PS/Int. EA 59-14
7th December 1959

COMPUTATION OF TRAJECTORIES IN THE C.P.S. MAGNET FIELD WITH A MERCURY PROGRAMME

As a first step in the development of a computer programme for the computation of particle trajectories in the C.P.S. magnet, a programme for trajectories in the median plane has been produced.

1. POSSIBILITIES AND LIMITATIONS.

It has been aimed at producing a versatile programme that will allow to compute most of the variety of trajectories along which protons and secondaries will cross the C.P.S. magnet. At the same time, the real configuration of the C.P.S. magnet field has been taken into account with only small exceptions. (cp. Appendix).

The programme permits the computations of the trajectories of particles of any charge and momentum, starting from any point in a magnet sector or its leakage field and moving in the forward direction (i.e., under an angle $< \pm 90^{\circ}$ with respect to the equilibrium proton orbit). The trajectory can be traced through any number of magnet sectors in the sequence indicated in the Appendix, sub. a. It will be stopped automatically if the radial excursion exceeds the value of 0.54 m which has been adopted for the radial limit of the leakage field.

The integration step width can be chosen arbitrarily. The print-out step can be any integral multiple of the integration step width. Provisions have been made such that any number of trajectories can be computed successively in one run.

Magnetic field measurements are available for average field levels of 1.186 Wb/m^2 and 1.419 Wb/m^2, corresponding to 24.9 GeV and 29.8 GeV total energy of the circulating protons. A facility for linear extrapolation from the given values is provided; the precision will of course depend on the width of the extrapolation interval.

The programme is not aware of magnet yokes, coil supports, and other hardware that may be in the way of many trajectories. The user has to decide himself what particle trajectories are possible.

As the field free sections are computed in a single step it will not be possible to start or to stop a trajectory in such sections. A slide rule manipulation can fill this gap.

2. THE METHOD.

All magnet sectors being curved, the equation of motion in polar coordinates is used :

$$r'' = 1/r \cdot \left[r^2 + 2\, r'^2 - c.B\, (r^2 + r'^2)^{3/2} \right]$$

where B is the flux density and c the reciprocal of the magnetic rigidity of the particle. It is integrated by the Runge-Kutta routine of the mercury computer. Appropriate corrections are applied in the azimuthal leakage field region, where the reference system is rectangular, but the programme continues in polar coordinates (cp. 3.1 f and App., sub. c; fig. 1).

The magnetic field data given in PS/Int. MM 59-5 which are the most complete data at present available, have been used. The results of measurements on the two types of magnet sectors (cp. App., sub. h) have been combined to form a unique table for half magnet sectors with a step width of 0.02 m in radial direction and an azimuthal step width of 0.04 m in the region of azimuthal inhomogeneity. This table is read backward or forward depending on the type of magnet sector being computed. The flux density in any point is found by a Bessel interpolation of third order in radial direction and by a linear interpolation, in azimuthal direction.

The magnetic field data extend in no direction to B = 0. They have been included up to such positions that the negative tail is to some extent compensated by the omitted small fraction of the positive flux. As shown in fig. 1, the field table extends 0.32 m beyond the end face of a magnet sector and 0.54 m from the reference line in radial direction.

The programme takes the following amounts of time :

a) Reading the programme and the field table takes 2.5 minutes.

b) The average computation time for one integration step is 0.4 seconds.

3. INPUT AND OUTPUT ARRANGEMENTS.

3.1. Input data.

The input of the field data and of the values characteristic to a certain trajectory have been separated. A first data tape contains the field table. A second one must contain, amongst other instructions, charge, momentum, and the initial conditions. On the latter tape, the following set of input data must be given in the indicated order for each trajectory:

a) A label of the trajectory. It is arranged that this label may consist of a number with 4 digits before and 3 digits behind the decimal point.

b) The maximum number of field free sections which may be passed by the particle. This number must be ≥ 1.

c) The sequence of long and short field free sections up to the above maximum number. A short section must be represented by a 0, a long one by a 1.

d) The reciprocal of the magnetic rigidity of the particle considered in units of $Wb^{-1}m$, with negative sign for negative particles.

e) The type of half magnet sector in which the trajectory shall start. For this purpose each of the four types of half magnet sectors has been labeled by a variable A as indicated in fig. 1.

f) The initial azimuth φ_o in radian, measured from the beginning of the half sector in which the programme starts (fig. 2). As indicated in fig. 1, the azimuthal leakage field is included in the magnet sector. In the case of sectors with $A < 0$ the initial azimuth must therefore be measured from a point 0.32 m from the end face of the magnet.

g) The initial distance y_o in metres from the reference line defined in the Appendix, sub. d (fig. 2).

h) The initial sloped y'_o in radian with respect to the reference line (fig. 2).

i) The total number of half magnet sectors to be traced.

k) The integration step width in rad.

l) The print-out step width in units of the integration step width. A number k results in printing of each $(k + 1)^{th}$ integration step.

m) The ratio Bex/Bm where Bex is the average field level at extraction time and Bm the average field level which corresponds to the used field table.

Several of such sets of values may be present on this data tape (the last being followed by an ⟶). The programme is designed so that having read and processed one set it will immediately proceed to the next and so on until halted by the ⟶ .

3.2. Data output.

The values of φ, y, y' are printed out at the prescribed intervals. In addition the initial values are printed as well as the values at the end of each half magnet sector and the values at the end of the computation. Like for the input data φ is given in radians, y is the distance in metres from the reference centre line, and y' the slope with respect to the reference line in radians. The azimuth φ is always measured from the beginning of the respective half magnet sector, it is set back to zero each time a new half magnet sector is entered.

Above the results of each trajectory, its label (cp 3.1 a) is printed.

4. FURTHER DEVELOPMENT.

An extension of the programme, such that also the vertical movement of the particles can be treated is now being worked on.

A programme of magnetic measurements is in progress to complete the magnetic field measurements in the median plane near the junction of half sectors.

The field is at present taken as changing there discontinuously from the pattern of a focusing to that of a defocusing sector.

Magnetic measurements until now have been made without poleface windings. For beam extraction at energies above 25 GeV field measurements on a magnet unit with powered poleface windings will be necessary.

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/fv

Distribution : (open)
Magnet Group
Parameter Committee
Executive Committee

APPENDIX. RELEVANT CHARACTERISTICS OF THE C.P.S. MAGNET.

The following properties of the C.P.S. magnet geometry (fig. 1) have been considered in setting up the programme :

a) The magnet has a periodic structure, the period being FOFDOD where F and D stand for half radially focusing respectively defocusing magnet sectors and O for field free sections. One F sector and one D sector are joined to form one "magnet unit".

b) All magnet sectors are curved, the radius of curvature being R = 70.079 m.

c) The effective bending length of a magnet unit is defined to be $2\pi R/100$ = 4.4032 m at the transition energy of the C.P.S., viz, at B = 0.284 Wb/m^2. The magnetic flux at the ends of course, decreases gradually and extends beyond this length. ("Azimuthal leakage field").

d) The reference center line of the C.P.S. aperture (equilibrium orbit) is composed of curved sections of 4.4032 m in length with a radius of curvature of 70.079 m. These are joined up by straight sections.

e) There exist two different straight sections. The "long" straight sections of 3.000 m length and "short" straight sections of 1.600 m length.

f) F - sectors are 0.008 m shorter than D - sectors.

g) F and D sectors have identical profiles. Small differences arising from the yoke reversal can be neglected for single traversal of particles. It is therefore possible to use a unique field table for extraction computations.

h) The magnet elements are separated by small air gaps. The corresponding small field dips have been neglected. The field is assumed to be constant over an azimuthal length of about 1.5 m inside a half magnet sector.

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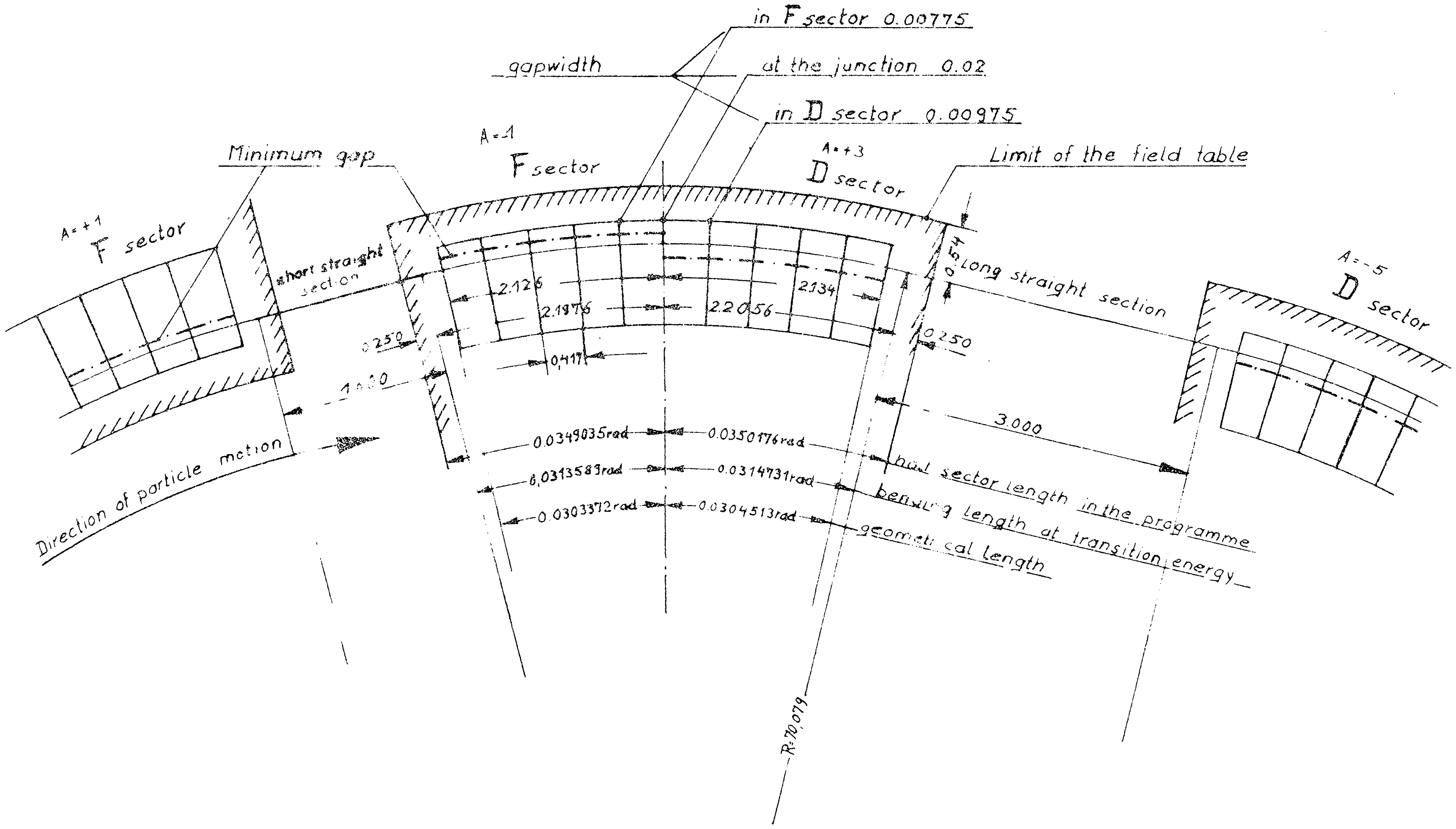
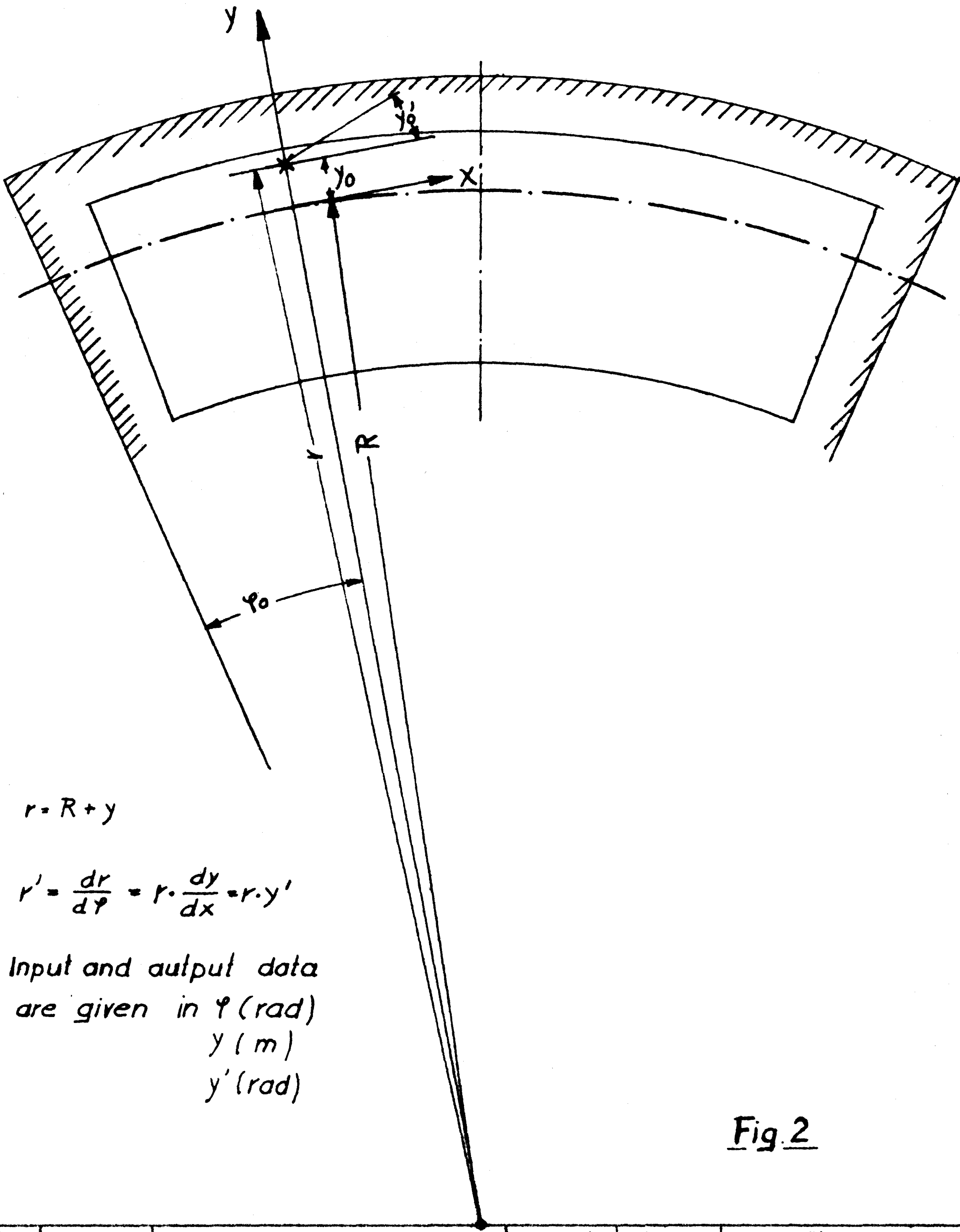


Fig. 1

Item Pos.	No. req. Nb. de p.	Descriptions	Material Matière	Pattern Modèle	Observations
1 11					
2 12					
3 13		Geometry of the C.P.S. magnet		Scale Echelle	CERN-PS GENÈVE
4 14		all dimensions in meter or radian			
5 15					
6 16		Modifications	Signatures		
7 17					
8 18					
9 19					
10 20					PS/Int FA 59-14



$$r = R + y$$

$$r' = \frac{dr}{d\varphi} = r \cdot \frac{dy}{dx} = r \cdot y'$$

Input and output data are given in φ (rad)

y (m)

y' (rad)

Fig. 2

Pos. Item	No. req. Nb. de p.	Descriptions	Material Matière	Pattern Modèle	Observations
1 11					
2 12					
3 13	III II I				
4 14					
5 15					
6 16					
7 17					
8 18					
9 19					
10 20					

The coordinates

Scale Echelle

CERN- PS GENÈVE

Modifications

Signatures

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