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NOTE ON VACUUM TANK LAYOUT

SUMMARY. Calculations have been made to design the vacuum system in the target area in such a way that secondary particles produced in a target could leave the system under very small scattering angles. It has been found that in addition to the main tank in section 1, a second smaller tank located in section 2 would be most useful to obtain scattering angles in the 1,5^{o} region. The vacuum chamber between those two tanks has to be of special construction.

I. DESCRIPTION.

The main tank in section 1 is a multipurpose tank. It has to accommodate the final bending magnet of the slow ejection system, idem for the fast ejection system and the target arrangement, which in principle should be able to probe the circulating beam from all sides and in all locations. The preliminary design of the main tank is presented on drawing P 29-101-0. The idea is to provide for a box with four removable cover plates. The four cover plates are free to mount on whatever experimental equipment is envisaged.

For the purpose of beams of particles produced on internal targets, a slit can be made in the cover plate facing the experimental area. Also a window will be provided for in the end plate downstream. One will note that the blind corner between window and slit is a constructional necessity, but nevertheless a target suitably located in the tank can be seen through window or slit under all angles larger than 3,2^{o}. For those angles there will be no worrying stray field and therefore all momenta are possible in this range. The extreme position of the target would be in the pump manifold, resulting in the lowest possible angle of scattering. However, in this case the stray field of the F section, following the tank, will probably bend inwards the positive particles of too low momentum, so that they are lost; the negative particles however stand a better chance to be guided into the subsequent transport system.

For angles smaller than 3^{o} the tank in section 2 is proposed. This tank is supposed to form an integral part with the vacuum system and for the time being no facilities are provided for to build in experimental equipment. The construction is therefore easy to realise. The all welded box has downstream two openings: one in line with the vacuum chamber of unit No. 2 and one window from which emerge the small-angle particles. Upstream there is a wide opening, which is connected up with the modified vacuum chamber of unit No. 1. Small port holes are provided for, which accommodate targets. The targets are supposed to be of simple construction: they can move only in the median plane in radial direction and at low velocity. A typical position of a target in operation would be in the middle of the magnet unit, 50 mm outwards from the geometrical centre line. From this position scattered particles can be "seen" through the window in a small interval of scattering angles which is a function of the momentum (see table 3). The overall result is that for positive particles the highest momenta can emerge down to scattering angle as low as $1{,}2^{o}$, whereas negative and neutral particles produced at a target include scattering angles of 0^{o} as well.

As will be discussed in chapter III, other target locations do not improve materially the range of scattering angles, nevertheless several additional port holes may be useful to simplify the beam transport system in the experimental area. The same applies to the variety of particles produced with targets located in tank 1, some of which will also go through the window in tank 2, but the range is inferior to the range already obtained with the target in the middle of the magnet unit. However, the window in tank 2 will be used for the beam produced with the slow ejection system. It is the intention that part of the transport system designed for the small-angle beams, will also acco modate the slow ejection beam.

The arrangement for the fast beam ejector is still under discussion. The problem is here that the ejected beam enters the F section of unit No. 1 80 to 120 mm radially outwards from the centre line, making the design of the vacuum chamber No. 1 even more difficult, and it is not known as yet whether it is possible to design chamber

No. 1 in such a way, that it can accommodate all the beams. Leaving this question for the time being unsolved, we proceed to discuss the equation of motion, enabling us to plot the trajectories of particles of any momentum or polarity.

II. EQUATION OF MOTION.

The formula used for piece wise construction of the trajectories is given by

$$\frac{d^2x}{ds^2} + \frac{1}{r_o} = \pm \frac{B}{B_oR_o} \qquad (1)$$

s is the distance measured along the geometrical centre line.
x is the distance towards the geometrical centre line and counted positive radially inwards.
r_o is the curvature of the geometrical centre line.
B is the vertical component of the magnetic field.
B_oR_o is the magnetic rigidity of the particle.
the plus sign is used for positive particles, the minus sign for negative particles.

Since d^2x/ds^2 is in good approximation the curvature of the orbit with respect to the rectified geometrical centre line, formula (1) expresses thus in so many words that the actual curvature of the particle is the sum of the curvature with respect to the chosen geometrical centre line plus the curvature of the geometrical centre line.

Assuming the magnetic field linear, we put

in F section $$B = B_o\,(1 - nx/r_o) \qquad (2)$$

in D section $$B = B_o\,(1 + nx/r_o) \qquad (3)$$

in which B_o is the field value belonging to the geometrical centre line and n is the modulus of the field index. F and D are conform to the nomenclature adopted for the PS machine. We note that for a proton of nominal energy $r_o = R_o$ so that (1) reduces to the well known form

$$\frac{d^2x}{ds^2} = - Kx \tag{1a}$$

Also in straight section $r_o = \infty$ and $B = 0$ so that (1) reduces to

$$\frac{d^2x}{ds^2} = 0 \tag{1b}$$

The solution of (1) in matrix notation, valid for piece wise uniform structures, is given by

$$\begin{bmatrix} x+a \\ x' \end{bmatrix}_{s_2} = M(s_2/s_1) \begin{bmatrix} x+a \\ x' \end{bmatrix}_{s_1} \tag{4}$$

Conform to the definition, R_o denotes the radius of curvature the particle in question would have in the magnetic field B_o. We have then:

$$\begin{aligned} a &= -(r_o - R_o)/n \quad \text{for positive particles in F section} \\ a &= (r_o - R_o)/n \quad \text{" positive " " D "} \\ a &= -(r_o + R_o)/n \quad \text{" negative " " F "} \\ a &= (r_o + R_o)/n \quad \text{" negative " " D "} \end{aligned} \tag{5}$$

The matrix M has the usual form, however K has to be modified according to :

$$K = \frac{n}{r_o R_o} \text{ and } \phi = K^{1/2}(s_2 - s_1) \tag{6}$$

we have then

$$M^+ = \begin{bmatrix} \cos\phi & K^{-1/2}\sin\phi \\ -K^{1/2}\sin\phi & \cos\phi \end{bmatrix} \text{ for } \begin{matrix}\text{pos} \\ \text{neg}\end{matrix} \text{ part. in } \begin{matrix}\text{F} \\ \text{D}\end{matrix} \text{ section}$$

and (7)

$$M^- = \begin{bmatrix} \text{ch}\,\phi & K^{-1/2}\,\text{sh}\,\phi \\ K^{1/2}\,\text{sh}\,\phi & \text{ch}\,\phi \end{bmatrix} \text{ for } \begin{matrix}\text{pos} \\ \text{neg}\end{matrix} \text{ part. in } \begin{matrix}\text{D} \\ \text{F}\end{matrix} \text{ section}$$

III. TRAJECTORIES.

In table I is presented the required elements for plotting trajectories for a variety of momenta. As parameter is chosen the ratio of the rigidity of primary and secondary particle, i.e. r_o/R_o. In drawing P 29-100-0 is shown in the median plane the contour of the vacuum system as it would appear with rectified geometrical centre line. The trajectories shown are also rectified, only the calculated points P,Q....T are exact, assuming the magnetic field linear.

In table II is considered a target located half way magnet unit 1, i.e. azimuth position (R) at 50 mm from the centre line outwards, in the median plane. With the scattering angle θ as variable and the momentum ratio r_o/R_o as parameter, the coordinates at the end of the magnet (S) are calculated. For the straight section 2 all particles with the same momentum have a virtual image point. Of this point the coordinates (p,q) are calculated with respect to this straight section and shown in the drawing.

In table III is given the limits of the scattering angles, allowing unhindered passage through the window, again with the momentum ratio as parameter. Also the range for neutral particles and γ's is mentioned. All this with the target in the above mentioned position. The chosen target position is the best for viewing the smallest scattering angles for positive particles in the momentum range considered. Approximately one finds that a target shift to the left by an amount Δs reduces the bending inwards by an angle $2\Delta s/3R_o$. The factor 2/3 results from the ratio of local snd nominal magnet fields. The loss in angle due to rotation is however $\Delta s/r_o$. Therefore up to $r_o/R_o = 1,5$ the loss in angle is higher than the gain. However the rotation effects also a shift towards the geometrical centre line so that even up to $r_o/R_o = 3$, the window edge would not be cleared by a target shift to the left under equal scattering angles. For lower momenta, $r_o/R_o > 3$, tank 1 is more interesting for direct viewing, or, if the most forward angles are important, one might envisage extracted positive beams into the North Hall. With respect to the negative particles, zero scattering angle is comprised in the momentum range up to $r_o/R_o = 3$. For still lower momenta a target shift to the left seems appropriate.

The calculations are based on a linear magnetic field. The error is not noticeable for the minimum scattering angles of both positive and negative particles. For the maximum scattering angles the error is less than 1 milliradian for the highest momenta and less than 3 milliradian for $r_o/R_o = 3$.

The window in tank II admits also the neutral particles and γ's produced in tank I at zero scattering angle. The direction of those with respect to tank II is obviously $2\pi/100$ radian. The general direction of the negative particles, produced with the target as discussed, happens to have about the same direction. The general direction of the positive particles is about half this value: $\pi/100$ radian, and coincides with neutrals produced with the same target at zero scattering angle. It also coincides with the general direction of the slow ejection beam.

The above remarks may serve as imput data for a discussion on the subsequent transport system.

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TABLE I

$r_o = 70$ m; $n = 288$; $s_2 - s_1 = 2,2$ m

r_o/R_o	$(r_o-R_o)/n$	$(r_o+R_o)/n$	$K = n/(r_o R_o)$	$\phi = K^{1/2} s$
1,0	0	0,486	0,05878	0,5334
1,2	0,041	0,446	0,07053	0,5843
1,5	0,081	0,405	0,08816	0,6532
2,0	0,122	0,365	0,1176	0,7543
3,0	0,162	0,324	0,1763	0,9238

$r_o/R_o = 1,0$ $$M^+ = \begin{bmatrix} 0,861 & 2,097 \\ -0,123 & 0,861 \end{bmatrix} \qquad M^- = \begin{bmatrix} 1,146 & 2,306 \\ 0,136 & 1,146 \end{bmatrix}$$

$r_o/R_o = 1,2$ $$M^+ = \begin{bmatrix} 0,834 & 2,077 \\ -0,146 & 0,834 \end{bmatrix} \qquad M^- = \begin{bmatrix} 1,176 & 2,327 \\ 0,164 & 1,176 \end{bmatrix}$$

$r_o/R_o = 1,5$ $$M^+ = \begin{bmatrix} 0,794 & 2,047 \\ -0,180 & 0,794 \end{bmatrix} \qquad M^- = \begin{bmatrix} 1,221 & 2,360 \\ 0,208 & 1,221 \end{bmatrix}$$

$r_o/R_o = 2,0$ $$M^+ = \begin{bmatrix} 0,729 & 1,997 \\ -0,235 & 0,729 \end{bmatrix} \qquad M^- = \begin{bmatrix} 1,298 & 2,415 \\ 0,284 & 1,298 \end{bmatrix}$$

$r_o/R_o = 3,0$ $$M^+ = \begin{bmatrix} 0,603 & 1,900 \\ -0,335 & 0,603 \end{bmatrix} \qquad M^- = \begin{bmatrix} 1,458 & 2,526 \\ 0,445 & 1,458 \end{bmatrix}$$

TABLE II

Target in middle of magnet unit No. 1 $\begin{bmatrix} x \\ x' \end{bmatrix}_R = \begin{bmatrix} -50 \text{ mm} \\ \theta \text{ m.rad} \end{bmatrix}$

$\frac{r_o}{R_o}$	positive particles $\begin{bmatrix} x \\ x' \end{bmatrix}_S$	p (m)	q (mm)	negative particles $\begin{bmatrix} x \\ x' \end{bmatrix}_S$	p (m)	q (mm)
1	$\begin{bmatrix} -57 + 2{,}30\,\theta \\ -6{,}8 + 1{,}15\,\theta \end{bmatrix}$	2,01	-43	$\begin{bmatrix} -111 + 2{,}10\,\theta \\ -53{,}6 + 0{,}86\,\theta \end{bmatrix}$	2,43	20
1,2	$\begin{bmatrix} -51 + 2{,}33\,\theta \\ -1{,}7 + 1{,}18\,\theta \end{bmatrix}$	1,98	-48	$\begin{bmatrix} -116 + 2{,}08\,\theta \\ -57{,}8 + 0{,}83\,\theta \end{bmatrix}$	2,49	29
1,5	$\begin{bmatrix} -43 + 2{,}36\,\theta \\ +6{,}5 + 1{,}22\,\theta \end{bmatrix}$	1,93	-56	$\begin{bmatrix} -123 + 2{,}05\,\theta \\ -63{,}9 + 0{,}79\,\theta \end{bmatrix}$	2,58	43
2	$\begin{bmatrix} -28 + 2{,}41\,\theta \\ +20{,}4 + 1{,}30\,\theta \end{bmatrix}$	1,86	-66	$\begin{bmatrix} -135 + 2{,}00\,\theta \\ -74 + 0{,}73\,\theta \end{bmatrix}$	2,74	68
3	$\begin{bmatrix} +1 + 2{,}53\,\theta \\ +50 + 1{,}46\,\theta \end{bmatrix}$	1,73	-86	$\begin{bmatrix} -159 + 1{,}90\,\theta \\ -92 + 0{,}60\,\theta \end{bmatrix}$	3,15	132

TABLE III

Limits of the scattering angles, in milliradians, to be seen through window.

$\frac{r_o}{R_o}$	positive θ_{max}	positive θ_{min}	negative θ_{max}	negative θ_{min}
1	-15	-54	+18	-32
1,2	-18	-56	+22	-29
1,5	-22	-59	+28	-26
2	-29	-65	+37	-20
3	-43	-73	+58	-7

neutral particles, γ's

θ_{max} = +5 m.rad

θ_{min} = -40 m.rad