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STRONGLY CURVED SUPER-CONDUCTING MAGNETS: BEAM OPTICS MODELING AND FIELD QUALITY*

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Abstract

Superconducting (SC) dipoles with a strong curvature (radius smaller than 2 meters, for an aperture of about 100 mm and a length of 1-3 meters) are required for applications where compactness is key, such as the synchrotron and gantry for Carbon-ion therapy developed within the European program HITRIplus. Such magnets challenge several assumptions in the field description and put to the test the range of validity of beam optics codes. In particular, the equivalence that holds for the straight magnets between the transverse multipoles description obtained from the Fourier analysis (used for magnet design and measurements) and the Taylor expansion of the vertical field component along the horizontal axis (used in beam optics) is not valid any longer. Proper fringe field modelling also becomes important due to the curved geometry and the aperture being large compared to the magnetic length. We explore the feasibility and the limits of modelling such magnets with optics elements (such as sector bends and multipoles), which allows parametric optics studies for optimization, field quality definition and fast long-term multi-pass tracking.

INTRODUCTION

In the design of SC magnets, the assumption that the magnet is "long enough" compared to its radius of curvature and its aperture is valid most of the time. This assumption implies that the Maxwell equations can be solved in 2D only and, historically, it has resulted in a magnetic field description and field quality definition in terms of harmonic expansion and normal and skew normalized multipoles (see e.g. [1]). Within this 2D approximation and straight geometry, there is a one-to-one correspondence between three different quantities:

- The Fourier coefficients of the radial magnetic field evaluated on a reference circle around the beam (obtained either from the analysis of simulated field patterns or from measurements with a rotating coil)
- The coefficients of the 2D multipole expansion of the transverse field pattern (describing the field to some approximation at any point within the aperture)

Table 1: HITRIplus Magnet Parameters

Parameters	Gantry	Synchrotron
B	4 T	3T
Aperture(D)	80 mm	80 mm
coils geometry	combined function	AG-CCT
ρ	1.65 m	1.89 m
angle	45^o	$60-90^o$
L	1.30 m	<2.6m
ratio D/L	0.06	0.03
ratio ρ/L	1.27	0.63

- The coefficients of the Taylor expansion of the vertical field components (used in beam dynamics and sufficient to describe the field if there is midplane symmetry).

This correspondence, which holds for most practical cases, may lead to the confusion of treating the three as equivalent.

The SC magnets considered for compact applications, such as the medical synchrotron and gantry for 430 MeV/u carbon ions, studied within the HITRIplus programme [2–5], have a maximum bending field of 4 T, resulting in a radius of curvature of 1.65 m. Their aperture is foreseen to be 80 mm and their length is about 1.3 m, for a bending angle of 45^o (gantry magnet). Moreover, for the synchrotron, the main bending magnets have nested Alternating-Gradient CCT (AG-CCT) coils [6, 7], and their field is therefore not constant in the body either.

These conditions are quite "extreme" and challenge not only the field measurement practices (a rotating coil - even if it fitted into the bending aperture - would not sample the field at the same "radial" position around the beam) but also the conventional magnetic field modelling and field quality definition.

For these reasons, within HITRIplus, a study group has been created, bringing together the magnet designers and the beam dynamics experts to understand how to define and correctly use the field quantities of interest (whether to minimise them or to correctly generate them with the coil design [8] and correctly treat them in the optics model).

Concerning the field measurement, the baseline is to use printed circuit boards with curved pickup coils, which measure the inductive voltage during ramp-up or ramp-down with an estimated precision at the level of 10^{-3} relative [9]. The experimental data is then compared to model predictions.

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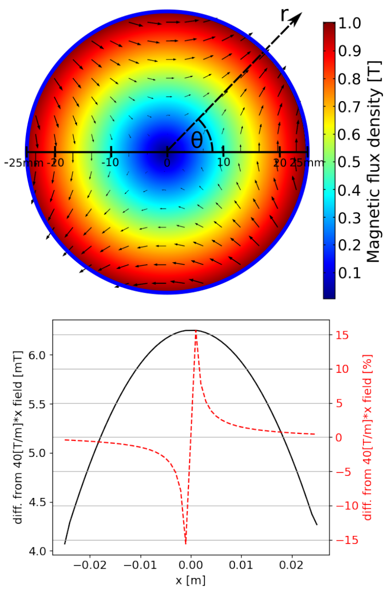

Figure 1: Difference with respect to a pure "quadrupole", in a curved geometry ©2022 IEEE. Reprinted, with permission, from [8].

Hall sensors at a few discrete points would complement these measurements, delivering field values in steady-state conditions as well. The detailed field distribution would then be taken from FEM simulations, cross-calibrated in this way. The measurement techniques will not be further discussed in this paper.

FIELD DESCRIPTION IN CURVED GEOMETRY

The description and handling of the magnetic field in curved geometry is not straightforward and has been approached in different ways by several groups. Many use toroidal harmonics [10, 11], but transforming them into useful beam dynamics quantities is not straightforward.

Beam dynamics codes use field derivatives $\partial^n B_y/\partial x^n(x = y = 0, s)$, where x and y are the transverse coordinates of the co-moving beam coordinate system, and s is the longitudinal coordinate along the beam reference trajectory (see, e.g. the MAD-X definition of local reference system [12]). These are sufficient to describe the magnetic field in the entire aperture [13], provided that the magnet has midplane-symmetry, which as a first approximation [8] is the case for our curved magnets.

Within HITRIplus, we adopted the beam dynamics definition and used the field derivatives to characterise the field. The field quality and the normalised multipoles are therefore defined(!) and computed in terms of field derivatives. The normal components b_n are defined as:

$$b_n = \frac{1}{B_0} R_{\mathrm{ref}}^{n-1} \frac{1}{(n-1)!} \frac{\partial^{(n-1)} B_y}{\partial x^{(n-1)}} \quad (1)$$

To demonstrate the breakdown of the equivalence of the three above-mentioned notions for strongly curved magnets, Figure 1 shows the errors in the field derivatives, when prescribing the boundary condition $B_r = B_0 \sin(2\theta)$ (with $B_0 = 1$ T) on the 2D circular cross section of a torus with major (bending) radius ρ=1.5 m and minor radius $R_{\mathrm{ref}} = 2/3$, R_{bore}=25 mm. This boundary condition produces a "quadrupole-only" Fourier-coefficient by definition also for a curved magnet, and would correspond to a perfect "quadrupole field" in a straight geometry, having only a single non-zero field derivative $\partial B_y/\partial x$ at the origin. However, in a curved geometry, the transverse field pattern can not be described by a single "naive multipole", nor by any linear combination of them, because they are not solutions of Maxwell's equations. In addition to the linear term, there are non-negligible additional components: a 6.25 mT background field and a significant second-order derivative alongside some smaller higher orders. At a distance of 10 mm from the beam, the field differs from the ideal value by 1.5%.

This example illustrates that the design of the coils must take into account the (curved) geometry, and vice versa that the Fourier analysis of the radial field on a reference circle in curved geometries could result in a significant misinterpretation of the field pattern. In both cases, it leads to a mismatch of field characterisation between beam optics and magnet design. Talking about a "quadrupole" in a curved magnet, therefore, makes no sense without exactly specifying what one means by "quadrupole field": it is more correct to talk about "gradient".

FRINGE MODELING

For short, large-aperture, strongly-curved magnets, the fringe fields need to be properly considered. The dipole hard-edge model is not enough, as one can see from Fig. 2 and Fig. 3. The description proposed by MAD-X and by its PTC libraries [12, 14, 15], based on [13], might also not be sufficient [16]: MAD-X implements the fringes with an effective, truncated (i.e. non-symplectic) map, PTC uses a modified, symplectic model which, however, includes only components up to second order in transverse coordinates.

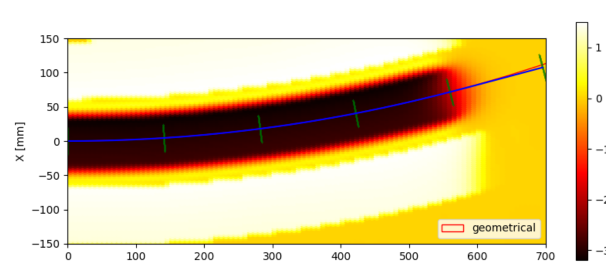

Figure 2: Field map, reference beam trajectory (blue), geometrical, i.e. arc, traectory (red) and multipole kick positions over the magnet(green).

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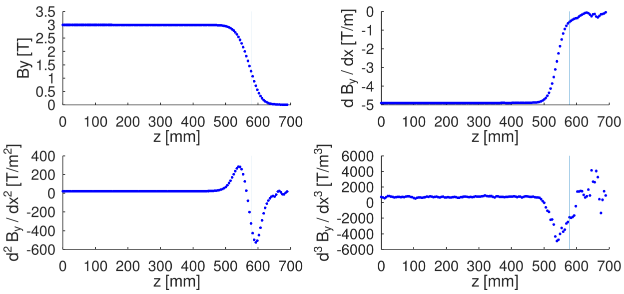

Figure 3: Higher order derivatives over the length of the magnet, showing the importance of the fringe field.

SINGLE-PASS MAGNETS (GANTRY)

For single-pass studies, two approaches were followed [17], both evaluating integrated quantities over the entire length of the magnet.

First of all, an 8$^{\text{th}}$ order Runge-Kutta algorithm was used to track through the 3D field map and the transfer matrix (i.e. the linear components) was extracted from the positions and angles at the beginning and the end of the magnet. As a second step, an equivalent sequence with a combined-function dipole, including the dipole's edges and two multipole lenses at the heads, was built in MAD-X/PTC. The non-linear coefficients of the lenses were fitted by minimising the difference in particle distribution between the PTC tracking and the one done in the 3D field map. This optimization led to a precision better than 10^{-2} mm (mrad) in the particle positions (divergence) at the exit of the magnet if compared to the tracking in the original 3D field map.

The second method, which does not involve tracking through the field map, consisted in computing the higher-order derivatives along the reference trajectory using the Taylor expansion series of the magnetic field, as shown in Fig. 3. Their integrals were then used to build another equivalent lattice in MAD-X, composed of a combined-function sector bend and two multipole lenses at the heads. These were then used for transporting the particle coordinates. The precision, in this case, was of the order of 10^{-1} mm (mrad).

Both methods were used to interact with the magnet designers and minimise the integrated higher-order components by modifying the magnet's heads [18].

MULTI-PASS MAGNETS (SYNCHROTRON)

In the compact synchrotrons studied in HITRIplus and NIMMS [19, 20], the phase advance along the main magnets, which contains significant focusing components, changes substantially. Considering higher-order components integrated along the entire magnet is therefore not sufficient for dynamic aperture and frequency map analysis, as demonstrated in [20]. Moreover, for long-term tracking in rings, one needs to guarantee symplecticity and compromise between the speed of the simulations and the accuracy of the description.

We used the tracking code RF-Track [21] to analyse the magnet. Full-ring tracking studies are the final goal, after benchmarking with codes from the cyclotrons and FFA communities. RF-Track is a code that allows the tracking of charged particles in two environments: one is called “Volume”, for integrating the equations of motion in time through any 3D field maps, and the other is “Lattice”, for integrating the equations of motion in space through matrix-based symplectic elements.

The Taylor expansion coefficients were computed along lines orthogonal to the beam reference trajectory. Then, instead of integrating them, we built a sequence composed of a number of combined-function sector bends, interleaved by multipole lenses [22].

The study aimed to find the minimum number of elements that models the magnet with enough accuracy. The error which quantifies the accuracy, χ^2, is computed from the covariance matrix of the difference of the final particle coordinates obtained by tracking through the 3D field map and through its Lattice representation, i.e. it is proportional to the sum in quadrature of the differences (in position and angle) in normalised coordinates. For the horizontal plane, it is defined as:

$$\chi^2 = \frac{\sum (\Delta x)^2}{\sigma_x^2} + \frac{\sum (\Delta p_x)^2}{\sigma_{p_x}^2} - 2\frac{\sigma_{xp_x}}{\sigma_x^2 \sigma_{p_x}^2} \sum (\Delta x)(\Delta p_x), \quad (2)$$

and it can be extended into 4D and eventually to 6D.

Figure 4 shows that by increasing the number of sector bends, both in the body and in the fringe of the magnet, one can increase the accuracy of the tracking.

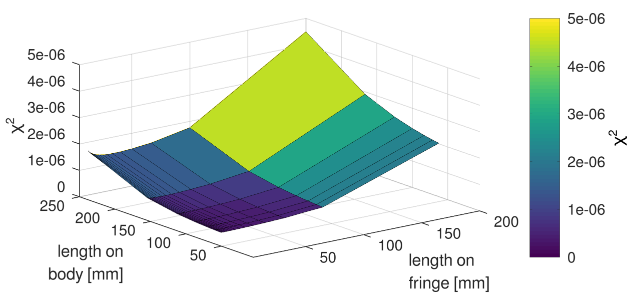

Figure 4: An estimate of the error, χ^2, occurring when one converts a realistic 3D field map to a matrix-based symplectic lattice.

CONCLUSIONS

Strongly-curved magnets challenge the current description of the field, based on the assumptions of straight geometry and ”well-behaving” fringe fields, both for the communication between beam-dynamics and magnet-design experts and for the modelling of the magnets themselves for tracking in lattice elements. The description in terms of Taylor expansion on the midplane allows a non-misleading communication between beam-dynamics and magnet-design experts, in the definition of the field components and the specifications of field quality. A different approach for considering the fringe field is required whether the magnet is single-pass or if long-term tracking is required.

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