Sudakov double logs in single-inclusive hadron production in DIS at small x from the Color Glass Condensate formalism

Tolga Altinoluk[a], Jamal Jalilian-Marian[a,b,c,d] and Cyrille Marquet[e]
[a] *Theoretical Physics Division, National Centre for Nuclear Research, Pasteura 7, Warsaw 02-093, Poland*
[b] *Department of Natural Sciences, Baruch College, CUNY, 17 Lexington Avenue, New York, NY 10010, USA*
[c] *City University of New York Graduate Center, 365 Fifth Avenue, New York, NY 10016, USA*
[d] *Theoretical Physics Department, CERN, 1211 Geneva 23, Switzerland*
[e] *CPHT, CNRS, École polytechnique, Institut Polytechnique de Paris, 91120 Palaiseau, France*

We investigate the high Q^2 (photon virtuality) limit of single-inclusive hadron production in DIS (SIDIS) at small x, using the color glass condensate formalism at next-to-leading order. We focus on the $\Lambda^2_{QCD} \ll \mathbf{p}_h^2 \ll Q^2$ kinematic regime where $\mathbf{p}_h$ is the produced hadron transverse momentum, and extract the Sudakov double logarithms. We further argue that compatibility between the CGC calculation and TMD factorization at one-loop order can only be achieved if the small-x evolution is kinematically constrained.

I. INTRODUCTION

The Color Glass Condensate (CGC) effective theory [1–3] is commonly used to describe the hadronic scattering processes at high collision energies in the Regge-Gribov limit. This effective theory is based on the gluon saturation phenomena which can be briefly described as follows; in the Regge-Gribov limit the increase in collision energy leads to a decrease of the longitudinal momentum fraction carried by the interacting partons. With decreasing x the gluon density of the interacting hadrons increases rapidly. This rapid increase in the density is tamed by nonlinear interactions of the emitted gluons and cause the above mentioned gluon saturation phenomena at sufficiently high energies. The non-linear functional evolution equation with increasing energy (or equivalently with rapidity) is given by Balitsky-Kovchegov / Jalilian-Marian-Iancu-McLerran-Wiegert-Leonidov-Kovner (BK-JIMWLK) equation [4–15].

Even though hints of gluon saturation phenomena have been seen in the experimental data from the Relativistic Heavy Ion Collider (RHIC) in the USA and the Large Hadron Collider (LHC) at CERN, a conclusive evidence is expected to be seen at the Electron Ion Collider (EIC) to be built in the USA. Deep inelastic scattering (DIS) on a dense target is one of the processes that will be at the focus of EIC to study the gluon saturation effects since it provides a clean environment to probe saturation. Theoretical computations of DIS related observables are frequently performed in the dipole factorization framework [16, 17], where the incoming lepton emits a virtual photon which splits into a quark-antiquark pair that scatters on the target. The splitting of the virtual photon into quark-antiquark pair is computed perturbatively while the interaction of the pair with the target is treated in the CGC framework by encoding the rescattering effects in the Wilson lines.

With the advent of the EIC, there have been a lot of efforts to increase the precision of the theoretical calculations of DIS related observables. Inclusive DIS [18–24] and its fits to HERA data [25] for massless quarks have been computed at next-to-leading order (NLO) in strong coupling α_s. Quark mass has been included in the NLO computations of inclusive DIS in [26–28] and fits of the results to HERA data have been performed in [29]. Single inclusive jet/hadron production have been studied both at leading order (LO) [30] and at NLO [31–33]. Inclusive dijet (and/or dihadron) production in DIS have been computed at NLO[1] in [39–46]. Finally, many different aspects of diffractive jet and dijet production have been studied in detail both at LO [47–57] and at NLO [58–63].

A remarkable aspect of dijet production is studied in [64, 65] where the equivalence between the CGC and transverse momentum dependent distributions (TMDs) have been shown once the appropriate limits are applied. Namely, the high energy limit of the dijet cross section computed in TMD factorization and the correlation limit (when the two jets are produced back-to-back) of the dijet cross section computed in the CGC framework (see [66, 67] for recent reviews). The back-to-back limit of the dijet production in DIS has been computed in [68, 69] at LO and both unpolarized and linearly polarized gluon TMDs are extracted in the CGC framework. The production of three-particule final states has also been considered in [70–74]. The equivalence between the CGC and TMD factorization frameworks are extended beyond the correlation limit for dijet production by resumming the kinematic twist corrections in [75–81] and this new framework is referred to as small-x improved TMD (iTMD) factorization that interpolates between the dilute limit of the CGC and the TMD limit of the CGC.

[1] An alternative way of increasing the precision of computations of DIS related observables is to relax the eikonal approximation that is routinely adopted in the CGC computations. Recently, these studies are performed for inclusive DIS in [34, 35] and dijet production in DIS in [36–38].

An important question whether the equivalence between the CGC and TMD frameworks holds beyond LO for dijet production in DIS in the back-to-back limit have been addressed in [40, 41]. It was shown that in order to get the correct Sudakov double logarithm that was conjectured in [82, 83], one should adopt a kinematically constrained BK evolution [84, 85] to properly resum the rapidity divergences that arise at NLO. More generally, the use of a kinematically-constrained non-linear small-x evolution is rapidly becoming unavoidable in CGC calculations, something which was realized long ago in the context of linear BFKL evolution [86, 87]. In addition, combining low-x and Sudakov resummation has been the subject of intensive research in various alternative approaches [88–110].

In this paper, we focus on the single inclusive hadron production in DIS. In section II, we start from the dihadron production cross section in DIS, integrate over the antiquark to get the single inclusive hadron production cross section at LO and discuss the kinematic region where one can expect the emergence of the Sudakov double logarithms once the next-to-leading order corrections to the cross section are included in the analysis. In section III, we include the next-to-leading order corrections to SIDIS cross section, identify the diagrams that will contribute to the Sudakov double logarithms in the appropriate kinematic region and discuss the divergences that appear. In section IV we discuss how to extract these double logarithms. Finally, in section V we present a brief discussion of our results.

II. SINGLE INCLUSIVE HADRON PRODUCTION IN DIS AT LEADING ORDER

The dominant channel for single inclusive particle production in the forward rapidity region in Deep Inelastic Scattering at small x is production of a quark antiquark pair, either of which can hadronize and be measured. At small x this is a two-step process; first the virtual photon splits into a quark antiquark pair which subsequently scatters on the target proton or nucleus. The cross section for this process can be written as [65]:

$$\begin{aligned}\frac{\mathrm{d}\sigma^{\gamma^* A\to q\bar{q}X}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}^2\mathbf{q}\,\mathrm{d}y_1\,\mathrm{d}y_2} = &\frac{e^2Q^2(z_1z_2)^2N_c}{(2\pi)^7}\delta(1-z_1-z_2)\int \mathrm{d}^8\mathbf{x}\,[S_{122'1'}-S_{12}-S_{1'2'}+1]\\ &e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}}\Bigg[4z_1z_2K_0(|\mathbf{x}_{12}|Q_1)K_0(|\mathbf{x}_{1'2'}|Q_1)+\\ &(z_1^2+z_2^2)\,\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2'}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2'}|}\,K_1(|\mathbf{x}_{12}|Q_1)K_1(|\mathbf{x}_{1'2'}|Q_1)\Bigg].\end{aligned} \tag{1}$$

where $(\mathbf{p}, y_1)$ and $(\mathbf{q}, y_2)$ are the transverse momentum and rapidity of the produced quark and antiquark respectively, and Q^2 is the virtuality of the incoming photon. The QED coupling e^2 should be understood as encompassing the various (massless) quark flavors: $e^2 = 4\pi\alpha_{em}\sum_f e_f^2$. The first (second) term inside the big square bracket corresponds to contribution of longitudinal (transverse) photons. Multiple scattering of the partons on the dense target are encoded in the dipole (S_{ij}) and quadrupole (S_{ijkl}) operators that are defined as

$$S_{ij} = \frac{1}{N_c}\mathrm{tr}\langle V_iV_j^\dagger\rangle\ \ ,\ \ S_{ijkl} = \frac{1}{N_c}\mathrm{tr}\langle V_iV_j^\dagger V_kV_l^\dagger\rangle \tag{2}$$

where the index i corresponds to the transverse coordinate $\mathbf{x}_i$ and the Wilson lines V_i are given in terms of the background field of the target A^- as

$$V_i = \hat{\mathcal{P}}\exp\Big(ig\int dx^+A^-(x^+,\mathbf{x}_i)\Big)\,. \tag{3}$$

Furthermore $\mathbf{x}_1(\mathbf{x}_2)$ is the transverse coordinate of the quark (antiquark) going through the target in the amplitude while the primed coordinates correspond to the same in the complex conjugate amplitude. We have defined $z_1 \equiv \frac{p^+}{l^+}$, $z_2 \equiv \frac{q^+}{l^+}$ as the momentum fractions carried by the final state quark and antiquark relative to the photon's longitudinal momentum l^+. Rapidity of a parton is related to its momentum fraction via $\mathrm{d}y_i = \frac{\mathrm{d}z_i}{z_i}$. We are also using the following definitions and short hand notations,

$$Q_i = Q\sqrt{z_i(1-z_i)},\quad \mathbf{x}_{ij} = \mathbf{x}_i-\mathbf{x}_j,\quad \mathrm{d}^8\mathbf{x} = \mathrm{d}^2\mathbf{x}_1\,\mathrm{d}^2\mathbf{x}_2\,\mathrm{d}^2\mathbf{x}_{1'}\,\mathrm{d}^2\mathbf{x}_{2'}. \tag{4}$$

In order to get the single inclusive production cross section one must integrate over either quark or antiquark in the final state. Both cases lead to identical results (this can also be shown to be true when we calculate the next to leading order corrections) so that it is enough to consider production of a quark only and multiply the final result by two, which is what we will do here.

$$\frac{\mathrm{d}\sigma^{\gamma^* A\to q(\mathbf{p},y)X}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_1} = 2\times\frac{e^2Q^2N_c}{(2\pi)^5}\int \mathrm{d}z_2\delta(1-z_1-z_2)\,(z_1^2\,z_2)\int \mathrm{d}^6\mathbf{x}\,[S_{11'}-S_{12}-S_{1'2}+1]\,e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}$$
$$\left\{4z_1z_2K_0(|\mathbf{x}_{12}|Q_1)K_0(|\mathbf{x}_{1'2}|Q_1)+(z_1^2+z_2^2)\,\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\,K_1(|\mathbf{x}_{12}|Q_1)K_1(|\mathbf{x}_{1'2}|Q_1)\right\}. \tag{5}$$

As hadronization of a colored parton is a genuinely non-perturbative phenomenon it is common to describe it by a parton-hadron fragmentation function when considering hadron production at moderate to high transverse momenta. We will follow this approach here and convolute the partonic cross section with a quark-hadron fragmentation function to get

$$\frac{\mathrm{d}\sigma^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h} = 2\frac{e^2N_c}{(2\pi)^5}\int_{z_h}^1\frac{dz_1}{z_h}D_{h/q}(z_h/z_1)\;z_1Q_1^2\int \mathrm{d}^6\mathbf{x}\,[S_{11'}-S_{12}-S_{1'2}+1]\,e^{i(z_1/z_h)\mathbf{p}_h\cdot\mathbf{x}_{1'1}}$$
$$\left\{\frac{4Q_1^2}{Q^2}K_0(|\mathbf{x}_{12}|Q_1)K_0(|\mathbf{x}_{1'2}|Q_1)+\left[z_1^2+(1-z_1)^2\right]\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\,K_1(|\mathbf{x}_{12}|Q_1)K_1(|\mathbf{x}_{1'2}|Q_1)\right\} \tag{6}$$

where z_h is fraction of the photon momentum carried by the produced hadron. We note that in collinear fragmentation the produced hadron and (massless) parton rapidities are the same.

In this work we are interested in particle production when the photon virtuality is large, so that we need the large Q^2 limit of our expressions. However taking the large Q^2 limit of the Bessel functions is a bit tricky; their argument depend on Q^2 through the combination with momentum fraction z which is integrated over so that the argument of Bessel functions $z_1(1-z_1)Q^2$ can go to zero even at very large Q^2. Clearly taking this limit requires some care; to accomplish this, we reformulate the procedure presented in [30] to extract the leading $1/Q^2$ behavior. We introduce a delta function of the form

$$Q_1^{2n}\,K_{(0,1)}(|\mathbf{x}_{12}|Q_1)\,K_{(0,1)}(|\mathbf{x}_{1'2}|Q_1)=\int_0^{Q^2/4}d\bar{Q}^2\;(\bar{Q}^2)^nK_{(0,1)}(|\mathbf{x}_{12}|\bar{Q})K_{(0,1)}(|\mathbf{x}_{1'2}|\bar{Q})\;\delta\left[\bar{Q}^2-z_1(1-z_1)Q^2\right] \tag{7}$$

and insert it in the various hadron-level cross-sections. We then do the z_1 integral using the delta function via

$$\delta\left[\bar{Q}^2-z_1(1-z_1)Q^2\right]=\frac{\delta(z_1-z_+)}{Q^2|1-2z_+|}+\frac{\delta(z_1-z_-)}{Q^2|1-2z_-|}\quad\text{with}\quad z_\pm=\frac{1}{2}\left(1\pm\sqrt{1-4\bar{Q}^2/Q^2}\right). \tag{8}$$

As we are interested in the high Q^2 limit, we keep only the leading power (in Q^2) terms, which comes from $z_1=z_+\sim 1.$, the so-called *aligned jet* configuration in which the quark that fragments into the measured hadron carries almost all of the photon longitudinal momentum. For the LO cross section this gives (transverse and longitudinal cases respectively):

$$\left.\frac{\mathrm{d}\sigma^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{T,LP} = \frac{1}{Q^2}\frac{e^2N_c}{(2\pi)^5}\frac{D_{h/q}(z_h)}{z_h}\int \mathrm{d}^6\mathbf{x}\,[S_{11'}-S_{12}-S_{1'2}+1]\,e^{i(\mathbf{p}_h/z_h)\cdot\mathbf{x}_{1'1}}$$
$$\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\int_0^\infty d\bar{Q}^2\bar{Q}^2\,K_1(|\mathbf{x}_{12}|\bar{Q})K_1(|\mathbf{x}_{1'2}|\bar{Q}). \tag{9}$$

$$\left.\frac{\mathrm{d}\sigma^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{L,LP} = \frac{1}{Q^2}\frac{e^2N_c}{(2\pi)^5}\frac{D_{h/q}(z_h)}{z_h}\int \mathrm{d}^6\mathbf{x}\,[S_{11'}-S_{12}-S_{1'2}+1]\,e^{i(\mathbf{p}_h/z_h)\cdot\mathbf{x}_{1'1}}$$
$$\frac{4}{Q^2}\int_0^\infty d\bar{Q}^2\bar{Q}^4K_0(|\mathbf{x}_{12}|\bar{Q})K_0(|\mathbf{x}_{1'2}|\bar{Q}). \tag{10}$$

Since the SIDIS production cross section via longitudinal photon is suppressed by a power of $1/Q^2$ in the high virtuality limit compared to the SIDIS production cross section via transverse photon, we restrict ourselves to the latter in this study. Moreover, one can rewrite the leading power expression of the transverse cross section by using the definition of the quark TMD distributions as [30, 99]:

$$\left.\frac{\mathrm{d}\sigma^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{T,LP} = \frac{\pi e^2}{Q^2}\frac{D_{h/q}(z_h)}{z_h}xq(x,\mathbf{p}_h/z_h) \tag{11}$$

where the small-x quark TMD distribution $xq(x, \mathbf{p})$ is defined as

$$xq(x, \mathbf{p}) \;=\; \frac{2N_c}{(2\pi)^6} \int \mathrm{d}^6\mathbf{x}\; e^{-i\mathbf{p}\cdot\mathbf{x}_{11'}}\;\; [S_{11'} - S_{12} - S_{1'2} + 1]\, \frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|} \int_0^\infty d\bar{Q}^2\bar{Q}^2\, K_1(|\mathbf{x}_{12}|\bar{Q})K_1(|\mathbf{x}_{1'2}|\bar{Q}) \quad (12)$$

where the x dependence on $xq(x)$ enters through the dipole amplitudes, or more precisely, through the rapidity scale choice at which they are to be evaluated, as will be discussed below. Let us finally introduce the so-called b-space TMD given by:

$$xq(x, \mathbf{p}) = \int \frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\; e^{-i\mathbf{p}\cdot\mathbf{x}_{11'}}\; x\tilde{q}(x, \mathbf{x}_{11'})\;. \quad (13)$$

Note that the b variable used in the TMD literature is the "dipole" size $\mathbf{x}_{11'}$ here in our work, where the dipole is made up of the quark in the amplitude at transverse position $\mathbf{x}_1$ and the quark in the conjugate amplitude at transverse position $\mathbf{x}_{1'}$, and *should not be confused with the impact parameter* $\mathbf{b}_\perp = (\mathbf{x}_1 + \mathbf{x}_1')/2$.

III. SINGLE INCLUSIVE HADRON PRODUCTION IN DIS AT NLO

Next to leading corrections to this leading order result have been calculated in [33]. In principle one must consider radiation of a gluon from either the quark or antiquark. This radiated gluon then can either be absorbed in the amplitude (virtual corrections) or in the complex conjugate amplitude (real corrections). In either case, the radiation can happen either after going through the target in which case only the original quark and antiquark scatter from the target, or before going through the target in which case all three partons scatter from the target. However as *we are interested only in terms in the the cross section which are enhanced by (Sudakov) double logs we will consider only the diagrams which contain a collinear divergence.* These are depicted in Fig. (1) where the labeling follows [44, 45],

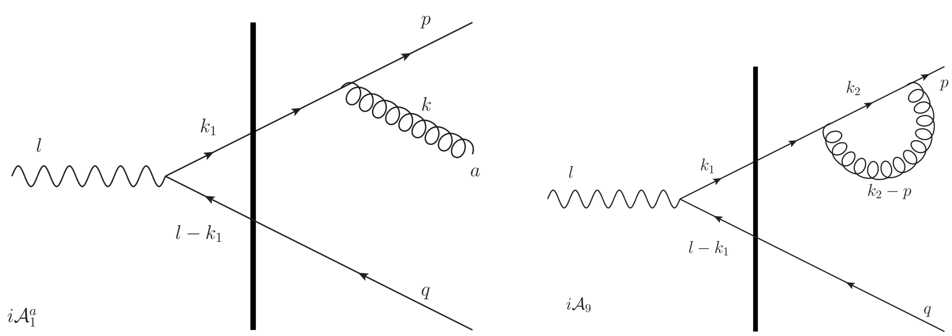

FIG. 1: The NLO real (left) and virtual (right) diagrams giving large double logs. The arrows on fermion lines indicate fermion number flow, all momenta flow to the right, *except* for gluon momenta. The thick solid line indicates interaction with the target.

In this work we will focus on transversely polarized photons since the cross section with longitudinal photons is suppressed by Q^2 (photon virtuality) as compared with transverse photons. Partonic production cross sections are then obtained by squaring the production amplitude and including the appropriate phase space and flux factors. Contribution of the real correction is then given by [45]

$$\frac{\mathrm{d}\sigma^T_{1\times 1}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}^2\mathbf{q}\,\mathrm{d}y_1\,\mathrm{d}y_2} = \frac{e^2g^2Q^2z_2^2(1-z_2)[z_1^2z_2^2 + (z_1^2+z_2^2)(1-z_2)^2 + (1-z_2)^4]}{(2\pi)^{10}z_1} \int \frac{\mathrm{d}z}{z}\delta(1-z-z_1-z_2)$$
$$\int \mathrm{d}^{10}\mathbf{x}\, K_1(|\mathbf{x}_{12}|Q_2)K_1(|\mathbf{x}_{1'2'}|Q_2)\, \frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2'}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2'}|}\, N_cC_F[S_{122'1'} - S_{12} - S_{1'2'} + 1]e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}}\,\Delta^{(3)}_{1'1}\, e^{i\frac{z}{z_1}\mathbf{p}\cdot\mathbf{x}_{1'1}} \quad (14)$$

where we have defined the radiation kernel

$$\Delta^{(3)}_{ij} = \frac{\mathbf{x}_{3i}\cdot\mathbf{x}_{3j}}{\mathbf{x}_{3i}^2\mathbf{x}_{3j}^2}. \quad (15)$$

while the virtual correction is given by

$$\frac{\mathrm{d}\sigma_9^T}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}^2\mathbf{q}\,\mathrm{d}y_1\,\mathrm{d}y_2} = \frac{-e^2g^2Q^2(z_1z_2)^2(z_1^2+z_2^2)}{2(2\pi)^8}\int \mathrm{d}^8\mathbf{x}\,K_1(|\mathbf{x}_{12}|Q_1)K_1(|\mathbf{x}_{1'2'}|Q_1)\,\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2'}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2'}|}\,\delta(1-z_1-z_2)$$
$$N_cC_F\left[S_{122'1'}-S_{12}-S_{1'2'}+1\right]e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}}\int_0^{z_1}\frac{\mathrm{d}z}{z}\left[\frac{z_1^2+(z_1-z)^2}{z_1^2}\right]\int\frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}\frac{1}{\left(\mathbf{k}-\frac{z}{z_1}\mathbf{p}\right)^2}. \tag{16}$$

Integrating over the antiquark phase space then gives contribution of real and virtual corrections to single inclusive quark production,

$$\frac{\mathrm{d}\sigma_{1\times1}^T}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_1} = \frac{e^2g^2Q^2}{(2\pi)^8}\int_0^{1-z_1}\frac{\mathrm{d}z}{z}\frac{(1-z-z_1)(z+z_1)}{z_1}\left[z_1^2(1-z-z_1)^2+\left(z_1^2+(1-z-z_1)^2\right)(z+z_1)^2+(z+z_1)^4\right]$$
$$\int \mathrm{d}^8\mathbf{x}\,K_1(|\mathbf{x}_{12}|Q_{1z})K_1(|\mathbf{x}_{1'2}|Q_{1z})N_cC_F\left[S_{11'}-S_{12}-S_{21'}+1\right]\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\,\Delta_{11'}^{(3)}\,e^{i\frac{z_1+z}{z_1}\mathbf{p}\cdot\mathbf{x}_{1'1}}. \tag{17}$$

$$\frac{\mathrm{d}\sigma_9^T}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_1} = -\frac{e^2g^2Q^2}{2(2\pi)^6}\int_0^{z_1}\frac{\mathrm{d}z}{z}(1-z_1)(z_1^2+(1-z_1)^2)\left[z_1^2+(z_1-z)^2\right]\int \mathrm{d}^6\mathbf{x}\,K_1(|\mathbf{x}_{12}|Q_1)K_1(|\mathbf{x}_{1'2}|Q_1)e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}$$
$$N_cC_F\left[S_{11'}-S_{12}-S_{21'}+1\right]\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\int\frac{\mathrm{d}^2\mathbf{x}_3}{(2\pi)^2}\frac{1}{\mathbf{x}_{31}^2} \tag{18}$$

with

$$Q_{1z}^2 \equiv (1-z-z_1)(z+z_1)Q^2\ . \tag{19}$$

Indeed, since we have integrated over z_2 the definition of Q_1 remains the same as before but Q_2 is now changed into Q_{1z}. For these NLO expressions, the extraction of the leading power in the large Q^2 limit can be achieved in a similar manner as in the previous section.

Let us start with the real-emission contribution. The hadron-level cross sections is

$$\frac{\mathrm{d}\sigma_{1\times1^T}^{\gamma^*A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h} = \frac{e^2g^2}{(2\pi)^8}N_cC_F\int_{z_h}^1\frac{dz_1}{z_h}\,D_{h/q}(z_h/z_1)$$
$$\times\int_0^{1-z_1}\frac{\mathrm{d}z}{z}\frac{Q_{1z}^2}{z_1}\left[z_1^2(1-z-z_1)^2+\left(z_1^2+(1-z-z_1)^2\right)(z+z_1)^2+(z+z_1)^4\right]$$
$$\times\int \mathrm{d}^8\mathbf{x}[S_{11'}-S_{12}-S_{21'}+1]K_1(|\mathbf{x}_{12}|Q_{1z})K_1(|\mathbf{x}_{1'2}|Q_{1z})\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}e^{i\frac{z_1+z}{z_1}\frac{z_1}{z_h}\mathbf{p}_h\cdot\mathbf{x}_{1'1}}\Delta_{1'1}^{(3)}. \tag{20}$$

and the leading-power contribution can be written as

$$\left.\frac{\mathrm{d}\sigma_{1\times1^T}^{\gamma^*A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = \frac{\pi e^2}{Q^2}\int\frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\,e^{-i(\mathbf{p}_h/z_h)\cdot\mathbf{x}_{11'}}\,x\tilde{q}(x,\mathbf{x}_{11'})$$
$$\times\frac{g^2C_F}{(2\pi)^3}\int_0^{1-z_h}\frac{\mathrm{d}z}{z(1-z)}\frac{D_{h/q}(z_h/(1-z))}{z_h}\left[1+(1-z)^2\right]\int \mathrm{d}^2\mathbf{x}_3\,\Delta_{1'1}^{(3)}. \tag{21}$$

where we have used the definition of the b-space TMD

$$x\tilde{q}(x,\mathbf{x}_{11'}) = \frac{2N_c}{(2\pi)^4}\int \mathrm{d}^4\mathbf{x}\left[S_{11'}-S_{12}-S_{1'2}+1\right]\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\int_0^\infty d\bar{Q}^2\bar{Q}^2\,K_1(|\mathbf{x}_{12}|\bar{Q})K_1(|\mathbf{x}_{1'2}|\bar{Q}) \tag{22}$$

with $\mathrm{d}^4\mathbf{x}\equiv\mathrm{d}^2\mathbf{b}_\perp\,\mathrm{d}^2\mathbf{x}_2$.

Now let us discuss the virtual contribution. At hadron-level it reads

$$\frac{\mathrm{d}\sigma_{9^T}^{\gamma^*A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h} = -\frac{e^2g^2}{2(2\pi)^6}N_cC_F\int_{z_h}^1\frac{dz_1}{z_1}\,\frac{D_{h/q}(z_h/z_1)}{z_h}[z_1^2+(1-z_1)^2]\int_0^{z_1}\frac{\mathrm{d}z}{z}\left[z_1^2+(z_1-z)^2\right]$$
$$\int \mathrm{d}^6\mathbf{x}\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}Q_1^2K_1(|\mathbf{x}_{12}|Q_1)K_1(|\mathbf{x}_{1'2}|Q_1)e^{i\frac{z_1}{z_h}\mathbf{p}_h\cdot\mathbf{x}_{1'1}}\left[S_{11'}-S_{12}-S_{21'}+1\right]\int\frac{\mathrm{d}^2\mathbf{x}_3}{(2\pi)^2}\frac{1}{\mathbf{x}_3^2}. \tag{23}$$

Before extracting the leading-power, one should recall that in the full calculation, the UV divergence in this diagram is canceled by the UV divergent part of another virtual diagram, as explained in [45] (labeled $\mathrm{d}\sigma_{14(1)}$ there). We shall take this into account by introducing the scale μ in the $\mathbf{x}_3$ intregral. Then, the high-Q^2 limit is given by

$$\left.\frac{\mathrm{d}\sigma_{9T}^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = \frac{\pi e^2}{Q^2}\frac{D_{h/q}(z_h)}{z_h}\int\frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}e^{-i(\mathbf{p}_h/z_h)\cdot\mathbf{x}_{11'}}\,x\tilde{q}(x,\mathbf{x}_{11'}) \times\left(-\frac{\alpha_s C_F}{4\pi^2}\right)\int_0^1\frac{\mathrm{d}z}{z}\left[1+(1-z)^2\right]\int_{|\mathbf{x}_3|\mu>1}\frac{\mathrm{d}^2\mathbf{x}_3}{\mathbf{x}_3^2}. \tag{24}$$

Adding the LO and NLO (we need to multiply σ_{9T} by 2 since that contribution is the product of diagram A_9 in Fig. 1 with the conjugate LO amplitude) terms we get

$$\left.\frac{\mathrm{d}\sigma_{LO+NLO}^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = \frac{\pi e^2}{Q^2}\frac{1}{z_h}\int\frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\,x\tilde{q}(x,\mathbf{x}_{11'})\Bigg\{\left[1-\frac{\alpha_s C_F}{2\pi^2}\int_0^1\frac{\mathrm{d}z}{z}\left[1+(1-z)^2\right]\int_{|\mathbf{x}_3|\mu>1}\frac{\mathrm{d}^2\mathbf{x}_3}{\mathbf{x}_3^2}\right]D_{h/q}(z_h) + \left[\frac{\alpha_s C_F}{2\pi^2}\int_0^{1-z_h}\frac{\mathrm{d}z}{z}\frac{\left[1+(1-z)^2\right]}{1-z}\int\mathrm{d}^2\mathbf{x}_3\,\Delta_{1'1}^{(3)}\right]D_{h/q}(\frac{z_h}{1-z})\Bigg\} \tag{25}$$

We next add (to virtual) and subtract (from the real part) the collinear divergence (for simplicity we also use μ as the factorization scale):

$$\left.\frac{\mathrm{d}\sigma_{LO+NLO}^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = \frac{\pi e^2}{Q^2}\frac{1}{z_h}\int\frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\,x\tilde{q}(x,\mathbf{x}_{11'}) \times\Bigg\{D_{h/q}(z_h)+\left[\frac{\alpha_s C_F}{2\pi^2}\left(\int_0^{1-z_h}\frac{\mathrm{d}z}{z}\frac{1+(1-z)^2}{1-z}D_{h/q}(\frac{z_h}{1-z})-\int_0^1\frac{\mathrm{d}z}{z}[1+(1-z)^2]D_{h/q}(z_h)\right)\right]\int_{|\mathbf{x}_3|\mu>1}\frac{\mathrm{d}^2\mathbf{x}_3}{\mathbf{x}_3^2} + \left[\frac{\alpha_s C_F}{2\pi^2}\int_0^{1-z_h}\frac{\mathrm{d}z}{z}\frac{\left[1+(1-z)^2\right]}{1-z}\left(\int\mathrm{d}^2\mathbf{x}_3\,\Delta_{1'1}^{(3)}-\int_{|\mathbf{x}_3|\mu>1}\frac{\mathrm{d}^2\mathbf{x}_3}{\mathbf{x}_3^2}\right)\right]D_{h/q}(\frac{z_h}{1-z})\Bigg\} \tag{26}$$

The first two terms in the curly bracket in Eq. (26) correspond to the Leading Order cross section convoluted with DGLAP-evolved quark-hadron fragmentation function. Indeed, defined as the expectation value of two bare field operators which becomes singular in the short distance limit, the fragmentation function gets renormalized (evolves) by loop corrections where the renormalized quark-hadron fragmentation function is defined as (see [45])

$$D_{h/q}(z_h,\mu^2)\equiv\left[D_{h/q}^0(z_h)+\frac{\alpha_s C_F}{2\pi}\left(\int_0^{1-z_h}\frac{\mathrm{d}z}{z}\frac{1+(1-z)^2}{1-z}D_{h/q}(\frac{z_h}{1-z})-\int_0^1\frac{\mathrm{d}z}{z}[1+(1-z)^2]D_{h/q}(z_h)\right)\int_{1/\mu^2}\frac{\mathrm{d}^2\mathbf{x}}{\mathbf{x}^2}\right] \tag{27}$$

Therefore, we can isolate the collinearly-divergent part of the cross section which leads to the standard DGLAP evolution of the fragmentation function (where the splitting function is defined with + prescription) which is then convoluted with the Leading Order cross section. The result can be then be written as

$$\left.\frac{\mathrm{d}\sigma_{LO+NLO}^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = d\sigma_{LO}\otimes D_{h/q}(z_h,\mu^2)+\frac{1}{Q^2}\frac{\pi e^2}{z_h}\int\frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\,x\tilde{q}(x,\mathbf{x}_{11'}) \times\left[\frac{\alpha_s C_F}{2\pi^2}\left(\int\mathrm{d}^2\mathbf{x}_3\,\Delta_{1'1}^{(3)}-\int_{|\mathbf{x}_3|\mu>1}\frac{\mathrm{d}^2\mathbf{x}_3}{\mathbf{x}_3^2}\right)\int_0^{1-z_h}\frac{\mathrm{d}z}{z}\frac{\left[1+(1-z)^2\right]}{1-z}D_{h/q}(\frac{z_h}{1-z})\right]\,. \tag{28}$$

In the second line of that equation, one can already point out the origin of the Sudakov logarithms. They will come from the integration over $\mathbf{x}_3$, in the range between and $1/\mu$ and $\mathbf{x}_{11'}$ where the virtual (subtracted, to be precise) term does not get canceled by the real-emisson one. Indeed, the combination inside the parenthesis is infrared finite, and in the real-emission integration, small dipole sizes are naturally cut-off by $x_{11'}$, while for the virtual term the integration over $\mathbf{x}_3$ extends down to $1/\mu$. From now on, since we are only after double logs, we may set the scale as $\mu = Q$, which is the natural choice in SIDIS. We note however that scale setting is more intricate for the extraction of the single logarithms.

The final step is to isolate the rapidity divergences which will result in the JIMWLK evolution of the dipole amplitude. To do so we introduce z_f, a factorization scale in z and break up the z integral in Eqs. (28) into two regions as follows,

$$\int_0^{1-z_h} \mathrm{d}z = \int_0^{z_f} \mathrm{d}z + \int_{z_f}^{1-z_h} \mathrm{d}z \tag{29}$$

where the first term contains the rapidity divergence while the second term is rapidity-finite and constitute part of the NLO corrections to the production cross section. We may now write:

$$\left.\frac{\mathrm{d}\sigma_{LO+NLO}^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = d\sigma_{LO}\otimes D_{h/q}(z_h,Q^2) + d\sigma_{NLO-rap-finite}$$
$$+\frac{\pi e^2}{Q^2}\frac{D_{h/q}(z_h)}{z_h}\int\frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\,e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\,x\tilde{q}(x,\mathbf{x}_{11'})\left[\frac{\alpha_s C_F}{\pi^2}\left(\int \mathrm{d}^2\mathbf{x}_3\,\Delta^{(3)}_{1'1} - \int_{|\mathbf{x}_3|Q>1}\frac{\mathrm{d}^2\mathbf{x}_3}{\mathbf{x}_3^2}\right)\int_0^{z_f}\frac{\mathrm{d}z}{z}\right] \tag{30}$$

It is customary in CGC calculations to absorb the entire second line of this expression into the leading-order term, making the dipole amplitudes z_f dependent and evolving according to the Leading Log (LL) BK-JIMWLK equation. That was shown explicitly for the double-inclusive cross-section – and consequently for SIDIS – in [33, 39, 40]. However, doing do leaves us without Sudakov logarithms, in contradiction with TMD factorization.

IV. EXTRACTION OF SUDAKOV LOGARITHMS

The extraction of Sudakov double logs was addressed recently for the case of back-to-back di-jet production, where LL JIMWLK evolution of dipoles and quadrupoles resulted in Sudakov double logs with the wrong sign. To remedy this problem and restore compatibility with TMD factorization, a kinematically constrained JIMWLK evolution was introduced. We shall implement a similar idea in the present case of SIDIS at large Q^2. The idea is that in addition to the restricting the + component of the gluon momentum, i.e. $k^+ < z_f\, l^+$ so that $z < z_f$, we must also constrain their - component $k^- > \tilde{z}_f\, l^-$ where $l^\pm$ is the momentum of the incoming virtual photon and it is natural to choose $z_f\,\tilde{z}_f = 1$. This constraint enforces the gluon lifetime $\sim 1/k^-$ to be small enough to participate in the small-x evolution of the target. This constraint on the gluon kinematica is naturally written in momentum space as

$$\Theta(\text{kin.const.}) = \Theta\left(z_f\frac{\mathbf{p}^2}{Q^2} - z\right), \tag{31}$$

and we will show that it restores the Sudakov double logarithms.

After absorbing a kinematically-constrained second-line of equation 30 into the LO cross-section, in which the dipole amplitudes shall now evolve according to kinematically constrained JIMWLK equation, we are left with the following z integral:

$$\int_0^{z_f}\frac{\mathrm{d}z}{z}\left[1-\Theta\left(z_f\frac{\mathbf{p}^2}{Q^2}-z\right)\right] = \int_0^{z_f}\frac{\mathrm{d}z}{z}\Theta\left(z - z_f\frac{\mathbf{p}^2}{Q^2}\right) = \Theta\left(\frac{Q^2}{\mathbf{p}^2}\right)\ln\left(\frac{Q^2}{\mathbf{p}^2}\right)\ . \tag{32}$$

We then write our coordinate-space expression (30) in momentum space using

$$\int \mathrm{d}^2\mathbf{x}_3\,\Delta^{(3)}_{1'1} - \int_{|\mathbf{x}_3|Q>1}\frac{\mathrm{d}^2\mathbf{x}_3}{\mathbf{x}_3^2} = \int\frac{\mathrm{d}^2\mathbf{p}}{\mathbf{p}^2}\ e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}} - \int_{Q>|\mathbf{p}|}\frac{\mathrm{d}^2\mathbf{p}}{\mathbf{p}^2} \tag{33}$$

and we get

$$\left.\frac{\mathrm{d}\sigma_{LO+NLO}^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = d\sigma_{LO}(z_f)\otimes D_{h/q}(z_h,Q^2) + d\sigma_{NLO-rap-finite} + \frac{\pi e^2}{Q^2}\frac{D_{h/q}(z_h)}{z_h}\int\frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\,e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\,x\tilde{q}(x,\mathbf{x}_{11'})$$
$$\times\left\{\frac{\alpha_s C_F}{\pi^2}\int^{Q^2}\frac{\mathrm{d}^2\mathbf{p}}{\mathbf{p}^2}\left(e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}-1\right)\ln\left(\frac{Q^2}{\mathbf{p}^2}\right)\right\} \tag{34}$$

The leading order term corresponds to Eq 11, but with the fragmentation function evaluated at the scale $\mu = Q$, and the quark TMD evaluated (though the dipole amplitudes) at the rapidity scale z_f, which we may now set to $z_f = x$, giving its meaning to our notation. The last term contains the large logarithms. The final step is to perform the $\mathbf{p}$ integral

$$\int^{Q^2} \frac{\mathrm{d}^2\mathbf{p}}{\mathbf{p}^2}\left[e^{-i\mathbf{p}\cdot\mathbf{x}_{11'}} - 1\right]\ln\left(\frac{Q^2}{\mathbf{p}^2}\right) = 4\pi\int_0^{Q|\mathbf{x}_{11'}|}\frac{\mathrm{d}\tau}{\tau}\left[J_0(\tau) - 1\right]\ln\left(\frac{Q|\mathbf{x}_{11'}|}{\tau}\right) \tag{35}$$

which can be evaluated by using the following generic results:

$$\int_0^X \frac{d\tau}{\tau}\Big[J_0(\tau) - 1\Big] = -\ln\left(\frac{X}{c_0}\right) + O\left(\frac{1}{\sqrt{X}}\right) \tag{36}$$

$$\int_0^X \frac{d\tau}{\tau}\ln(\tau)\Big[J_0(\tau) - 1\Big] = -\frac{1}{2}\Big[\ln(X)\Big]^2 + \frac{1}{2}\Big[\ln(c_0)\Big]^2 + O\left(\frac{1}{\sqrt{X}}\right) \tag{37}$$

in the $X \to +\infty$ limit with $c_0 = 2\, e^{-\gamma_E}$. Using these results, the integral in Eq. (35) can be written as

$$4\pi\int_0^{Q|\mathbf{x}_{11'}|}\frac{\mathrm{d}\tau}{\tau}\left[J_0(\tau) - 1\right]\ln\left(\frac{Q|\mathbf{x}_{11'}|}{\tau}\right) = -\frac{\pi}{2}\ln^2\left(Q^2\mathbf{x}_{11'}^2/c_0^2\right) + O\left(\frac{1}{\sqrt{Q|\mathbf{x}_{11'}|}}\right) \tag{38}$$

Then, we finally have "factorized" the contribution of the Sudakov double logs inside the "b-space" $\mathbf{x}_{11'}$ integral. Adding them to the LO cross-section gives the factor

$$1 - \frac{\alpha_s C_F}{2\pi}\ln^2\left(Q^2\mathbf{x}_{11'}^2/c_0^2\right) = 1 - S_{sud}(\mathbf{x}_{11'}) \tag{39}$$

Further assuming exponentiation of the Sudakov logs we get the following result for Sudakov-resummed single inclusive hadron production in DIS,

$$\left.\frac{\mathrm{d}\sigma_{LO+NLO}^{\gamma^* A\to h(\mathbf{p}_h, y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = \frac{\pi e^2}{Q^2}\frac{D_{h/q}(z_h, Q^2)}{z_h}\int\frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\, e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\, x\tilde{q}(x, \mathbf{x}_{11'})\, e^{-S_{sud}(\mathbf{x}_{11'})} + d\sigma_{NLO-rap-finite} \tag{40}$$

where we have used the fact that $D_{h/q}(z_h, Q^2) \simeq D_{h/q}(z_h) + O(\alpha_s)$ to replace the bare fragmentation function with the DGLAP evolved one. This restore compatibility with TMD factorization [99].

V. CONCLUSIONS AND OUTLOOK

In this work we have studied the single inclusive hadron production in DIS at NLO focusing on the limit of large photon virtuality, i.e. large Q^2. In order to study this processes, we started from the dihadron production in DIS at NLO [33, 44], integrate over the antiquark to get the cross section for single inclusive hadron production in DIS. As is known, in the large photon virtuality limit, production via longitudinally-polarized photons is suppressed by $1/Q^2$ when compared to production via transversely-polarized photons, and thus we focused on the transverse case.

In the case of dijet production at NLO, one gets the connection between the CGC and standard TMD factorization frameworks in the so called correlation limit where the two jets are produced almost back-to-back [40, 41]. In this kinematic region, the hard scale is set by the large transverse momentum of the produced jets while the semi-hard scale is set by the vector sum of the transverse momenta of the produced jets. In [40, 41], it has been shown that in the case dijet production in DIS, in order to get the correct sign of the large double logarithms of the ratio of hard to soft scales, which is known as the double Sudakov logarithms, one has to adopt kinematically constrained BK-JIMWLK evolution equation instead of the standard one when resumming the rapidity divergences. Extraction of the Sudakov double logarithms in the case of single inclusive hadron production in DIS is quite different from the back-to-back dihadron production. First of all, when considering the single inclusive hadron (or jet) production in DIS, the hard scale that appears in the argument of Sudakov logarithms is provided by the virtuality of the incoming photon. Therefore, these logarithms become large and require resummation only in the large Q^2 limit which we consider in this paper. The second and most remarkable outcome of this study is that in this process, double Sudakov logarithms emerge only when the kinematically constraint BK-JIMWLK evolution equation is adopted for resumming the rapidity divergences.

Finally, in this work we only focus on double Sudakov logarithms and understand the role of the kinematically constrained rapidity evolution on the emergence of these double logarithms. A natural continuation of this work is to study the single Sudakov logarithms in single inclusive hadron production in DIS which we leave for a future work.

ACKNOWLEDGMENTS

We thank G. Beuf, R. Boussarie, P. Caucal, E. Iancu, P. Taels, B.W. Xiao and F. Yuan for useful discussions. TA and JJM thank CPhT, Ecole Polytechnique for hospitality during the visits when this work was initiated. JJM thanks CERN Department of Theoretical Physics for hospitality and financial support. This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, within the framework of the Saturated Glue (SURGE) Topical Theory Collaboration. JJM is supported by ULAM program of NAWA No. BPN/ULM/2023/1/00073/U/00001 and by the US DOE Office of Nuclear Physics through Grant No. DE-SC0002307. This work has been performed in the framework of MSCA RISE 823947 "Heavy ion collisions: collectivity and precision in saturation physics" (HIEIC) and has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No. 824093.

---

[1] F. Gelis, E. Iancu, J. Jalilian-Marian, and R. Venugopalan, Ann. Rev. Nucl. Part. Sci. **60**, 463 (2010), 1002.0333.
[2] J. L. Albacete and C. Marquet, Prog. Part. Nucl. Phys. **76**, 1 (2014), 1401.4866.
[3] J.-P. Blaizot, Rept. Prog. Phys. **80**, 032301 (2017), 1607.04448.
[4] I. Balitsky, Nucl. Phys. B **463**, 99 (1996), hep-ph/9509348.
[5] Y. V. Kovchegov, Phys. Rev. D **60**, 034008 (1999), hep-ph/9901281.
[6] Y. V. Kovchegov, Phys. Rev. D **61**, 074018 (2000), hep-ph/9905214.
[7] J. Jalilian-Marian, A. Kovner, L. D. McLerran, and H. Weigert, Phys. Rev. D **55**, 5414 (1997), hep-ph/9606337.
[8] J. Jalilian-Marian, A. Kovner, A. Leonidov, and H. Weigert, Nucl. Phys. B **504**, 415 (1997), hep-ph/9701284.
[9] J. Jalilian-Marian, A. Kovner, A. Leonidov, and H. Weigert, Phys. Rev. D **59**, 014014 (1998), hep-ph/9706377.
[10] J. Jalilian-Marian, A. Kovner, and H. Weigert, Phys. Rev. D **59**, 014015 (1998), hep-ph/9709432.
[11] A. Kovner, J. G. Milhano, and H. Weigert, Phys. Rev. D **62**, 114005 (2000), hep-ph/0004014.
[12] H. Weigert, Nucl. Phys. A **703**, 823 (2002), hep-ph/0004044.
[13] E. Iancu, A. Leonidov, and L. D. McLerran, Nucl. Phys. A **692**, 583 (2001), hep-ph/0011241.
[14] E. Iancu, A. Leonidov, and L. D. McLerran, Phys. Lett. B **510**, 133 (2001), hep-ph/0102009.
[15] E. Ferreiro, E. Iancu, A. Leonidov, and L. McLerran, Nucl. Phys. A **703**, 489 (2002), hep-ph/0109115.
[16] J. D. Bjorken, J. B. Kogut, and D. E. Soper, Phys. Rev. D **3**, 1382 (1971).
[17] N. N. Nikolaev and B. G. Zakharov, Z. Phys. C **49**, 607 (1991).
[18] I. Balitsky and G. A. Chirilli, Phys. Rev. D **83**, 031502 (2011), 1009.4729.
[19] I. Balitsky and G. A. Chirilli, Phys. Rev. D **87**, 014013 (2013), 1207.3844.
[20] G. Beuf, Phys. Rev. D **85**, 034039 (2012), 1112.4501.
[21] G. Beuf, Phys. Rev. D **94**, 054016 (2016), 1606.00777.
[22] G. Beuf, Phys. Rev. D **96**, 074033 (2017), 1708.06557.
[23] B. Ducloué, H. Hänninen, T. Lappi, and Y. Zhu, Phys. Rev. D **96**, 094017 (2017), 1708.07328.
[24] H. Hänninen, T. Lappi, and R. Paatelainen, Annals Phys. **393**, 358 (2018), 1711.08207.
[25] G. Beuf, H. Hänninen, T. Lappi, and H. Mäntysaari, Phys. Rev. D **102**, 074028 (2020), 2007.01645.
[26] G. Beuf, T. Lappi, and R. Paatelainen, Phys. Rev. D **104**, 056032 (2021), 2103.14549.
[27] G. Beuf, T. Lappi, and R. Paatelainen, Phys. Rev. Lett. **129**, 072001 (2022), 2112.03158.
[28] G. Beuf, T. Lappi, and R. Paatelainen, Phys. Rev. D **106**, 034013 (2022), 2204.02486.
[29] H. Hänninen, H. Mäntysaari, R. Paatelainen, and J. Penttala, Phys. Rev. Lett. **130**, 192301 (2023), 2211.03504.
[30] C. Marquet, B.-W. Xiao, and F. Yuan, Phys. Lett. B **682**, 207 (2009), 0906.1454.
[31] P. Caucal, E. Ferrand, and F. Salazar (2024), 2401.01934.
[32] F. Bergabo and J. Jalilian-Marian (2024), 2401.06259.
[33] F. Bergabo and J. Jalilian-Marian, JHEP **01**, 095 (2023), 2210.03208.
[34] R. Boussarie and Y. Mehtar-Tani, Phys. Lett. B **831**, 137125 (2022), 2006.14569.
[35] R. Boussarie and Y. Mehtar-Tani, JHEP **07**, 080 (2022), 2112.01412.
[36] T. Altinoluk, G. Beuf, A. Czajka, and A. Tymowska, Phys. Rev. D **107**, 074016 (2023), 2212.10484.
[37] T. Altinoluk, N. Armesto, and G. Beuf, Phys. Rev. D **108**, 074023 (2023), 2303.12691.
[38] P. Agostini, T. Altinoluk, and N. Armesto (2024), 2403.04603.
[39] P. Caucal, F. Salazar, and R. Venugopalan, JHEP **11**, 222 (2021), 2108.06347.
[40] P. Taels, T. Altinoluk, G. Beuf, and C. Marquet, JHEP **10**, 184 (2022), 2204.11650.
[41] P. Caucal, F. Salazar, B. Schenke, and R. Venugopalan, JHEP **11**, 169 (2022), 2208.13872.
[42] P. Caucal, F. Salazar, B. Schenke, T. Stebel, and R. Venugopalan, JHEP **08**, 062 (2023), 2304.03304.
[43] P. Caucal, F. Salazar, B. Schenke, T. Stebel, and R. Venugopalan, Phys. Rev. Lett. **132**, 081902 (2024), 2308.00022.
[44] F. Bergabo and J. Jalilian-Marian, Phys. Rev. D **107**, 054036 (2023), 2301.03117.
[45] F. Bergabo and J. Jalilian-Marian, Phys. Rev. D **106**, 054035 (2022), 2207.03606.
[46] E. Iancu and Y. Mulian, JHEP **07**, 121 (2023), 2211.04837.
[47] T. Altinoluk, N. Armesto, G. Beuf, and A. H. Rezaeian, Phys. Lett. B **758**, 373 (2016), 1511.07452.

[48] Y. Hatta, B.-W. Xiao, and F. Yuan, Phys. Rev. Lett. **116**, 202301 (2016), 1601.01585.
[49] H. Mäntysaari, N. Mueller, and B. Schenke, Phys. Rev. D **99**, 074004 (2019), 1902.05087.
[50] F. Salazar and B. Schenke, Phys. Rev. D **100**, 034007 (2019), 1905.03763.
[51] E. Iancu, A. H. Mueller, and D. N. Triantafyllopoulos, Phys. Rev. Lett. **128**, 202001 (2022), 2112.06353.
[52] Y. Hatta, B.-W. Xiao, and F. Yuan, Phys. Rev. D **106**, 094015 (2022), 2205.08060.
[53] E. Iancu, A. H. Mueller, D. N. Triantafyllopoulos, and S. Y. Wei, JHEP **10**, 103 (2022), 2207.06268.
[54] E. Iancu, A. H. Mueller, D. N. Triantafyllopoulos, and S. Y. Wei, Eur. Phys. J. C **83**, 1078 (2023), 2304.12401.
[55] B. Rodriguez-Aguilar, D. N. Triantafyllopoulos, and S. Y. Wei, Phys. Rev. D **107**, 114007 (2023), 2302.01106.
[56] T. Kar, A. Kovner, M. Li, and V. V. Skokov (2023), 2312.04493.
[57] S. Hauksson, E. Iancu, A. H. Mueller, D. N. Triantafyllopoulos, and S. Y. Wei (2024), 2402.14748.
[58] G. Beuf, H. Hänninen, T. Lappi, Y. Mulian, and H. Mäntysaari, Phys. Rev. D **106**, 094014 (2022), 2206.13161.
[59] G. Beuf, T. Lappi, H. Mäntysaari, R. Paatelainen, and J. Penttala (2024), 2401.17251.
[60] M. Fucilla, A. Grabovsky, E. Li, L. Szymanowski, and S. Wallon, JHEP **02**, 165 (2024), 2310.11066.
[61] R. Boussarie, A. V. Grabovsky, L. Szymanowski, and S. Wallon, JHEP **11**, 149 (2016), 1606.00419.
[62] R. Boussarie, A. V. Grabovsky, L. Szymanowski, and S. Wallon, Phys. Rev. D **100**, 074020 (2019), 1905.07371.
[63] M. Fucilla, A. V. Grabovsky, E. Li, L. Szymanowski, and S. Wallon, JHEP **03**, 159 (2023), 2211.05774.
[64] F. Dominguez, B.-W. Xiao, and F. Yuan, Phys. Rev. Lett. **106**, 022301 (2011), 1009.2141.
[65] F. Dominguez, C. Marquet, B.-W. Xiao, and F. Yuan, Phys. Rev. D **83**, 105005 (2011), 1101.0715.
[66] E. Petreska, Int. J. Mod. Phys. E **27**, 1830003 (2018), 1804.04981.
[67] R. Boussarie et al. (2023), 2304.03302.
[68] C. Marquet, E. Petreska, and C. Roiesnel, JHEP **10**, 065 (2016), 1608.02577.
[69] C. Marquet, C. Roiesnel, and P. Taels, Phys. Rev. D **97**, 014004 (2018), 1710.05698.
[70] A. Ayala, M. Hentschinski, J. Jalilian-Marian, and M. E. Tejeda-Yeomans, Phys. Lett. B **761**, 229 (2016), 1604.08526.
[71] A. Ayala, M. Hentschinski, J. Jalilian-Marian, and M. E. Tejeda-Yeomans, Nucl. Phys. B **920**, 232 (2017), 1701.07143.
[72] T. Altinoluk, N. Armesto, A. Kovner, M. Lublinsky, and E. Petreska, JHEP **04**, 063 (2018), 1802.01398.
[73] T. Altinoluk, R. Boussarie, C. Marquet, and P. Taels, JHEP **07**, 079 (2019), 1810.11273.
[74] T. Altinoluk, R. Boussarie, C. Marquet, and P. Taels, JHEP **07**, 143 (2020), 2001.00765.
[75] P. Kotko, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, and A. van Hameren, JHEP **09**, 106 (2015), 1503.03421.
[76] A. van Hameren, P. Kotko, K. Kutak, C. Marquet, E. Petreska, and S. Sapeta, JHEP **12**, 034 (2016), [Erratum: JHEP 02, 158 (2019)], 1607.03121.
[77] H. Fujii, C. Marquet, and K. Watanabe, JHEP **12**, 181 (2020), 2006.16279.
[78] T. Altinoluk, C. Marquet, and P. Taels, JHEP **06**, 085 (2021), 2103.14495.
[79] T. Altinoluk and R. Boussarie, JHEP **10**, 208 (2019), 1902.07930.
[80] T. Altinoluk, R. Boussarie, and P. Kotko, JHEP **05**, 156 (2019), 1901.01175.
[81] R. Boussarie, H. Mäntysaari, F. Salazar, and B. Schenke, JHEP **09**, 178 (2021), 2106.11301.
[82] A. H. Mueller, B.-W. Xiao, and F. Yuan, Phys. Rev. Lett. **110**, 082301 (2013), 1210.5792.
[83] A. H. Mueller, B.-W. Xiao, and F. Yuan, Phys. Rev. D **88**, 114010 (2013), 1308.2993.
[84] G. Beuf, Phys. Rev. D **89**, 074039 (2014), 1401.0313.
[85] E. Iancu, J. D. Madrigal, A. H. Mueller, G. Soyez, and D. N. Triantafyllopoulos, Phys. Lett. B **744**, 293 (2015), 1502.05642.
[86] J. Kwiecinski, A. D. Martin, and P. J. Sutton, Z. Phys. C **71**, 585 (1996), hep-ph/9602320.
[87] J. Kwiecinski, A. D. Martin, and A. M. Stasto, Phys. Rev. D **56**, 3991 (1997), hep-ph/9703445.
[88] F. Hautmann and H. Jung, JHEP **10**, 113 (2008), 0805.1049.
[89] M. Deak, F. Hautmann, H. Jung, and K. Kutak, JHEP **09**, 121 (2009), 0908.0538.
[90] H. Jung et al. (CASCADE), Eur. Phys. J. C **70**, 1237 (2010), 1008.0152.
[91] P. Sun, B.-W. Xiao, and F. Yuan, Phys. Rev. D **84**, 094005 (2011), 1109.1354.
[92] M. Deak, F. Hautmann, H. Jung, and K. Kutak, Eur. Phys. J. C **72**, 1982 (2012), 1112.6354.
[93] S. Dooling, F. Hautmann, and H. Jung, Phys. Lett. B **736**, 293 (2014), 1406.2994.
[94] J. Zhou, JHEP **06**, 151 (2016), 1603.07426.
[95] M. Hentschinski, A. Kusina, and K. Kutak, Phys. Rev. D **94**, 114013 (2016), 1607.01507.
[96] M. Bury, A. van Hameren, H. Jung, K. Kutak, S. Sapeta, and M. Serino, Eur. Phys. J. C **78**, 137 (2018), 1712.05932.
[97] M. Hentschinski, A. Kusina, K. Kutak, and M. Serino, Eur. Phys. J. C **78**, 174 (2018), 1711.04587.
[98] D. Boer, P. J. Mulders, J. Zhou, and Y.-j. Zhou, JHEP **10**, 196 (2017), 1702.08195.
[99] B.-W. Xiao, F. Yuan, and J. Zhou, Nucl. Phys. B **921**, 104 (2017), 1703.06163.
[100] A. Stasto, S.-Y. Wei, B.-W. Xiao, and F. Yuan, Phys. Lett. B **784**, 301 (2018), 1805.05712.
[101] J. Zhou, Phys. Rev. D **99**, 054026 (2019), 1807.00506.
[102] C. Marquet, S.-Y. Wei, and B.-W. Xiao, Phys. Lett. B **802**, 135253 (2020), 1909.08572.
[103] D.-X. Zheng and J. Zhou, JHEP **11**, 177 (2019), 1906.06825.
[104] E. Blanco, A. van Hameren, H. Jung, A. Kusina, and K. Kutak, Phys. Rev. D **100**, 054023 (2019), 1905.07331.
[105] A. van Hameren, P. Kotko, K. Kutak, and S. Sapeta, Phys. Lett. B **795**, 511 (2019), 1903.01361.
[106] A. van Hameren, P. Kotko, K. Kutak, and S. Sapeta, Phys. Lett. B **814**, 136078 (2021), 2010.13066.
[107] M. Hentschinski, Phys. Rev. D **104**, 054014 (2021), 2107.06203.
[108] M. Nefedov, Phys. Rev. D **104**, 054039 (2021), 2105.13915.
[109] D. Boer, Y. Hagiwara, J. Zhou, and Y.-j. Zhou, Phys. Rev. D **105**, 096017 (2022), 2203.00267.
[110] P. Caucal and E. Iancu (2024), 2406.04238.