

# Supplementary material for LHCb-PAPER-2024-007

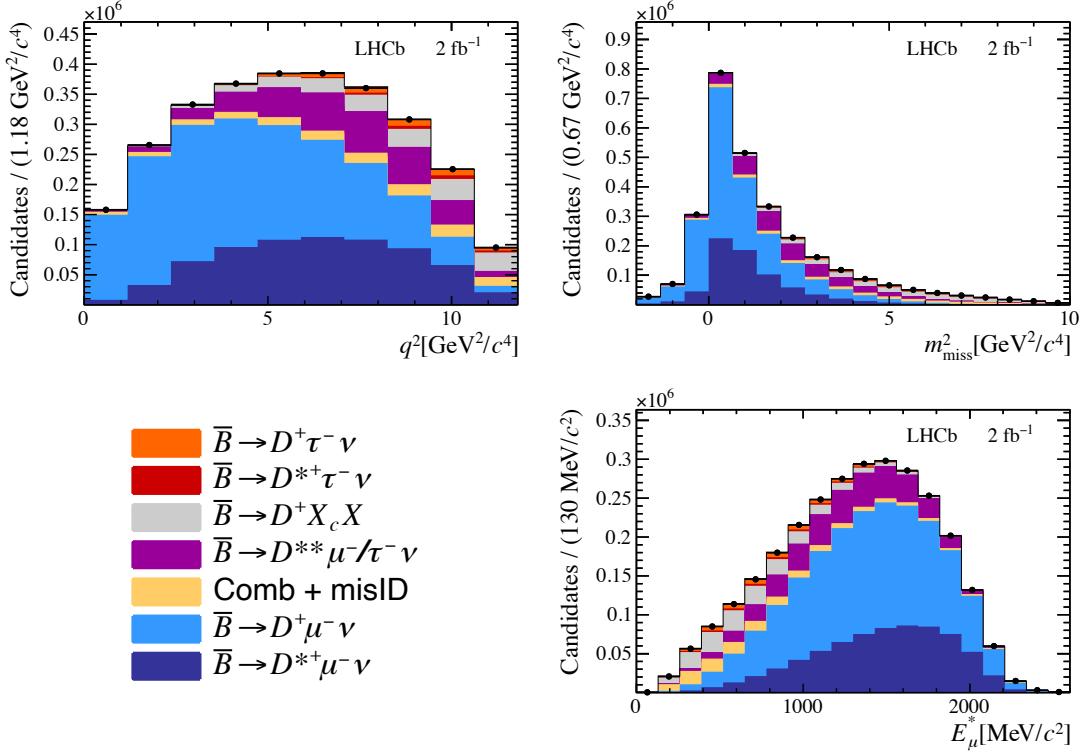


Figure 2: Distributions of  $q^2$ , squared missing mass and lepton energy in the signal isolation region. Projections of the fit results are overlaid.

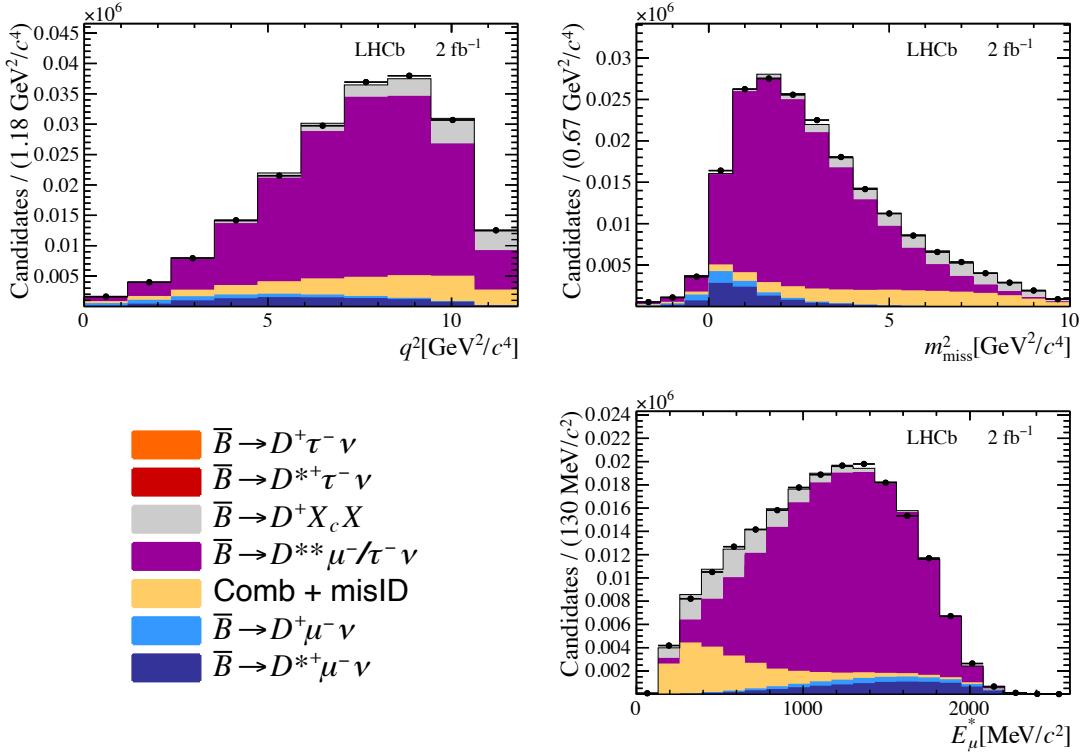


Figure 3: Distributions of  $q^2$ , squared missing mass and lepton energy in the one-pion region. Projections of the fit results are overlaid.

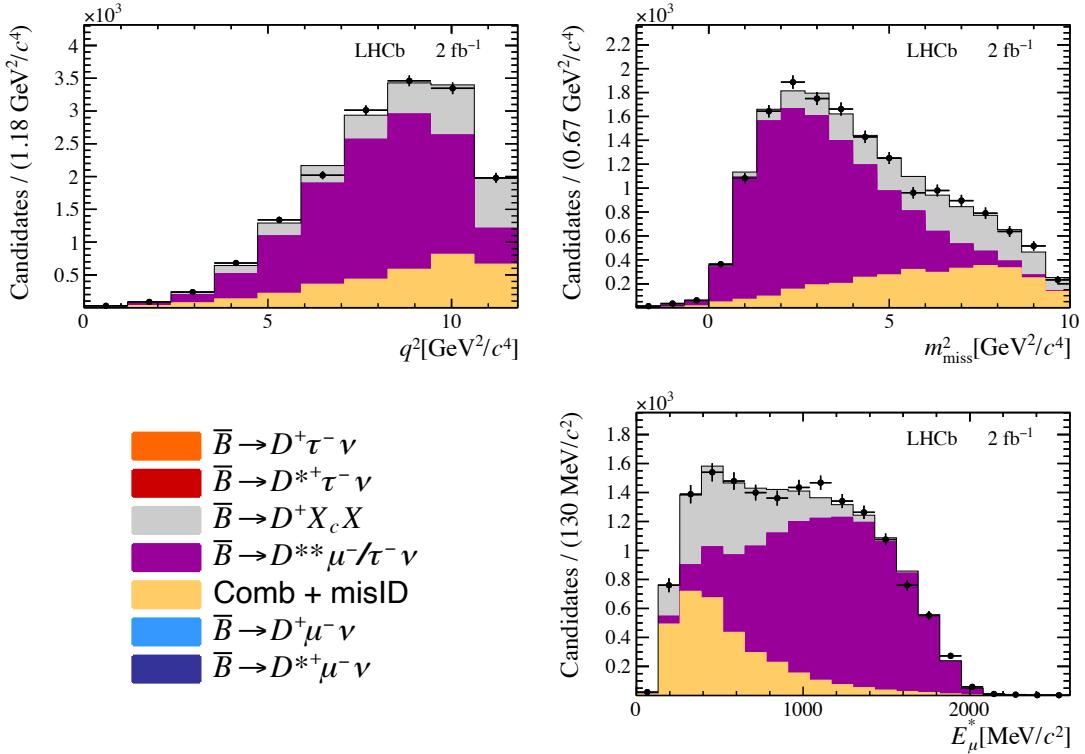


Figure 4: Distributions of  $q^2$ , squared missing mass and lepton energy in the two-pion region. Projections of the fit results are overlaid.

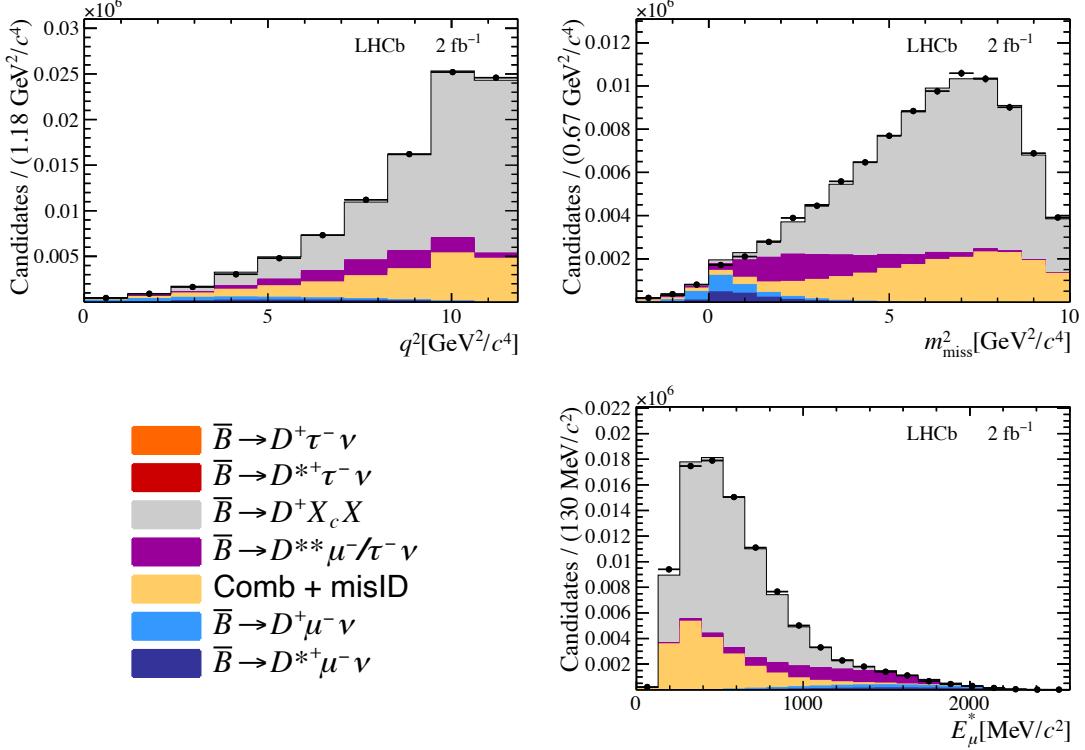


Figure 5: Distributions of  $q^2$ , squared missing mass and lepton energy in the one-kaon region. Projections of the fit results are overlaid.

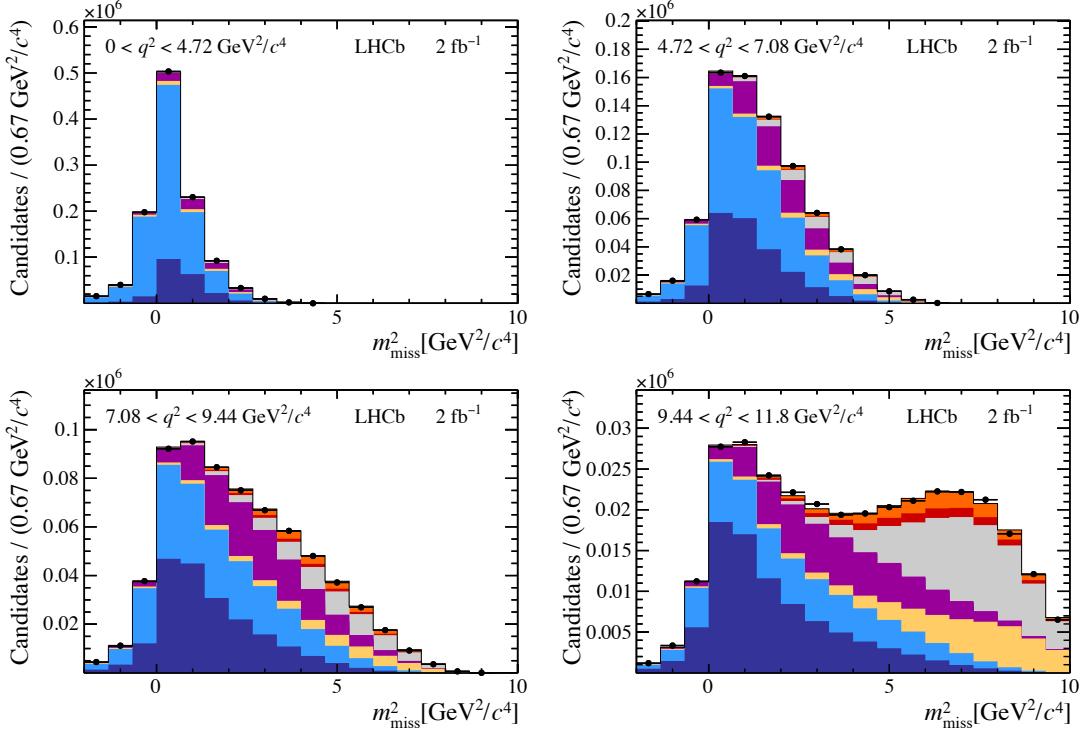


Figure 6: Distributions of squared missing mass in the signal isolation region in bins of  $q^2$ . Projections of the fit results are overlaid.

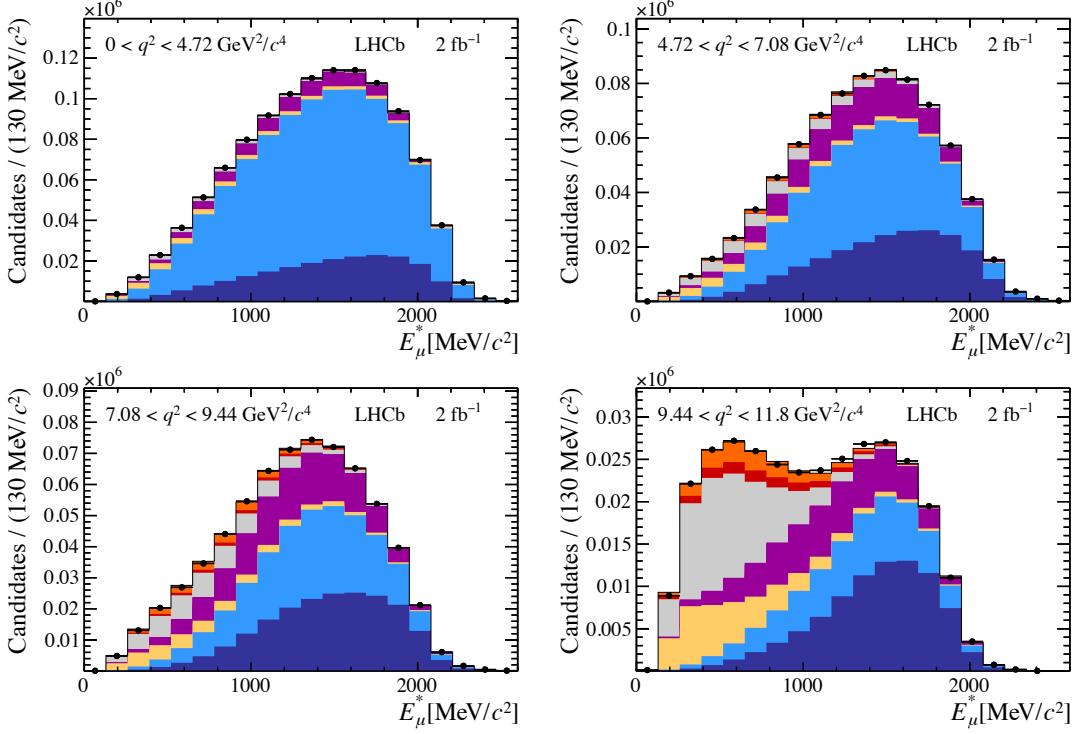


Figure 7: Distributions of the lepton energy in the signal isolation region in bins of  $q^2$ . Projections of the fit results are overlaid.

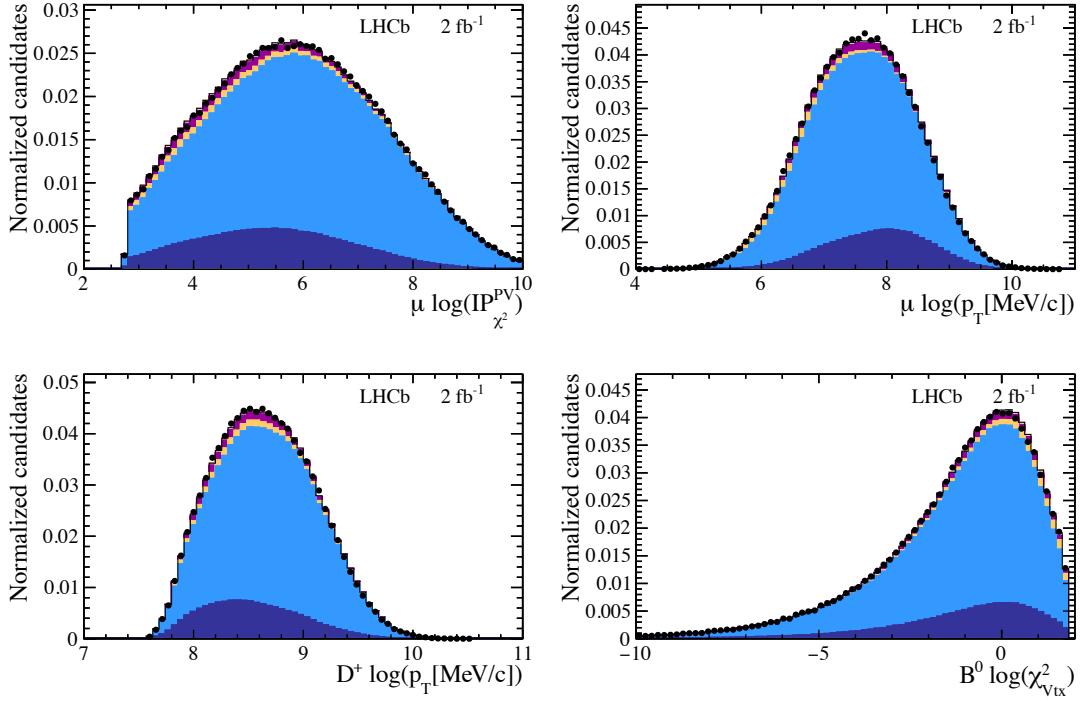


Figure 8: Comparison between data and simulation for the (top left) muon impact parameter, (top right) muon transverse momentum, (bottom left)  $D^+$  transverse momentum and (bottom right)  $B^0$  vertex quality criteria. The data is selected in a low missing-mass region  $m_{\text{miss}}^2 < 2 \text{ GeV}^2/c^4$  to enrich the sample with normalisation decays.

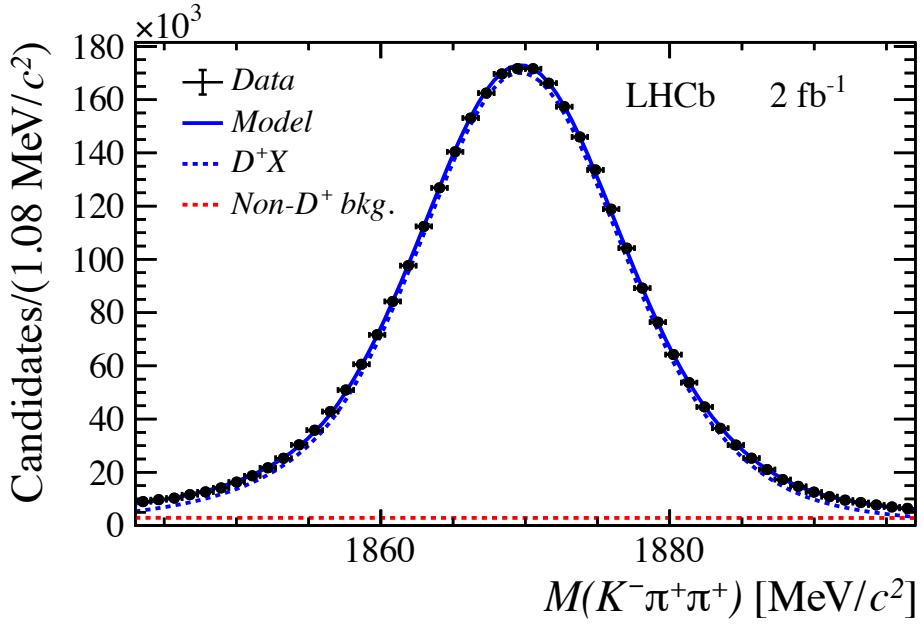


Figure 9: Distribution of the  $K^-\pi^+\pi^+$  invariant mass, overlaid with the fit result to discriminate  $\bar{B} \rightarrow D^+\mu^-\bar{\nu}_\mu X$  decays from background.

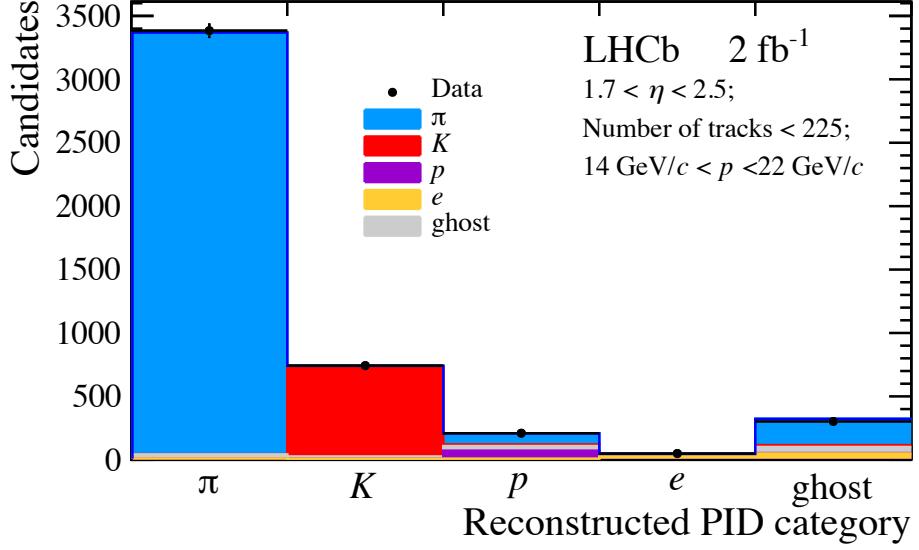


Figure 10: Fit to the distribution of reconstructed misID categories for events with less than 225 tracks, and with pseudo-rapidity of the probe track between 1.7 and 2.5 and momentum between  $14 \text{ GeV}/c$  and  $22 \text{ GeV}/c$ . Fits are performed in kinematic and multiplicity bins and used to determine the fraction of different particle types in the misID sample. Distributions of true pions, kaons, protons, electrons and fake tracks (ghosts) across the reconstructed misID categories are derived from control data and simulated samples, and are shown with different fill colours.