitation were performed during the LHC's second run. A detailed review of the methodology for AC-dipole based RDT measurement in the LHC is provided in.[16,17] RDTs of the driven motion were successfully observed for sextupole, octupole and decapole errors.[8,9,12,16,17] Observations of feed-down to skew-octupole RDTs were also achieved.[12,13,16,17] Direct beam-based correction of forced skew-octupolar RDTs was demonstrated in the LHC during 2018 commissioning,[16,17,20] while sextupole and normal octupole RDT measurements were also used during LHC commissioning to validate corrections based on other observables.[8,9]

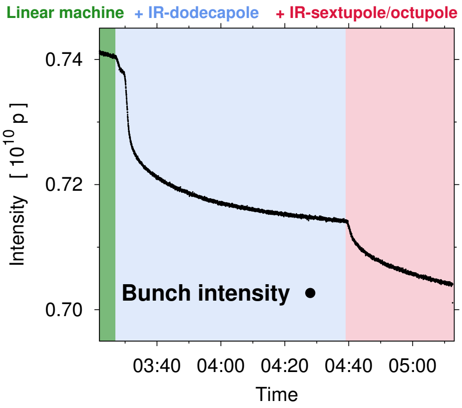
Feed-down, detuning- and RDT-based measurement techniques have been developed, which proved effective in the LHC. These methods are however indirectly associated to dynamic aperture, which will be a key figure of merit to HL-LHC operation. Direct DA measurement techniques based on losses following single-kicks are impractical due to the slow machine cycle. An alternative technique, based on observing beam-loss of bunches heated to large emittance with the Transverse Damper (ADT), was demonstrated in the LHC at injection,[24] and later applied at 6.5 TeV.[25] Figure 7 (left) shows beam loss from DA observed firstly as dodecapole sources (representative of those possible at HL-LHC end-of-squeeze) are introduced (blue region), and then as corrections for sextupole/octupole errors in LHC IRs are removed (red region). DA shifts on the scale of expected errors in HL-LHC were clearly measurable, and could be associated with expected behaviours in simulation,[12,13,25] implying direct measurement of DA is a viable observable to validate nonlinear optics corrections in HL-LHC. As described in Section 1, the DA of forced AC-dipole oscillations also represents a challenge to successful HL-LHC operation. Equally however, beam-loss via forced-DA represents a potential observable for nonlinear correction quality. During dedicated tests it was demonstrated that shifts in forced-DA could also be clearly measured for changes in nonlinear corrector powering.[2,12,13,23]

A broad range of observables viable for study of the nonlinear optics at top energy in the LHC and HL-LHC have been developed. No individual technique was exclusively employed for study of a given multipole however, and in practice a combination of these observables were utilized for beam-based study and correction: for example normal octupole corrections were defined by a combination of detuning and feed-down studies, then validated with RDT measurements, while skew octupole corrections determined from RDT observations could be cross-checked via the quadratic feed-down to linear coupling.[8,9] The breadth of measurement techniques now available at top energy was thus of significant benefit to the commissioning process.

5. Implications of LHC Experience to HL-LHC Commissioning

Experience from the LHC has several implications in regard to nonlinear optics correction at HL-LHC. LHC experience clearly demonstrated the importance of beam-based techniques for measurement and correction in the experimental insertions. Furthermore, while attention in regard to the nonlinear optics has traditionally (and justifiably) been focused towards preserving dynamic aperture and lifetime, LHC experience also highlighted the importance of nonlinear optics quality to the successful control of linear optics and luminosity imbalance, to the performance of beam instrumentation, and to control of Landau damping and instabilities. A particular challenge may arise if the impact from such additional effects also limit operation, since optimal corrections may differ between different figures of merit (for example between feed-down from a multipole and its directly-driven RDTs, due to different dependency on the optics functions and orbit).

The question of residual errors following correction may also be especially pertinent for some of these additional figures of merit. Figure 1 (center) showed histograms of simulated β-beating generated at HL-LHC end-of-squeeze by feed-down from nonlinear errors in IR1/5. Blue histograms demonstrate that while optics errors were reduced upon application of the ideal model-based sextupole corrections, significant optics errors could still remain ($\frac{\Delta\beta}{\beta} \leq 5\,\%$). While this baseline sextupole correction may be sufficient for dynamic aperture, a 5 % residual beta-beat may still be unacceptable in regard to luminosity imbalance. During LHC commissioning similar residual beta-beating also remained after sextupole correction, which required additional iterations of linear optics corrections in order to achieve an acceptable luminosity imbalance. Figure 7 (right) shows histograms of simulated cross-term amplitude-detuning due to octupole errors (over 60 instances of the errors encompassing expected tolerances) at HL-LHC end-of-squeeze before (red) and after (blue) application of the ideal model-based correction. For context, the maximum detuning generated by octupole errors at $\beta^* = 0.15\,\text{m}$ is as large as the maximum detuning generated by the Landau octupoles. Even after corrections are applied in simulation, a large cross-term detuning remains. In some cases residual detuning after correction is still larger than any uncorrected detuning with which the LHC has been commissioned (typically $\sim 40 \times 10^3\,\text{m}^{-1}$). Such residual detuning remaining after correction may still be large enough to



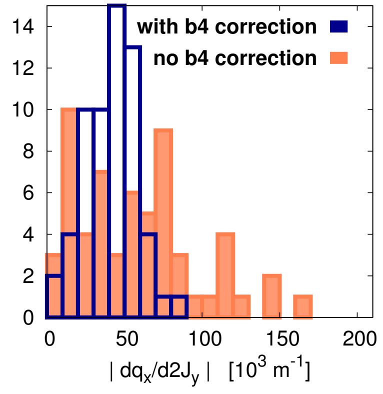


Fig. 7. Left: Beam-loss from a large emittance bunch due to reduction of DA as nonlinear errors are introduced into the LHC IRs. Right: histogram over 60-seeds of HL-LHC amplitude detuning at flat-orbit end-of-squeeze, without (orange) and with (blue) normal-octupole corrections applied in low-β^* IRs.

cause deterioration in the performance of beam-instrumentation (as discussed in Section 3) which can impede linear optics commissioning, and dependent on the β^* may also become relevant to the Landau damping of instabilities.

LHC experience has made clear that commissioning of the linear and nonlinear optics are intrinsically linked. The potential for direct contributions to linear optics errors from feed-down, as well as detrimental effects on the performance of the AC-dipole and beam-instrumentation due to nonlinear errors, mean there is no guarantee linear optics commissioning will succeed at very small β^* without nonlinear corrections already in place. Equally, a reliable linear optics model is a necessary pre-requisite to the calculation of both model- and beam-based nonlinear corrections. It is anticipated that annual checks and refinement of the nonlinear optics corrections will be performed in parallel with the regular linear optics commissioning, with progressive optimization of the corrections also performed as the minimum β^* is reduced.

LHC experience also demonstrated the importance of alignment and orbit errors in the IRs to the nonlinear optics corrections. Such alignment issues meant the various orders of nonlinear multipole corrections in the LHC could not be considered independently, for example necessitating repeated recommissioning of the sextupole corrections to account for changing feed-down from the higher-orders. Optics commissioning of the HL-LHC will require an iterative approach, both between linear and nonlinear optics corrections, and between the multipole orders. In the HL-LHC the Full Remote Alignment System (FRAS) will be used to control the alignment of IR elements with respect to the detector inner tracker and to compensate for ground motion during the year. Large changes to the alignment (of order 1 mm) are only anticipated during the commissioning phase, while during the year only small movements are expected with the aim of maintaining the magnets at their original locations. Further iterations of the nonlinear corrections may also be required during the commissioning period if any large changes to the alignment with the FRAS are performed after the initial optics commissioning.

Finally it is worth highlighting that nonlinear optics commissioning in HL-LHC assumes correction of significantly more multipole species than are currently corrected in LHC. Dedicated LHC machine studies show promise in regard to compensation of normal and skew decapoles and normal dodecapoles, however as yet no direct beam-based observable has been demonstrated for skew-dodecapole compensation.

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