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EXPERIMENTAL STUDY OF STOCHASTIC COOLING OF PROTONS IN NAP-M

E.N. Dementiev, N.I. Zinevitch, A.S. Medvedko,
V.V. Parkhomchiuk, D.V. Pestrikov

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Abstract

Results of experiments on stochastic cooling of the energy dispersion in a beam are presented. Correlations were studied between damping decrement and the beam phase space density, number of harmonics involved and feedback gain. The final dispersion in a beam was found to be determined by the noise of the electronics. A coherent instability, associated with a beam shift in a pick-up was found. The limitations of the method due to the noise of electronics and to collective effects in intense beams are discussed.

Introduction

The method of stochastic cooling, proposed by S. van der Meer[1], is based on the use of a system of wideband feedback to cool a beam of heavy particles in a storage ring. In the simplest case, such a system consists of a pick-up electrode, measuring the deviation of a particle from its equilibrium position and a correcting element (kicker) to which the signal from the pick-up is applied through a wideband amplifier. As it was shown by Ya. S. Derbenev and S.A. Heifets[2], the damping effect is associated with a self-interaction of a particle through the feedback. Interaction of different particles of a beam through the feedback leads to the effect of "screening" and increases the cooling time proportionally to the number N of particles in a beam. An attractive feature of the method of stochastic cooling is its low dependency of cooling time on the emittance of the beam to be cooled.

A series of experiments was performed at CERN from 1974 to 1978 on the ISR and on ICE, to study experimentally stochastic cooling[3]. In these experiments the cooling effect was demonstrated and it was also found that the cooling time increases with the number of particles in the beam.

It is regrettable that despite of the large amount of experimental data, important dependencies of e.g. the cooling time or beam size versus feedback loop parameters and dispersion of revolution frequency of the particles in the beam were not given. This makes it difficult to apply the results[3] to different machines. The importance of finding dependencies experimentally is emphasized by the fact that the process of stochastic cooling is described by a non-linear equation, the solution of which can be estimated only.

Here we present results of experiments on stochastic cooling of the energy dispersion in a proton beam in NAP-M. The goal of the experiments was to estimate the possibility of using this method of cooling in machines for antiproton accumulation and to study peculiarities of the cooling process.

1. Estimation of cooling decrements

Let us consider the case of cooling a beam, of uniform azimuthal distribution, with the help of a system shown in Fig. 1. A balanced pick-up consisting of open-ended strip lines was used as a signal source for a feedback loop (FB). The level of the differential signal induced by a particle is :

$$U(t) = \frac{evZ_0}{2\pi Ap} \cdot \psi \cdot \Delta p \sum_{n=-\infty}^{\infty} e^{-in\omega_p t} \, \frac{1 + e^{2in\theta_0\beta} - 2e^{in\theta_0(1+\beta)}}{2} \qquad (1)$$

where $v = \beta c$, ωp and e are the velocity, revolution frequency and charge of the particle, resp., $p = M\gamma v$ = momentum of equilibrium particle, Δp = deviation of momentum of the particle from equilibrium value, ψ = dispersion function of the storage ring, $\theta_0 = L/R_0$ = azimuthal length of the sensor plate, A = its aperture, Z_0= impedance of the strip line.

The differential signal is applied through a transformer to a feedback amplifier and then to the kicker, representing itself a drift tube with a length equal to that of the sensor. If the gain of the amplifier K_0 is constant within the band $|n| \leq n_{max}$ and is zero outside that frequency band, then the voltage at the kicker is :

$$U(t) = \frac{evZ_0}{2\pi Ap} \cdot \psi \, \Delta p \, K_0 \sum_{n=-\infty}^{\infty} e^{-in\omega_p(t-\tau_0)} \frac{1 + e^{2in\theta_0\beta} - 2e^{in\theta_0(1+\beta)}}{2} \tag{2}$$

where τ_0 is the feedback loop delay. The change of particle momentum after passing through a drift tube for optimum values τ_0 and pick-up length θ_0 equals :

$$\delta(\Delta p) \cong -3 \frac{e^2 Z_0 \psi}{2\pi Ap} \cdot \Delta p \cdot K_0 \cdot n_{max}$$

The last equation allows us to determine the single particle decrement :

$$\lambda_0 = \frac{\omega_0}{2\pi} \cdot \frac{\delta(\Delta p)}{\Delta p} = \frac{3Z_0\psi r_p c}{2\pi A\gamma\beta} \cdot K_0 \cdot W \tag{3}$$

Here $r_\ell = e^2/Mc^2$ is the classical radius of the particle, $W = f_0 n_{max}$ = amplifier band width. As one can see from expressions (2) and (3) the value of the decrement λ_0 is increasing with the bandwidth W as long as $n_{max}\Delta f/f \leq 1$ (Δf is the revolution frequency spread in a beam). If one increases W further, the particle and the correcting pulse will not reach the ends of the drift tube simultaneously and this will lead to the decrease of λ_0.

In addition to self-interaction, leading to a decrease of the momentum spread in a beam, particles are subject to random kicks, associated with the thermal noise of the amplifier. This random kicks define the final spread in a beam. Assuming that the amplifier noise is defined by its input resistance, the mean square of the amplifier output is, according to the Nyquist formula :

$$\overline{U_m^2} = 4kT \cdot Z_0 \, W \, K_0^2 \tag{4}$$

where T is the temperature and k the Boltzmann factor. With this, the rate of the change of momentum spread in a beam is given by the following equation :

$$\frac{d}{dt} \langle \Delta p^2 \rangle = -2\lambda_0 \langle \Delta p^2 \rangle + \frac{e^2 \overline{U_m^2}}{v^2} f \, [1 - g(W\tau)],$$
$$1 - g(W\tau) = \frac{1}{n_{max}} \sum_{n=0}^{n_{max}} |1 - \exp(in\omega_0\tau)|^2 \cong \frac{W^2\tau^2}{3}, \tag{5}$$

where $G(w\tau)$ is the correlation function of the noise of the amplifier and τ is the flight time through the accelerating tube. From this equation one can see that the square of the final momentum spread increases linearily with the increase of the gain K_0. That is why the decrement of damping to a given final spread is restricted by the value :

$$\lambda_0 = \frac{e^2 v \; Z_0 W}{kT \; R_0} \cdot \left[\frac{R_0 \psi}{A} \left(\frac{\Delta p}{p} \right)_{st} \right]^2 \cdot \frac{3\pi}{4W^2\tau^2}, \tag{6}$$

R_0 = radius of equilibrium orbit.

Another limitation on the damping rate is associated with the self-interaction of the beam particles through the feedback loop. Because of the finite amplifier bandwidth, a signal, induced by a particle, affects adjacent particles, shifting them to the opposite direction, and this makes the cooling decrement smaller. This effect has been studied in many works[2-6] and can be described by the formula (6) :

$$\frac{1}{\lambda} = \frac{1}{\lambda_0} + \frac{N}{\Delta f\ n^2_{max}} \equiv \tau_0 + \tau_{equiv} \qquad (7)$$

where N is the number of particles in a beam. As one can see from (7), the described screening effect leads to the increase of cooling time of intense beams with small momentum dispersion.

It was mentioned above that the maximum number of harmonics which can effectively participate in cooling is defined by the spread of the revolution frequency $n_{max} \leq f_0/\Delta f$. Therefore, if there are no technical limitations on the bandwidth of the systems'operation frequencies, the value of τ_{eq} at the initial stage of cooling may approach the value :

$$\tau_{eq} = \frac{N}{W} ; \qquad \frac{\Delta f}{f_0} \sim \frac{f_0}{W}$$

As one can see from the above condition, in order to increase W it is necessary to decrease the revolution frequency spread in the beam and therefore to increase the "hardness" of a storage ring, provided the value of $\Delta p/p_0$ is given. The minimum possible spread of the revolution frequency is defined in this case by the condition of the coherent stability of a beam.

2. Description of the cooling system NAP-M

The block diagram of the cooling system is given in Fig. 1. The signals from two radial plates of the pick-up are applied to a differential transformer, then a differential signal, after amplification in pre-amplifier and power amplifier, is applied to the kicker. A variable attenuator and a delay line are included in the feedback loop in order to be able to change the amplitude and delay time of the feedback signal. Pick-up plates together with the inner surface of the ring's vacuum chamber form 50 Ohm open-ended strip lines. The kicker is made of four matched strip lines. Signals for the kicker are taken from four differential outputs of the power amplifier. The influence of such a kicker on the beam is equivalent to that of an accelerating gap. The sensor and the kicker have the same length $\beta c\ W/4$ equal to that of the wavelength in a beam and corresponding to the upper frequency of the feedback bandwidth.

In addition to this system, which we will refer to as wideband, a cooling system with a resonant filter on its input was also studied. Coaxial cables were used as a filter. They form together with the strip lines of the pick-up two resonant lines short circuited at the end by the low input resistance of amplifiers. The amplified signals of both channels are substracted then on the transformer. The lines have the electrical lengths corresponding to 1/4 wavelength of beam revolution frequency. The filter being transparent for the working spectrum of the beam, suppresses the spectral components of the noise of the input amplifiers and of the quarter-wavelength lines outside their bandwidth. This allows to have high feedback gain at a given power of the output amplifier. On the other hand, the signal-to-noise ratio may be increased if the quality factor of the filter is sufficiently high.

It should be noted that a system with a filter passing only half the number of harmonics of the beam revolution frequency makes it possible to study experimentally the influence of particles on the cooling decrement when the number of working harmonics is varied.

Below are given the characteristics of the cooling systems :

	Wideband	With filter
Working frequencies	100 - 300 MHz	100 - 300 MHz
Maximum gain	$0.75 \cdot 10^6$	$4. \cdot 10^6$
Output power, Watt	4 x 0.5	4 x 0.5
Number of harmonics	100	100
Noise factor, DB	2.5	3 - 4

Beam observation during cooling was done by measurement of the beam's Schottky noise spectrum. A continuous beam, coasting in a ring, induces in the pick-up a signal which is associated with the finite number of particles in a beam. This signal is essentially a Schottky noise with a spectrum concentrated around harmonics of the average revolution frequency. The spectral width of the n-th harmonic is defined by the spread of revolution frequencies $N\Delta\omega$, which is proportional to the energy spread of the particles in the beam. The power of such a signal i.e. the spectral integral around a given harmonic, is determined by the total number of particles in a beam and does not depend on the revolution frequency dispersion.

Noise spectra were taken at the 8th harmonic of the revolution frequency with the help of the integrating pick-up and using the system described in (7). After double down frequency conversion, the initial signal was converted into digital form and sent to the computer. It was then analyzed with fast Fourier transformation. Thus one determined the width of the spectrum of the longitudinal noise in the beam and the spectral integral, containing information on the number of particles in a beam.

3. Experimental results

NAP-M is a proton accelerator, designed for experiments on electron cooling. Its mean radius is 7.5 m, the injection energy about 1.5 MeV. Experiments were performed at an energy of 62 MeV with a revolution frequency equal to 2.21 MHz. After acceleration, the beam size was rather small. Therefore, in order to be able to see the effect of stochastic cooling, we had to increase beforehand the momentum dispersion of the beam. To do this, a frequency modulated voltage with a mean frequency equal to the beam revolution frequency was applied to the cavity of the ring. The modulation frequency was chosen to be about 20 Hz. Frequency deviation and level of the voltage applied to the cavity were set such that the momentum dispersion was increased without substantial increase of the amplitudes of betatron oscillations. After initial heating, before switching on the feedback, the beam had $\Delta p/p \cong \pm 3 \; 10^{-4}$ ($\Delta f/f_0 \cong \pm 3 \; 10^{-5}$).

The optimum feedback loop time delay was chosen observing the decrease of the width of the noise spectrum. A typical change of noise spectrum is shown in Fig. 2. Spectra are normalized on their own amplitude. In Fig. 3, beam behaviour is compared when positive and negative feedback is applied. One can see that for practically the same rate of change of the number of particles in the beam (ln S), spectrum width is decreasing when negative feedback is applied and increasing when one applies a positive one. In the last case the maximum width of the spectrum was determined mainly by aperture limitations.

In our experiments, the width of spectrum of the beam being cooled has been decreasing to a final level determined by the noise of the electronics in feedback circuit. In Fig. 4, dependencies are shown of the final spread of revolution frequencies in a beam as a function of single-particle damping decrement. The dependencies are given for filter and wideband systems. The fact that in the wideband system with the same decrement a smaller final size is achieved can be explained possibly by better noise matching of the input circuit, a higher number of working harmonics in the wideband system and also by a low quality factor of the filter in the resonant system.

In Fig. 5, the dependence of the damping decrement is shown vs. the gain of the sensor-kicker circuit in the middle of the working frequency band. Data are presented for wideband and resonant systems. Straight lines 1 and 3 correspond to measurements with a small number of particles ($N \cong 2\ 10^7$). The substantial difference (about 2 times) in decrements λ for both systems at the same gain is explained by the two times smaller number of harmonics participating in the cooling.

The difference in harmonics number shows up in the screening effect. In a wideband system at $N \cong 10^8$, the joint influence of particles is small (curve 2). Halving the number of harmonics leads to a substantial increase of cooling time of the intense beam (curve 4). It should be noted that starting from $K_0 \cong 0.8\ 10^6$ the decrement practically does not change staying at the level of $\lambda \cong 1/8$ min.

As was pointed out before (7), the damping time constant, when the joint interaction of the particles is taken into account, is defined by the following expression :

$$\tau = \tau_0 + \frac{N}{\Delta f\ n^2}$$

where n = number of harmonics, τ_0 = single-particle damping time constant. The experimental dependence of the damping time constant on the value of $N/\Delta f\ n^2$ is given in Fig. 6 (τ_0 was the same for both systems). One can see that the results obtained are in good agreement with theoretical predictions.

4. Longitudinal beam instability

After applying feedback to a beam with a small spread of revolution frequency, a longitudinal instability has been seen. A coherent signal from the integrating pick-up was seen during several seconds and disappeared afterwards. When the feedback was switched on for the second time, no coherent signal was seen. In Fig. 7, a typical behaviour of noise power and spectrum width is shown at the moment of instability. At the moment of switching, S is sharply increasing and Δf decreasing. This corresponds to the monochromatisation of the signal.

During instability development, S is getting smaller and Δf higher, until they reach values corresponding to a new stable state. It is shown in Fig. 8 that the magnitude of the revolution frequency spread Δf_{st}, which is observed after the instability, is proportional to the square root of beam current I_p.

$$\Delta f_{st} = A\sqrt{I_p} \ .$$

It was found that at a given current and gain factor, the instability may be initiated by :

1) electron cooling of the beam below Δf_{st} spread,
2) displacing the beam position in the feedback pick-up.

Reducing the gain factor or beam current led to disappearance of instability. The stability condition for a beam interacting with a stochastic cooling system may be obtained with the help of the linearized Vlasov equation. Assume that stationary state of a beam is described by a distribution function $F_0(p)$. When a collective movement is excited in a beam, the distribution gets non-stationary :

$$F(p,\theta,t) = F_0(p) + \sum_{n\neq 0} F_n(p) e^{in\theta - i\omega t} \ , \tag{8}$$

which causes to appear on the kicker an accelerating voltage with Fourier amplitude :

$$U_{n\omega} = \frac{evZ_0\psi}{Ap} K_0 \left[1 - e^{in\theta_0(1+\beta)}\right] \cdot \frac{1 + e^{2in\theta_0\beta} - 2e^{in\theta_0(1+\beta)}}{2} \cdot e^{-in\omega_0\tau} \int_{-\infty}^{\infty} dp \ p \ F_{n\omega}(p) \tag{9}$$

Using the Vlasov equation we can write :

$$F_{n\omega} = -\frac{iNe}{2\pi} \cdot \frac{U_{n\omega}}{2\pi R_0} \cdot \frac{1}{\omega - n\omega_p} \cdot \frac{\partial F_0}{\partial p} \ , \tag{10}$$

where the number of particles in the beam, N, is separated out from F_0.

Picking out in (10) the moment $\int_{-\infty}^{\infty} dp \ p \ Fn_{\omega}(p)$, we get the dispersion equation for finding the eigen-frequencies ω :

$$1 = -\Omega_n \int d\Delta p \cdot \frac{\partial F_0}{\partial \Delta p} \cdot \frac{\Delta p}{\omega - n\omega_p} \ , \tag{11}$$

where

$$\Omega_n = i \frac{N \ \tau_p \ c \ f_0 \ Z_0 \ \psi}{2\pi \ A\gamma\beta} \cdot K_0 e^{-in\omega_0\tau} \frac{1 + e^{2in\theta_0\beta} - 2e^{in\theta_0(1+\beta)}}{2} \cdot \left[1 - e^{in\theta_0(1+\beta)}\right] \tag{12}$$

is the complex coherent frequency shift near $n\omega_0$.

One can see from equation (11) that the reason for beam instability may be either some displacement relative to the centre of the pick-up or incomplete substraction of the plate signals, the latter being also equivalent to a displacement of the beam in the pick-up.

For a beam with finite momentum spread, this instability may be stabilized by Landau damping. In order to write down the stability criterion it is more convenient to write equation (11) in the following form :

$$1 = -\Omega_n \cdot \frac{\Delta_n + n\Delta\omega_0}{n\omega_p'} \cdot \int d\Delta p \cdot \frac{\partial F_0}{\partial \Delta p} \cdot \frac{1}{\Delta_n - n\omega_p' \Delta p} \qquad (13)$$

where $\Delta_n = \omega - n\omega(\Delta p_0)$, $\Delta\omega_0 = \omega_p' \Delta p_0 = \eta(\Delta p_0/p)$, Δp_0 corresponds to the beam displacement in the pick-up, $\eta = \gamma^{-2} - \gamma_{tr}^{-2}$, $\gamma_{tr}Mc^2$ is the transition energy of the storage ring. If one changes Δ_n along the real axis ($\Delta_n \to \Delta_n + i0$), then equation (13) in its parametric form gives the limit of the stability region in the plane of complex variable Ω_n. The detailed behaviour of the limit curve depends on the equilibrium distribution $F_0(p)$. One can see, however, that if the displacement of the beam in the pick-up is big enough, ($|\Delta\omega_0| > \Delta\omega$, $\Delta\omega$ = modulation frequency spread in the beam), then the limit of the instability region has usually a pear-like form along the axis Re Ω_n. Therefore the stability criterion may be written in the form :

$$\Delta\omega^2 > \left|\Delta\omega_0 \frac{\Omega_n}{n}\right| \quad , \qquad |\omega_0| \gg \Delta\omega \qquad (14)$$

It should be stressed that since instability is defined by the difference in sign of Re Ω_n and η, condition (14) is a necessary one, because stability must be provided for all harmonics in the working band.

Stability condition (14) limits the cooling rate. We will compare this limitation with that obtained from the screening effect. It should be noted that in the band of harmonics giving the most important contribution to the cooling rate the value of $|\Omega_n|$ may be approximately expressed in the following form :

$$|\Omega_n| \cong \frac{N\lambda_0}{n_{max}} \cdot 2\pi \quad , \qquad (15)$$

where λ_0 is the damping decrement for single particle. The screening effect becomes important when the coherent shift $|\Omega_n|$ is getting closer, or higher than the dispersion of the beam revolution frequency :

$$\lambda_0 \leq \lambda_{eq} = \frac{\Delta\omega n_{max}^2}{2\pi\ N} \qquad (16)$$

Taking into account (15) and (16), the condition of coherent stability may be rewritten in the form :

$$\lambda_0 < \frac{\Delta\omega}{|\Delta\omega_0|} \cdot \lambda_{eq} \qquad (17)$$

It can be seen from this that for a beam being cooled near the centre of the pick-up, the limitation on the cooling rate caused by screen effect is more rigid (16). Anyway, when storing in longitudinal phase space, one can get a situation where a beam with relatively small spread $\Delta\omega$ is far from the centre of the storage region $|\Delta\omega_0| \gg \Delta\omega$. In this case, to provide coherent beam stability according to (17), it is necessary to decrease the decrement λ_0 and, therefore, to decrease the filling rate.

Conclusions

In the performed experiments, the influence of the main factors (thermal noise of electronics and joint interaction of particles) on the effectiveness of stochastic cooling was studied. The cooling time achieved for a low intensity beam was equal to 150 s, while the spread had changed from 3 10^{-4} to 2 10^{-4}. This corresponds well to the limitation due to thermal noise : formula (6), which gives 170 s. One of the methods to make this limitation less severe was proposed in CERN[3] and consists of using many parallel cooling systems. In that case, the final size is the same as in the case of only one system, but the cooling time is decreasing in proportion to the number of systems. So, in experiments on ICE, using 12 pick-ups and 12 kickers, a cooling time of 15 s[3] was reached, which approximately corresponds to results of our measurements.

Let us estimate the power of thermal noise at the amplifier output in a system providing cooling in time τ. Assuming the aperture in pick-up as small as possible, $A = R_0\psi(\Delta p/p)_{in}$ where $(\Delta p/p)_{in}$ is the initial momentum spread, and combining with formulas (3), (4) we will get :

$$P = \frac{4kT}{\tau^2 Wq} \cdot \left(\frac{\Delta p}{p}\right)^2_{in} \left(\frac{2pR_0}{e^2 Z_0}\right)^2 , \qquad (18)$$

q is the number of parallel systems. From formula (18) one can see that the power needed to cool a beam in time τ increases as the square of the spread $\Delta p/p$ and as the 4th power of momentum (at a fixed field in the storage ring). So, for 100 systems cooling a beam with momentum pc = 4 GeV and spread $\Delta p/p = \pm 3\ 10^{-2}$ in a machine with $R_0 = 2.5\ 10^3$ cn, Z_0 = 50 Ohm, W = 500 MHz, T_0 = 300°K in a time τ = 4 s, the total power of the amplifiers is $P = 1.5\ 10^5$ W.

That is why for the antiproton accumulator project AA[8] it was proposed to use a beam with much smaller momentum spread $\Delta p/p = 0.75\ 10^{-2}$, τ = 2.2 s, which allows to limit the power of the amplifiers to 25 kW.

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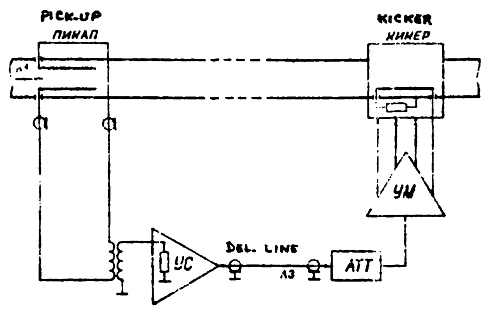

Fig. 1 Block diagram of wideband system for stochastic cooling.

ATT = attenuator,
YM = power amplifier.

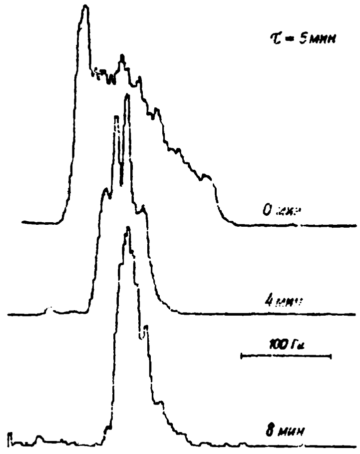

Fig. 2 Wideband system.

Longitudinal beam noise spectra.

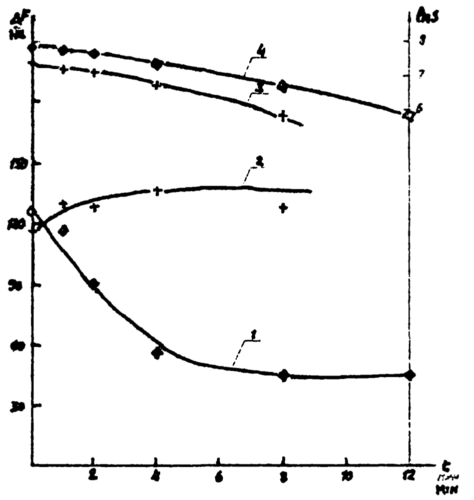

Fig. 3 Wideband system.

Spectrum width ΔF and current S, rate of change for different signs of feedback.

1,2 : spectrum,
3,4 : current,
♦ : negative FB,
\+ : positive FB.

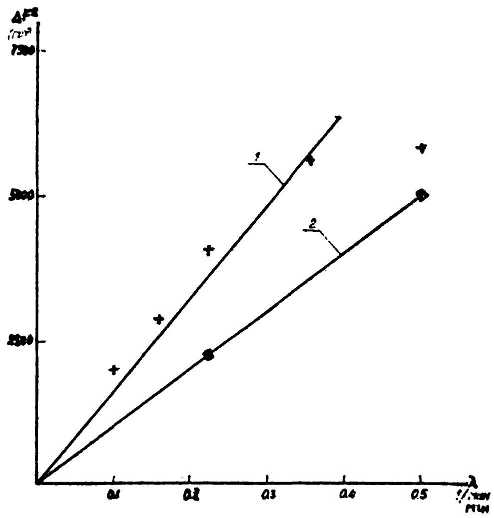

Fig. 4 Square of final spread of revolution frequency versus cooling time.

1 : system with filter,
2 : wideband system.

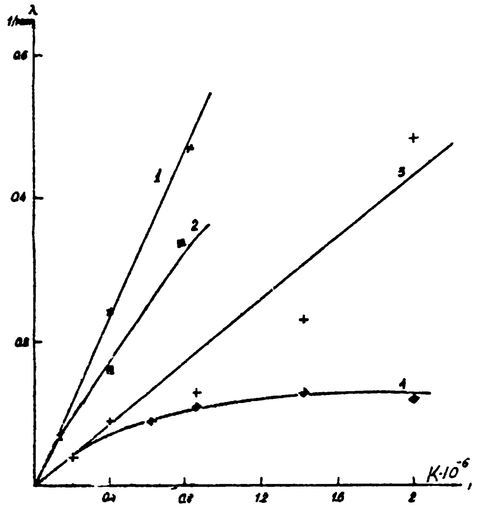

Fig. 5 Cooling decrement vs. gain for different numbers of particles.

x, + : N = 2 10^7
⊠ ◆ : N = 10^8
1, 2 : wideband system,
3, 4 : with filter.

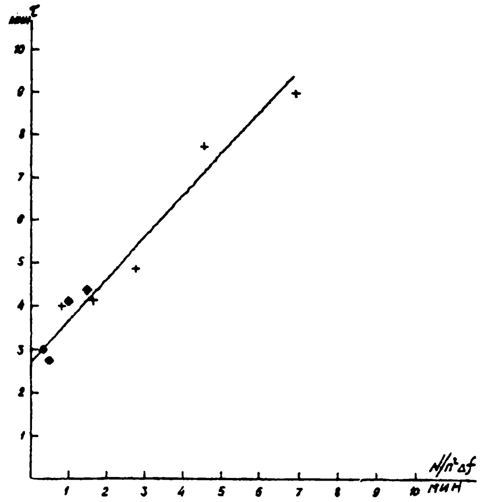

Fig. 6 Cooling time vs. phase density of particles divided by square of number of harmonics.

+ : system with filter,
◆ : wideband system.

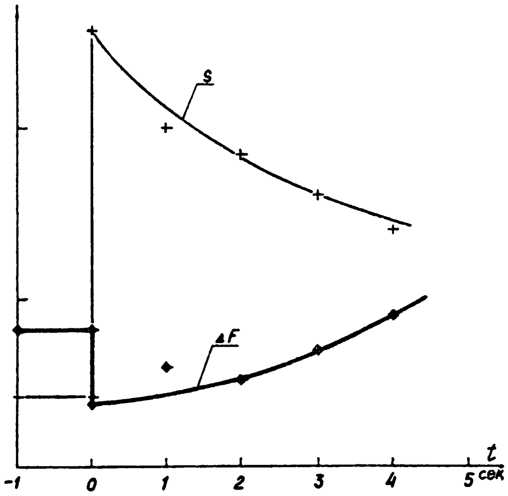

Fig. 7 Development of instability at the moment when stochastic cooling was switched on.

S : integral power of noise,
ΔF : width of spectrum.

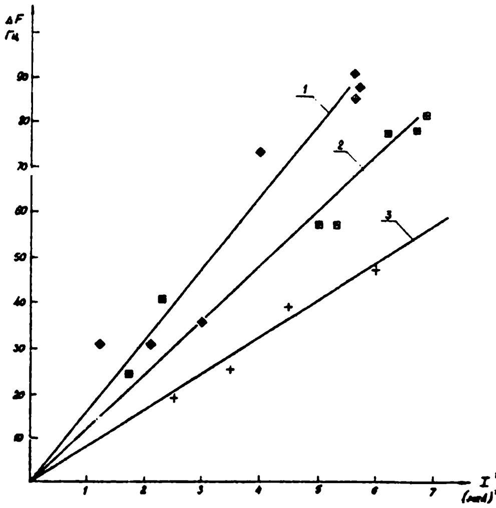

Fig. 8 Final size ΔF after instability vs. current I for different feedback gains.

1 : K = 2 10^6
2 : K = 1.4 10^6
3 : K = 0.5 10^6