Probing New Gauge Forces with a High-Energy Muon Beam Dump

Cari Cesarotti,* Samuel Homiller,† Rashmish K. Mishra,‡ and Matthew Reece§
Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

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We propose a new beam-dump experiment at a future TeV-scale muon collider. A beam dump would be an economical and effective way to increase the discovery potential of the collider complex in a complementary regime. In this Letter, we consider vector models such as the dark photon and L_μ-L_τ gauge boson as new physics candidates and explore which novel regions of parameter space can be probed with a muon beam dump. We find that for the dark photon model, we gain sensitivity in the moderate mass (MeV–GeV) range at both higher and lower couplings compared to existing and proposed experiments, and gain access to previously untouched areas of parameter space of the L_μ-L_τ model.

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Introduction.—The standard model (SM) of particle physics has proven to be remarkably successful. However, there is an abundance of empirical evidence that it is incomplete. While there are a variety of channels we can explore to discover new physics, colliders provide a clean and controlled experimental environment to identify the particle content of beyond the SM phenomena.

As we look to advance the energy and intensity frontier of collider physics, a possibility with growing interest is the construction of a TeV-scale $\mu^+\mu^-$ collider [1–10]. Such a muon collider (MuC) is a particularly compelling option as it affords a complementary physics program to that of a high-energy hadron collider like the LHC. For example, with a MuC we gain access to direct couplings of both electroweak-mediated and second-generation processes [11–22]. Additionally, with increased available center-of-mass energy, we can expand our discovery prospects for massive new physics.

Since the cost of a MuC—or any future high-energy collider—is substantial, it is prudent to consider possible auxiliary experiments that extend the physics program of the collider facility. An economical extension with remarkable and complementary discovery potential is a beam dump. In a beam-dump experiment, the high energy muon beam is "dumped" into a dense material to greatly increase the total rate of interaction at the price of center-of-mass energy. This experimental setup can therefore test couplings too small to be probed at the main collider by several orders of magnitude in a slightly lower mass range [23–30].

In this Letter we propose the construction of a beam-dump experiment to be included in the design of a future MuC. We consider benchmark models of moderate-mass, weakly coupled new vector particles that are inaccessible at any other terrestrial experiment. First, we consider the dark photon scenario [31–34], for which similar past proposals have focused on electron beams; see, e.g., [23,35,36]. Here, the main novelty of a proposed MuC is high energy, which can provide access to a different range of masses and (due to the large boost) lifetimes than lower-energy electron beam dumps. We also consider a model for which a muon beam is uniquely well suited, namely, the gauged flavor symmetry L_μ-L_τ [37–41]. In this case, the gauge boson is produced much more copiously from a muon beam than an electron or proton beam. We present the projected reach of such an experiment for several generic experimental configurations.

Production from a high energy muon beam.—Preliminaries: In what follows we restrict our attention to the new physics scenario of a new vector particle, which we generically call Z', coupling to a current including muons. The effective Lagrangian of interest for a new $U(1)'$ gauge boson is

$$\mathcal{L} \supset \mathcal{L}_{\rm SM} - \frac{1}{4} Z'_{\mu\nu} Z'^{\mu\nu} + \frac{1}{2} m_{Z'}^2 Z'^{\mu} Z'_{\mu} - \sum_{l\in e,\mu,\tau} (igQ_l \bar{l}\gamma^\mu Z'_\mu l + igQ'_l \nu_l^\dagger \bar{\sigma}^\mu Z'_\mu \nu_l). \quad (1)$$

We consider two $U(1)$ models: dark photons $(Q_l = 1, Q'_l = 0, g = \epsilon e)$ and the gauged flavor symmetry L_μ-L_τ $(Q_{\mu,\tau} = Q_{\mu,\tau}{}' = \pm 1)$. In the latter case, kinetic mixing will also be present (at least through loops of muons and taus; see, e.g., [42]), but the effect is both small and dependent on details of the UV completion, so we will neglect it in our discussion.

The photon–dark photon interaction is often defined via kinetic mixing in a basis of quantized charges, but the above formulation can be derived through the field redefinition $A_\mu \to A_\mu + \epsilon Z'_\mu$ to generate a coupling to the electromagnetic current. Both the dark photon and gauged flavor symmetry models are of interest at a high-energy muon collider, although they are differently motivated. Dark photons can be produced directly at any charged particle collider, but the extended reach of our proposed experiment comes from the increased center-of-mass energy. The L_μ-L_τ model, on the other hand, would benefit uniquely from a muon collider, as this would be the first experiment capable of direct production at high energies.

Cross section:—The dominant production mechanism is the $2 \to 3$ bremsstrahlung process shown in the top of Fig. 1, where the incoming high-energy muon exchanges a virtual photon with a nucleon in the target and radiates a Z' [23,43–45]. To compute this cross section, we use the Weizsäcker-Williams approximation [46,47]. For relativistic incoming muons, the exchanged photon is nearly on shell. We can therefore approximate the full scattering process $[\mu(p) + N(P_i) \to \mu(p') + N'(P_f) + Z'(k)]$ with the $2 \to 2$ process $[\mu(p) + \gamma(q) \to \mu(p') + Z'(k)]$ shown in the bottom of Fig. 1, evaluated at minimum virtuality $t_{\min} \equiv -q^2_{\min}$, weighted by the effective photon flux. The cross section in the lab frame is

$$\frac{d\sigma(p + P_i \to p' + k + P_f)}{dE_{Z'}\, d\cos\theta_{Z'}} = \left(\frac{\alpha_{\rm EM}\chi}{\pi}\right)\left(\frac{xE_0\beta_{Z'}}{(1-x)}\right) \times \frac{d\sigma(p + q \to p' + k)}{d(p\cdot k)}\bigg|_{t=t_{\min}}, \qquad (2)$$

where E_0 is the energy of the incoming muon beam, $x \equiv E_{Z'}/E_0$ is the fraction of energy of the Z', $\theta_{Z'}$ is the angle of emission, and $\beta_{Z'} \equiv \sqrt{1 - m^2_{Z'}/E^2_{Z'}}$. The effective photon flux is parameterized by χ, defined as

$$\chi \equiv \int_{t_{\min}}^{t_{\max}} dt \frac{t - t_{\min}}{t} G_2(t), \qquad (3)$$

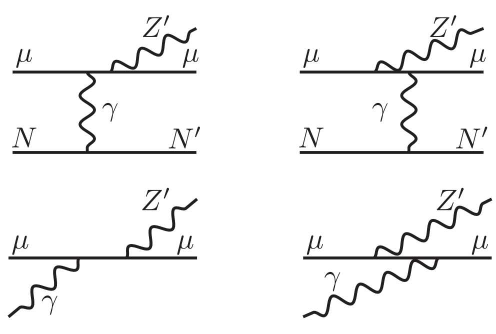

FIG. 1. Top: the dominant bremsstrahlung production for a vector particle Z' at a muon beam dump. Bottom: the same production process in the Weizsäcker-Williams approximation.

where $G_2(t)$ is the electric form factor of the target atom, including both atomic and nuclear, as well as elastic and inelastic effects, following the approximation in [23].

This approximation scheme is valid in the regime of highly relativistic beam particles and emitted vector particles:

$$\frac{m_\mu}{E_0}, \qquad \frac{m_{Z'}}{xE_0}, \qquad \theta_{Z'} \ll 1. \qquad (4)$$

After integrating out the angular dependence, the differential cross section in x is

$$\frac{d\sigma(2\to 3)}{dx} = \frac{8\alpha^2_{\rm EM}\alpha_g Q^2_\mu \chi \beta_{Z'}}{m^2_{Z'}\frac{1-x}{x} + m^2_\mu x}\left(1 - x + \frac{x^2}{3}\right), \qquad (5)$$

where $\alpha_g \equiv (g^2/4\pi)$. For all values of mass, we find that the probability of emission has support primarily in the highly relativistic regime. Note that in the context of an electron beam-dump experiment, the term proportional to the beam particle mass in the denominator of Eq. (5) can be neglected. Here, on the other hand, we are interested in $m_{Z'}$ near the muon mass, so we cannot drop this term.

Signature:—The signal of interest for this experimental setup is a dilepton final state (e^+e^- or $\mu^+\mu^-$). From the Lagrangian defined in Eq. (1), the decay rate to massive leptons is

$$\Gamma(Z' \to l^+ l^-) = \frac{g^2 Q^2_l}{12\pi} m_{Z'}\left(1 + \frac{2m^2_l}{m^2_{Z'}}\right)\sqrt{1 - \frac{4m^2_l}{m^2_{Z'}}}. \qquad (6)$$

We restrict our attention to this signature for ease of detection prospects.

Before we proceed with the details, we can check that a multi-TeV muon collider provides the right environment to extend the search boundaries of these models. Taking the dark photon as a benchmark, the approximate decay length [estimated from Eq. (6)]

$$l_{Z'} \equiv c\gamma\tau_{Z'} \approx x\left(\frac{E_0}{\rm TeV}\right)\left(\frac{\rm GeV}{m_{Z'}}\right)^2\left(\frac{10^{-7}}{\epsilon}\right)^2 \times 10\ {\rm m}, \qquad (7)$$

where $x \sim 1$. This suggests that with a modest-size experiment, a TeV beam dump can dramatically expand the reach of these vector models.

Number of signal events:—The differential number of signal Z' events per energy fraction x and position z along the beamline is given by the equation

$$\frac{dN}{dxdz} = N_\mu \frac{X_0}{m_T}\int_{E_{Z'}}^{E_0} dE_1 \int_0^T dt\, I(E_1; E_0, t) \times \left(\frac{E_0}{E_1}\frac{d\sigma}{dx'}\right)_{x' = \frac{E_{Z'}}{E_1}} \frac{dP(z - \frac{X_0}{\rho}t)}{dz}, \qquad (8)$$

where N_μ is the number of incident muons on target, m_T is the mass of a target atom, and X_0 and ρ are the unit radiation length and density of the target, respectively [48]. We parameterize the position along the length of the target in terms of the dimensionless parameter t from 0 to T, such that the full length of the target is $L_{\rm tar} = (X_0/\rho)T$.

The energy of the beam particle after radiative losses in the material is modeled by the function $I(E_1; E_0, t)$. However, the effective radiation length of a muon in reasonable target materials (e.g., water or lead) is 50 m to 1 km [49]. For the proposed experimental setup, we can safely assume zero radiative losses and replace

$$I(E_1; E_0, t) = \delta(E_1 - E_0). \tag{9}$$

The differential probability of Z' decay is given by

$$\frac{dP(z)}{dz} = \frac{1}{l_{Z'}} e^{-z/l_{Z'}}. \tag{10}$$

As indicated in Eq. (7), the decay length $l_{Z'}$ is a function of the Z' mass $m_{Z'}$ and energy xE_0. After integrating over the muon beam energy and target thickness, Eq. (8) simplifies to

$$\frac{dN}{dx} = N_\mu \frac{\rho l_{Z'}}{m_T} \frac{d\sigma}{dx} \times (e^{L_{\rm tar}/l_{Z'}} - 1) e^{-(L_{\rm tar}+L_{\rm sh})/l_{Z'}} (1 - e^{-L_{\rm dec}/l_{Z'}}), \tag{11}$$

where the various length scales are illustrated in Fig. 2.

Experimental setup.—The experimental setup of a beam-dump experiment is shown in Fig. 2. The high energy muon beam is dumped into a solid material target of length $L_{\rm tar}$. Immediately after the target is a shield of length $L_{\rm sh}$ dedicated to removing the residual beam and any background. This includes both a region with a strong magnetic field of length l_m and a shielding mechanism to remove any remaining particles. Beyond the shielding is the fiducial decay region of length $L_{\rm dec}$ before the detector. To be detected, signal events must be produced in the target and decay in the fiducial region.

From Eq. (11) it is clear that the length scales of the various components of the experiment have dramatic impact on the number of signal events observed. New particles that are produced too early get shielded and are missed, but particles that live too long escape the experimental hall altogether.

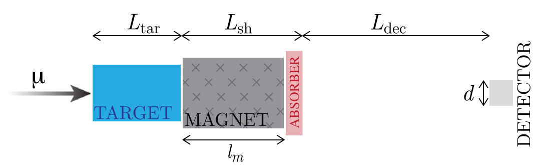

FIG. 2. Schematic of a muon beam-dump experiment. This diagram is not drawn to scale, but the lengths will be on the order of $L_{\rm tar}, L_{\rm sh}, l_m \sim 10$ m, $L_{\rm dec} \sim 100$ m, and $d \sim 10$ cm.

For concreteness, we choose experimental parameters such as length scales at reasonable orders of magnitude. The purpose of this Letter is to demonstrate the feasibility of beam-dump experiments at future muon colliders. Since we cannot know the available technology or experimental design constraints at this point, we do not attempt to optimize the experimental design.

Target: In this Letter, we consider a target made of water at standard temperature and pressure, with a length of $L_{\rm tar} = 10$ m. To avoid overheating and evaporating the target, the geometric cross section of the tank and therefore volume of water should be adjusted. For the number of muons on target considered here, a cross section of 10^{-2} to 1 $\rm m^2$ target would be sufficient. Other economical and common choices for a beam dump target include pressurized water, lead, or tungsten. Higher density materials will increase the cross section as the effective photon flux increases, but may be too expensive or infeasible to install at the beam dump site.

Shielding: In order to realize a clean dilepton signal, various sources of background particles must be removed. We propose a two-stage shielding region that prevents both the high-energy beam muons and any secondary particles from reaching the detector: a magnet to deflect the beam followed by a block of dense material to absorb any other emitted particles.

Since the Z' and its subsequent decay products are highly boosted, it is imperative to remove the central muon beam (and electrons it decays to) from the geometric acceptance of the detector. Since high-energy muons are extremely difficult to absorb, we instead propose to deflect the beam using a strong magnetic field. The size of the detector is set by the opening angle of the dilepton decay products of the boosted Z'. The decay products are produced extremely forward with an emission angle in the lab frame of $\theta_{\rm max} \lesssim m_{Z'}/xE_0$. Note that, in the parameter space of interest where $m_{Z'} > m_\mu$, the maximum angle of Z' emission $\theta_{\rm max}^{Z'} \sim (\sqrt{m_{Z'} m_\mu}/E_0)$ is parametrically smaller and therefore negligible.

The shielding magnet must be sufficiently strong to divert a roughly TeV-scale muon beam at a greater angle $\theta_{\rm mag} > \theta_{\rm max}$. If we assume the magnetic field is constant over a length l_m just after the target, the field strength B must be approximately

$$\frac{B}{\rm Tesla} \sim \theta_{\rm max} \times \frac{E_0}{\rm GeV} \times \frac{\rm meter}{l_m}. \tag{12}$$

For the parameter space of interest, this corresponds to sub-Tesla magnetic fields over a few meters, which is comparable to or smaller than similar shielding mechanisms proposed in other future beam-on-target experiments [50].

Since bending a high-energy beam of charged particles will emit some radiation, there must also be a shield beyond the magnetic field. This shield will stop lower-energy electrons and photons, and can therefore be made of a dense material to absorb these particles, such as iron or lead. It may also be necessary to include a charged particle tracker region either immediately after the shielding or before the detector in order to reject any residual high-energy leptons with production vertices outside of the target. We assume that the total length of this shielding region will be contained in $L_{\rm sh} = 10$ m, but we discuss the consequences of a larger $L_{\rm sh}$ later with the results.

Detector: Once a Z' has been produced in the target and propagates beyond the shielding region, it must decay in the fiducial region before the detector in order to be observed. As previously mentioned, our signal of interest is a dilepton resonance (e^+e^- or $\mu^+\mu^-$). This signal can be observed with a relatively simple detector setup: a charged particle and muon tracker. Again, as the state-of-the-art detection technology is not yet known, we do not provide a detailed description of the system. The size of the detector d must be roughly

$$d \sim \theta_{\rm max} L_{\rm dec}. \quad (13)$$

From Eq. (11), it is clear that a longer experimental hall increases the number of Z' particles that decay in acceptance. As a reasonable benchmark we set $L_{\rm dec} = 100$ m, which corresponds to a detector size of $d \lesssim 10$ cm for the relevant range of masses. Note that for this exploratory study, we consider a minimally instrumented scenario. However, one could envision more sophisticated detection scenarios that involve instrumenting along the fiducial region and accounting for missing energy signals as well.

Reach for new gauge forces.—We present the reach of both a dark photon and L_μ-L_τ Z' model. For concreteness we consider the reach with a 1.5 TeV beam (corresponding to a 3 TeV collider), a standard benchmark in MuC literature [4].

For the dark photon scenario, we show the existing constraints from e^+e^- or $\mu^+\mu^-$ resonance searches at *BABAR* [51], NA48 [52], the A1 Experiment at the Mainz Microtron [53], KLOE [54–57], and LHCb [58]; previous beam-dump experiments, such as E141 [59] and E137 [60–62] at SLAC, E774 at Fermilab [63], CHARM [64,65], and NuCal [66–68]; as well as constraints from Supernova 1987A [69] in gray. Additionally we plot the projected reach from other future experiments including Belle-II [70], LHCb [71,72], SHiP [73], and AWAKE [74].

The projected sensitivity of the MuC beam dump to a dark photon Z' is shown in Fig. 3. Since the number of muons delivered to target cannot be known at this stage, we provide three reach curves reflecting conservative to optimistic projections of N_μ. A discussion of these choices can be found in the Supplemental Material [6,75,76]. The experiment would expand the reach in parameter space not only beyond existing constraints, but also in complementary

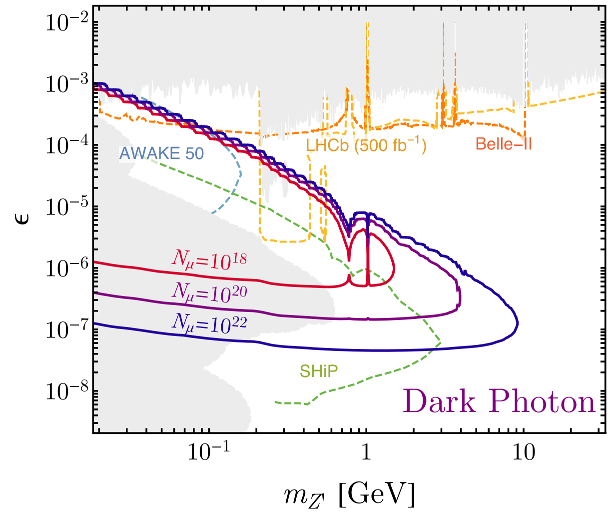

FIG. 3. Contour plots indicating five signal events detected with $N_\mu = 10^{18}, 10^{20}, 10^{22}$ with a beam energy $E_0 = 1.5$ TeV. The dips in the contours near $m_{Z'} = 1$ GeV occur when there is resonant production in the $Z' \to$ hadron decay channel, thus reducing the dilepton branching ratio.

regions to other future experiments. The gain in coverage occurs mainly in the directions of larger coupling and mass. This is due to the high energy of the beam, and therefore production of highly boosted Z' particles.

The boundary of the discovery region at large couplings occurs when the Z' decays too early and the decay products are caught by the shielding. However, a relativistic Z' will live longer in the lab frame and therefore can decay in the fiducial region. At this unprecedented beam energy, Z's at higher masses than before are sufficiently boosted to survive past the shield. If the shielding length $L_{\rm sh}$ is extended, then sensitivity degrades in the large coupling regime while leaving the bottom edge unmoved. However, if the beam energy E_0 is increased, both the upper and lower boundaries are shifted upwards to higher couplings.

The parameter space of the L_μ-L_τ model is notably unconstrained in the region $g \lesssim 10^{-3}$ and $m_{Z'} \gtrsim 10$ MeV. Existing constraints come from measurements of the primordial abundances of light nuclei [77], from observations of SN1987A [78], from measurements of the anomalous magnetic moment of the muon [79], limits from neutrino trident production [82,83], and searches for $e^+e^- \to \mu^+\mu^- Z'(\mu^+\mu^-)$ at *BABAR* [84]. The current bounds are shown in gray in Fig. 4. We also show the projected limits from other muon beam experiments, M^3 [85] and NA64μ [29]. Note that other proposed experiments such as Ref. [86] might have comparable reach to NA64μ and M^3. The reach plot, drawn with the same values of N_μ and beam energy E_0, is shown in Fig. 4. The coverage is completely separated from other constraints on this model due to the novelty of both the beam of muons and the energy of the beam.

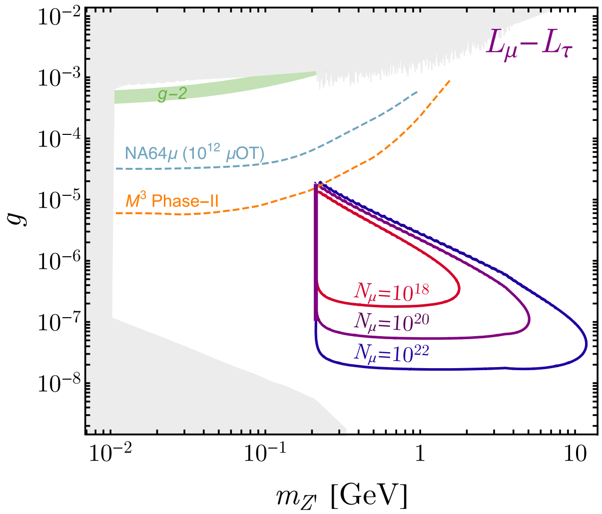

FIG. 4. Same as Fig. 3 but for the L_μ-L_τ model. The sensitivity is bounded at lower $m_{Z'}$ by the dimuon production threshold, since the electron channel is not open in this model.

Conclusions and future work.—A future multi-TeV muon beam-dump experiment would provide a window into previously unexplored parameter space for a variety of motivated new physics models. The sensitivity improvements beyond other similar proposed experiments, such as Refs. [23–29], stem from two features of such an experimental setup: the increased beam energy and direct coupling to muons.

As discussed, the unprecedented beam energy translates to boosted new particles with extended lifetimes. Therefore couplings that would otherwise be too large to be detected at previous beam-dump experiments would be accessible. Additionally, a muon collider is uniquely well suited to study models with couplings to muons. In this Letter we computed the reach of the gauged L_μ-L_τ symmetry as a motivated example, but this broadly applies to more general dark sectors with nonuniversal fermion interactions.

In the proposed detection strategy, we have taken a minimalist approach to instrumentation. However, in the event of an observed resonance, additional detectors to identify the rate into taus could be used to determine the underlying theory. If this experiment were to confirm the existence of a gauged flavor symmetry, this would be significant for several areas of particle physics. A gauged L_μ-L_τ symmetry could explain the near-maximal mixing between muon neutrinos and tau neutrinos [39]. It has recently been observed that an $SU(3)$ extension of this group can give rise to a complete model of lepton masses [87]. Finally, we cannot resist noting that every measured gauge coupling to date is an $O(1)$ number, while Fig. 4 shows that a discovery of L_μ-L_τ at the beam dump would necessarily imply a tiny gauge coupling $\lesssim 10^{-5}$, which would become a powerful constraint on UV physics [88].

While we have focused on models with new vectors, the improved reach would be similarly impressive for other dark sectors with nonvector mediators. Additional searches should include models with new muonphilic scalars or pseudoscalars such as axions. These particles could be produced either directly from the muon or from photon fusion by effective interactions. A detailed study of the reach for these models will be presented in a future publication.

A beam-dump experiment is particularly compelling for a future muon collider due to the short lifetime of the beam particles. As muons decay and produce a shower of high-energy electrons and neutrinos throughout the accelerator ring, mitigation strategies must be introduced to protect the instrumentation and ensure a high-quality beam at the collision. The beam is estimated to lose approximately a millionth of its energy per meter. Unlike at hadron or electron colliders, it therefore must be dumped quickly after some fraction of particles decay. While the exact threshold beyond which a beam must be dumped requires further studies, it seems likely that $\mathcal{O}(10\%)$ of the beam is dumped [89]. This estimate is consistent with 10^{20} muons delivered to the beam dump facility per year.

Since a muon collider is far from guaranteed, we emphasize that a complete muon collider is not required to improve our sensitivity. In the necessary research and development phase of a potential future muon collider, there will be test beams on the energy scale of $\mathcal{O}(100)$ GeV. Even though this is far from the multi-TeV energy design of the main collider, a dedicated muon beam at this lower energy could still improve our reach especially for the nonuniversal fermion models. Thus, the success of a muon beam dump program is not strictly tied to the completion of a muon collider.

In the current era of particle physics, no stone can be left unturned when searching for new phenomena. A future muon collider and a corresponding beam-dump experiment would greatly enhance our sensitivity in novel and complementary regimes to our current and past experimental program.

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[*]ccesar@mit.edu
[†]shomiller@g.harvard.edu
[‡]rashmishmishra@fas.harvard.edu
[§]mreece@g.harvard.edu

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