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DISPERSION SUPPRESSOR
FOR
PHASE SHIFT WHICH IS NOT EXACTLY 60°

S.X. Fang

1. INTRODUCTION

As mentioned by E. Keil[1] and J.P. Delahaye[2], a particularly simple dispersion suppressor can be obtained for a $\mu = 60°$ regular lattice structure by just leaving out the bending magnets in one of the two regular cells. But in most practical cases the phase advance per cell cannot be just equal to 60°, then two different types of bending magnets must be introduced in this dispersion suppressor. This complicates the manufacture of magnets.

In this paper, it will be shown that if the phase advance per cell is not far from 60°, for example in the range 60° ±15°, the dispersion suppressor still can keep the above simple form, but the lengths of the straight sections, in which the bending magnets are emitted, are different from the lengths between quadrupoles in the regular cell. Meanwhile, the modulation of the β function is rather small, in most cases less than 10%.

2. TRANSFER MATRIX OF THE DISPERSION SUPPRESSOR

Suppose the dispersion suppressor is composed of two cells : one the regular cell, another, we call the special cell. They have the same quadrupole strengths as in the regular cell. However, the straight section lengths between the quadrupoles where the bending magnets are omitted are different (Fig. 1).

The horizontal transfer matrix of such a dispersion suppressor can be described as follows :

$$M = \begin{bmatrix} \cos\mu_0 + \alpha_0 \sin\mu_0 & \beta_0 \sin\mu_0 & (1 - \cos\mu_0 - \alpha_0 \sin\mu_0)D_0 - \beta_0 \sin\mu_0 D_0' \\ +\gamma_0 \sin\mu_0 & \cos\mu_0 - \alpha_0 \sin\mu_0 & -\gamma_0 \sin\mu_0 D_0 + (1 - \cos\mu_0 + \alpha_0 \sin\mu_0)D_0' \\ 0 & 0 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} \cos\mu_1 + \alpha_1 \sin\mu_1 & \beta_1 \sin\mu_1 & 0 \\ \gamma_1 \sin\mu_1 & \cos\mu_1 - \alpha_1 \sin\mu_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (1)$$

where μ is the horizontal phase advance per cell, α, β, γ are the Twiss parameters. Subscripts "0" and "1" denote the corresponding value in regular cell and the cell without bending magnets. D_0 and D_0' are the dispersion value and its derivative at the end point of the regular cell.

Then the conditions for suppressing dispersion are :

$$\overline{M} \times \begin{pmatrix} D_0 \\ D_0' \\ 1 \end{pmatrix} = 0 \, , \tag{2}$$

that is :

$$\begin{aligned}(C_0C_1 + \alpha_0\alpha_1S_0S_1 + \alpha_0S_0C_1 + \alpha_1C_0S_1 + \beta_0\gamma_1S_0S_1 + 1 - C_0 - \alpha_0S_0)D_0 \\ + (\beta_1C_0S_1 + \alpha_0\beta_1S_0S_1 + \beta_0S_0C_1 - \alpha_1\beta_0S_0S_1 - \beta_0S_0)D_0' = 0\end{aligned} \tag{3}$$

$$\begin{aligned}(\gamma_0S_0C_1 + \alpha_1\gamma_0S_0S_1 + \gamma_1C_0S_1 - \alpha_0\gamma_1S_0S_1 - \gamma_0S_0)D_0 \\ + (C_0C_1 + \alpha_0\alpha_1S_0S_1 - \alpha_0S_0C_1 - \alpha_1S_1C_0 + \gamma_0\beta_1S_0S_1 + 1 - C_0 + \alpha_0S_0)D_0' = 0 \, .\end{aligned} \tag{4}$$

If the end (or starting) point of the cell coincides with the middle or end of a quadrupole, then $D_0'/D_0 = -\alpha_0/\beta_0$, and (3) and (4) become :

$$\begin{aligned}\left[\cos(\mu_0+\mu_1) + 1 - \cos\mu_0\right] - \alpha_0\left(\frac{\Delta\beta}{\beta_0} - \frac{\Delta\alpha}{\alpha_0}\right) \sin\mu_1 \cos\mu_0 - \frac{1}{\alpha_0}\frac{\Delta\beta}{\beta_0} \sin\mu_1 \cos\mu_0 - \left(\frac{\Delta\alpha}{\alpha_0}\right) \sin\mu_0 \sin\mu_1 \\ + \frac{\sin(\mu_0+\mu_1)}{\alpha_0} - \frac{\sin\mu_0}{\alpha_0} = 0\end{aligned} \tag{5}$$

$$\frac{\sin(\mu_0+\mu_1)}{\alpha_0} - \frac{\sin\mu_0}{\alpha_0} + \sin\mu_0 \sin\mu_1 \left(\frac{\Delta\alpha}{\alpha_0} - \frac{\Delta\beta}{\beta_0}\right) - \frac{1}{\alpha_0}\frac{\Delta\beta}{\beta_0} \sin\mu_0 \cos\mu_0 = 0 \tag{6}$$

where $\Delta\beta = \beta_1 - \beta_0$, $\Delta\alpha = \alpha_1 - \alpha_0$, and we suppose $\Delta\alpha/\alpha_0 \ll 1$ and $\Delta\beta/\beta_0 \ll 1$. (5) and (6) can be simplified further :

$$\cos\mu_1 - 1 + \cos\mu_0 + \alpha_0\left(\frac{\Delta\alpha}{\alpha_0} - \frac{\Delta\beta}{\beta_0}\right) \sin\mu_1 = 0 \tag{7}$$

$$\sin\mu_1 - \sin\mu_0 - \frac{\Delta\beta}{\beta_0} \sin\mu_1 = 0 \tag{8}$$

Obviously, by chosing two of the three parameters μ_1, $\Delta\beta$, $\Delta\alpha$, one can easily fit the above dispersion suppressing conditions.

3. RESULTS FOR FODO LATTICE

Let us use the FODO lattice structure as an example. For simplicity, we choose the middle of F quadrupole as the end point of the cell, and suppose the absolute values of the focusing strength in all F and D quadrupole are the same. Using thin lens approximation, we have :

$$\cos\mu_0 = 1 - 2\left(\frac{\delta L}{4}\right)^2, \qquad \alpha_0 = 0, \qquad \beta_0 \sin\mu_0 = L\left(1 + \frac{\delta L}{4}\right) ;$$

$$\cos\mu_1 = 1 - 2\left(\frac{\delta L_1}{4}\right)^2 - \frac{1}{2}(\delta\Delta)^2, \qquad \Delta\alpha \sin\mu_1 = \delta\Delta, \qquad \beta_1 \sin\mu_1 = L\left(\frac{L_1}{L} + \frac{\delta L}{4}\right) - \delta\Delta^2 .$$

where δ is the product of focusing strength K and the length of quadrupoles ℓ_Q, L and L_1 are the total length of the regular cell and the special cell, respectively. 2Δ is the length difference between D to F and F to D (see Fig. 1), i.e. $\ell_1 - \ell_2 = 2\Delta$, $\ell_1 + \ell_2 = L_1$.

Then (7) and (8) can be rewritten as follows :

$$1 - 2\left(\frac{\delta L_1}{4}\right)^2 - 2\left(\frac{\delta L}{4}\right)^2 - \frac{1}{2}(\delta\Delta)^2 + \delta\Delta = 0 \qquad (9)$$

$$\frac{\delta L_1}{4}\left(1 - \frac{\delta L_1}{4}\right) - \frac{\delta L}{4}\left(1 - \frac{\delta L}{4}\right) + \frac{1}{4}(\delta\Delta)^2 = 0 \ . \qquad (10)$$

due to $\delta\Delta \ll 1$, so one can find $\delta L_1/4 = (1 - \delta L/4)$ from (10) and $\delta\Delta = -1 + 2(\delta L_1/4)^2 + 2(\delta L/4)^2$ from (9).

Table 1 and Figs 2 and 3 show the results for different phase shifts in a regular cell.

Table 1 – The straight section length of the special cell for different phase shifts in a regular cell.

μ_0	45°	50°	55°	57.5°	60°	62.5°	65°	70°	75°
μ_1	69.6°	67.6°	64.4°	62.3°	60°	57.4°	54.3°	47.2°	38.0°
L_1/L	1.61	1.37	1.17	1.08	1	0.93	0.86	0.74	0.64
$\delta\Delta$	0.055	0.024	0.0058	0.0015	0	0.0014	0.0056	0.0022	0.047
ΔB	0.178	0.088	0.022	0.0054	0	0.0056	0.025	0.106	0.284

One can see from Table 1, that the orbit dispersion can be easily suppressed by slightly changing the straight section lengths in the special cell for $45° < \mu_0 < 75°$ and the difference of the two lengths $\delta\Delta$ is rather small. Another interesting phenomenon is $\mu_1 + \mu_0 \approx 120°$. The phase shift in the special cell seems to compensate the deviation of the regular cell from 60°.

Obviously, when using the above technique one cannot avoid the modulation of the β function in the lattice, the maximum increment of the beat $\Delta\beta/\beta$ depends on the particular lattice structure of a ring. For example, if the machine is composed of one super-period which contains n regular cells and one dispersion suppressor (Fig. 4), then $\Delta\beta_A$ value at the point A will be :

$$\Delta\beta_A = \Delta B \ . \ \cos(2n\mu_0 + \mu_1 - \theta)\ \frac{\sin\mu_1}{\sin[(2n+1)\mu_0 + \mu_1]} \ ,$$

where $\theta = \tan^{-1}\left(\Delta\alpha/(\Delta\beta/\beta_0)\right)$ and $\Delta B = 2\beta_0\sqrt{(\Delta\beta/\beta_0)^2 + (\Delta\alpha)^2}$. The $\Delta\beta_i$ increment in any other points will be proportional to B so it can be used as a quantity describing how serious the β function modulation will be. Table 1 also shows the results of B for different μ_0. Fortunately, the B values are rather small ($\Delta B \leqq 10\%$) in most cases.

It is worthwhile to point out that it is impossible to eliminate the increment B in horizontal plane by increasing the number of the variables for matching the β function in the end of special cell, because there are only three free parameters in (7) and (8).

For a small machine, the edge focusing of bending magnets is important, then a rather large beat of β function in vertical plane will happen along due to leaving out the bending magnets in the special cell. (We suppose that straight bending magnets are adopted). This beating of β function will be decreased for $\mu_0 \leqq 60°$ and increased for $\mu_0 \geqq 60°$ when above technique is applied.

4. CONCLUSION

If a regular lattice structure phase shift per cell μ is in the range 60° ±15°, a simple dispersion suppressor can be obtained by leaving out the bending magnets in one of the two regular cells and changing the lengths of these straight sections. The corresponding modulation of the β function is rather small.

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REFERENCES

1) E. Keil, CERN 77-13 (1977).

2) J.P. Delahaye, AA Long Term Note No. 26.