Universal Pattern in Quantum Gravity at Infinite Distance

Alberto Castellano* and Ignacio Ruiz†
Instituto de Física Teórica UAM-CSIC and Departamento de Física Teórica, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain

Irene Valenzuela‡
Instituto de Física Teórica UAM-CSIC and Departamento de Física Teórica, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain and CERN, Theoretical Physics Department, 1211 Meyrin, Switzerland

(Received 21 December 2023; revised 8 March 2024; accepted 11 April 2024; published 2 May 2024)

Quantum gravitational effects become significant at a cutoff that can be much lower than the Planck scale whenever there is a large number of light fields. This is expected to occur at any perturbative limit of an effective field theory coupled to gravity, or equivalently, at infinite distance in the field space of the UV completion. In this note, we present a universal pattern that links the asymptotic variation in field space of the quantum gravity cutoff $\Lambda_{\rm sp}$ and the characteristic mass of the lightest tower of states m: $(\vec{\nabla}m/m)\cdot(\vec{\nabla}\Lambda_{\rm sp}/\Lambda_{\rm sp}) = [1/(d-2)]$, with d the spacetime dimension. This restriction can be used to make more precise several Swampland criteria that constrain any effective field theory which can be consistently coupled to quantum gravity.

DOI: 10.1103/PhysRevLett.132.181601

Introduction.—Effective field theories (EFTs) are very useful in high energy physics to describe the physical phenomena of our world. However, they are characterized by having a finite regime of validity, meaning that there is some cutoff energy scale at which the EFT breaks down and must be modified to incorporate, e.g., new physical degrees of freedom. In this Letter, we are interested in the quantum gravity (QG) cutoff at which an EFT weakly coupled to semiclassical gravity breaks down. In other words, what is the scale at which quantum gravitational effects become significant?

The first naive guess is to set this scale to be the Planck mass (around 10^{19} GeV), since this determines the strength of gravitational interactions. However, in certain cases, QG effects can become significant at a cutoff scale much lower than the Planck scale. This occurs, for instance, when we have many light fields weakly coupled to gravity (termed *species*), which renormalize the graviton propagator and lower the QG cutoff to $\Lambda_{\rm sp} = M_{\rm Pl}/\sqrt{N}$, N being the number of species. Above this energy scale (known as the species scale), quantum gravitational effects kick in and it is no longer possible to have a local EFT description weakly coupled to gravity. This scale is further motivated by black hole physics [1,2], unitarity of scattering amplitudes [3–8], and string theory [9–12].

Clearly, the species scale can be made arbitrarily small $\Lambda_{\rm sp} \ll M_{\rm Pl}$ whenever we get a parametrically large number of species. This is known to happen whenever we get an infinite tower of states becoming light (either because they open up some extra dimension, or they correspond to oscillator modes of a weakly coupled higher dimensional object like a string). From a string theory perspective, the presence of a light tower of states is a universal feature that occurs whenever we try to engineer an exact global symmetry, since this pushes us to the boundaries of the field space. However, the existence of these towers acting as a censorship mechanism to restore global symmetries is expected to be a general feature of QG (even beyond string theory) and plays a central role in the Swampland program [13–25]. The absence of exact global symmetries in QG has been shown using AdS/CFT [26–28] and black hole physics [29–31], in addition to the stringy evidence. Understanding in a quantitative way the behavior of the tower of states would allow us to quantify how approximate a global symmetry can be and put sharp bounds on the value of gauge couplings and axionic decay constants (since the limit of a vanishing gauge coupling is equivalent to restoring a global symmetry). This can have important phenomenological implications for beyond standard model particle physics and cosmology.

In this note we present a precise constraint relating the asymptotic variation rates of the characteristic mass of the

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

leading (i.e., lightest) tower of states, $m_{\rm t}$, and the species scale $\Lambda_{\rm sp}$, as follows:

$$\frac{\vec{\nabla} m_{\rm t}}{m_{\rm t}} \cdot \frac{\vec{\nabla}\Lambda_{\rm sp}}{\Lambda_{\rm sp}} = \frac{1}{d-2}, \qquad (1)$$

where d is the number of macroscopic dimensions in our theory and every quantity is measured in Planck units. In the context of string theory, all coupling constants in the EFT are controlled dynamically by the vacuum expectation values (vevs) of scalar fields, so the derivatives are taken with respect to those, as we explain below. Since by definition, $m_{\rm t} \leq \Lambda_{\rm sp}$, we will see that this pattern leads to sharp bounds on how fast the tower can become light and the value of the species scale at which QG effects become significant. Therefore, the pattern (1) can also constrain the scalar field variation that can be accommodated by an EFT weakly coupled to gravity, yielding bounds of phenomenological interest for cosmological models of inflation or quintessence [32], as well as other dynamical proposals to explain the electro-weak hierarchy problem like cosmological relaxation [33].

In a companion paper [34] we present strong evidence on its favor by analyzing a plethora of string theory constructions in different dimensions and with all allowed levels of supersymmetry. This pattern makes manifest an underlying structure behind all different string theory examples that highly constrains the structure of the possible towers of states and helps make more precise the distance conjecture [35,36] in the Swampland program. In this Letter, we introduce the pattern as well as some of its most immediate consequences, highlighting the key ingredients that make it work and providing the first steps towards a bottom-up explanation for the constraint.

Systematics of the pattern.—We consider in what follows a d-dimensional EFT containing some light scalars (known as moduli if exactly massless), weakly coupled to gravity as follows:

$$\mathcal{L}_{\rm EFT} \supset \frac{1}{2\kappa_d^2}\left(\mathcal{R} + \mathsf{G}_{ij}(\phi)\partial\phi^i \cdot \partial\phi^j\right), \qquad (2)$$

where $\mathcal{M}_\phi$ is the moduli space of the theory, namely, the space of physically distinct vacua parametrized by the scalar field vevs $\langle\phi^i\rangle$. When probing any infinite distance boundary of $\mathcal{M}_\phi$, the distance conjecture [35] requires from the appearance of at least one infinite tower of states becoming exponentially light. Therefore, in terms of the traversed *geodesic* distance, which is defined by

$$\Delta_\phi = \int_\gamma {\rm d}\sigma \sqrt{\mathsf{G}_{ij}(\phi)\frac{{\rm d}\phi^i}{{\rm d}\sigma}\frac{{\rm d}\phi^j}{{\rm d}\sigma}}, \qquad (3)$$

with γ denoting some geodesic path and σ an affine parameter, there should exist a tower whose mass scale decreases as $m \sim e^{-\lambda\Delta_\phi}$ for $\Delta_\phi \gg 1$ (in Planck units), with λ some $\mathcal{O}(1)$ factor.

Note that (2) only displays the relevant piece of the action needed to check the condition (1) in a given EFT. This does not mean, however, that our analysis here is restricted to flat spacetime and exactly massless scalars, since the existence of the light tower also extends to more general setups, such as 4d $\mathcal{N}=1$ theories with, e.g., runaway potentials [34].

In the presence of several moduli, it is useful to define a ζ vector for each tower becoming light, whose components read

$$\zeta^i \coloneqq -\mathsf{G}^{ij}\frac{\partial}{\partial\phi^j}\log m = -\partial^i \log m. \qquad (4)$$

These are denoted *scalar charge-to-mass vectors* [36–38], and they encode information about how fast the associated tower becomes light. In particular, for any given asymptotically geodesic direction in moduli space characterized by some normalized tangent vector $\hat{T}$, the decay rate of the tower can be determined as the projection $\lambda = \vec{\zeta}\cdot\hat{T}$. Moreover, for any such limit, we denote by $\vec{\zeta}_{\rm t}$ the scalar charge-to-mass vector of the leading one.

Relatedly, the presence of an increasing number of light particle species in the theory implies a drastic breakdown of the original EFT. The maximum energy scale at which such description holds is known as the species scale $\Lambda_{\rm sp}$ (see, e.g., [39] and references therein), which in general is given by

$$\Lambda_{\rm sp} \simeq \frac{M_{{\rm Pl};d}}{N^{\frac{1}{d-2}}}, \qquad (5)$$

where N is roughly the number of species at or below the species scale itself. Notice that whenever N grows large at infinite distance, the species scale goes to zero (exponentially) in Planck units. To characterize how this occurs, one analogously introduces a $\mathcal{Z}$ vector as follows [40]

$$\mathcal{Z}^i_{\rm sp} \coloneqq -\partial^i \log\Lambda_{\rm sp}, \qquad (6)$$

thus providing the asymptotic decay rate of the species scale.

In principle, there is some degree of independence between the vectors $\vec{\zeta}_{\rm t}$ and $\vec{\mathcal{Z}}_{\rm sp}$, meaning that one can get different casuistics upon exploring distinct asymptotic corners of moduli space. This is so since $\Lambda_{\rm sp}$ knows *a priori* about *all* towers of (sufficiently) light states, such that the leading one does not always determine it alone. Nevertheless, the idea that we want to put forward in the present Letter is that there seems to be a very precise link between these two quantities via the relation

$$\vec{\zeta}_{\rm t} \cdot \vec{\mathcal{Z}}_{\rm sp} = \mathsf{G}^{ij}(\partial_i \log m_{\rm t})(\partial_j \log \Lambda_{\rm sp}) = \frac{1}{d-2}, \quad (7)$$

valid in the *strict infinite* field distance limit [41]. This pattern holds universally regardless of the nature of the infinite distance limit that is explored as well as the microscopic interpretation of the towers. Using (5), we can equivalently rewrite the above relation as follows:

$$\frac{\vec{\nabla} m_{\rm t}}{m_{\rm t}} \cdot \frac{\vec{\nabla} N}{N} = -1, \quad (8)$$

where the product is again taken with respect to the field space metric. Hence, the more fields we get, the slower they become light, in a very concrete way that is even independent of the spacetime dimension.

In the following, we will explain via some (realistic) toy model how this comes about, as well as commenting on the consequences that derive immediately from (7). In a companion paper [34], we present string theory evidence in a wide range of scenarios supporting the claim.

A simple toy model.—Let us first show how the pattern works in simple single-field examples. We consider two cases: First, when dealing with a Kaluza-Klein decompactification of n internal dimensions, we find a KK tower with characteristic mass $m_{{\rm KK},n}$ yielding infinitely many states becoming light, with a spectrum of the form $m_k = k^{1/n} m_{{\rm KK},n}$, where $k = 1, \ldots, \infty$ [42]. Its associated species scale is the higher-dimensional Planck mass, which is given by $M_{{\rm Pl};d+n} = M_{{\rm Pl};d}(m_{{\rm KK},n}/M_{{\rm Pl};d})^{[n/(d+n-2)]}$. Hence, the relevant charge-to-mass and species vectors, which can be obtained via dimensional reduction [43], read as follows [40]:

$$\zeta_{{\rm KK},n} = \sqrt{\frac{d+n-2}{n(d-2)}}, \quad \mathcal{Z}_{{\rm KK},n} = \sqrt{\frac{n}{(d+n-2)(d-2)}}. \quad (9)$$

It is easy to check that these always reproduce the pattern (7), regardless of the number of dimensions decompactifying.

Second, we can also get an infinite tower of states when having a higher dimensional object (like a weakly coupled string) becoming tensionless. In this case, the tower of string oscillator modes behaves as $m_k = \sqrt{k} m_{\rm osc}$ where $k = 1, \ldots, \infty$, with an exponential degeneracy of states per level k. This leads to the following relevant vectors [38]

$$\zeta_{\rm osc} = \frac{1}{\sqrt{d-2}} = \mathcal{Z}_{\rm osc}, \quad (10)$$

since the species scale coincides with the string scale [39]. From this, it automatically follows that $\zeta_{\rm osc} \cdot \mathcal{Z}_{\rm osc} = [1/(d-2)]$, in agreement with (7).

However, this is not yet enough to show that the pattern holds in full generality, since when dealing with theories with more than one scalar field and more than one tower, the vectors $\vec{\zeta}_{\rm t}$ and $\vec{\mathcal{Z}}_{\rm sp}$ will not be necessarily parallel to each other. Still, the structure of the towers and the angles subtended by the vectors are always such that (7) is satisfied. For example, consider the case where several KK towers (associated to different internal cycles) become light. Then, the species scale is always given by some higher dimensional Planck mass, as in the one-modulus example above. For simplicity, we focus on a two-dimensional slice spanned by two KK towers decompactifying to $d+n$ and $d+n'$ dimensions, respectively, with associated canonically normalized volume moduli $\hat{\rho}$ and $\hat{\rho}'$. The ζ vectors are given by [36]

$$\vec{\zeta}_{{\rm KK},n} = \left(0, \sqrt{\frac{d+n-2}{n(d-2)}}\right),$$
$$\vec{\zeta}_{{\rm KK},n'} = \left(\sqrt{\frac{d+n+n'-2}{n'(d+n-2)}}, \sqrt{\frac{n}{(d+n-2)(d-2)}}\right), \quad (11)$$

while the relevant $\mathcal{Z}$ vectors are [40]

$$\vec{\mathcal{Z}}_{{\rm KK},n} = \frac{n}{d+n-2}\vec{\zeta}_{{\rm KK},n},$$
$$\vec{\mathcal{Z}}_{{\rm KK},n'} = \frac{n'}{d+n'-2}\vec{\zeta}_{{\rm KK},n'},$$
$$\vec{\mathcal{Z}}_{{\rm KK},n+n'} = \frac{n\vec{\zeta}_{{\rm KK},n} + n'\vec{\zeta}_{{\rm KK},n'}}{d+n+n'-2}. \quad (12)$$

The species scale will correspond to the *lightest* Planck scale for any chosen asymptotic trajectory $\hat{T}$ (i.e., that with the largest value of the exponential rate $\lambda_{\rm sp} = \vec{\mathcal{Z}} \cdot \hat{T}$). Hence, it will always be given by the Planck scale associated to full decompactification, $\vec{\mathcal{Z}}_{{\rm KK},n+n'}$, unless we move parallel to either one of the individual species vectors. The leading tower, on the other hand, will always be one of the two individual KK towers unless we move precisely parallel to $\vec{\mathcal{Z}}_{{\rm KK},n+n'}$, where all the internal geometry decompactifies homogeneously. In any event, the pattern is always satisfied for any intermediate direction, since one can check that

$$\vec{\zeta}_{{\rm KK},n} \cdot \vec{\mathcal{Z}}_{{\rm KK},n+n'} = \vec{\zeta}_{{\rm KK},n'} \cdot \vec{\mathcal{Z}}_{{\rm KK},n+n'} = \frac{1}{d-2}. \quad (13)$$

Similarly, when exploring some perturbative string limit in higher dimensions, the species scale is given by the string scale, as in (10), but the leading tower might be a KK tower rather than that of string oscillator modes. Upon restricting again to a 2D slice parametrized by the overall volume modulus and the d-dimensional dilaton, one finds the following relevant vectors (in a flat frame) [36,40]:

$$\vec{\zeta}_{\rm osc} = \vec{\mathcal{Z}}_{\rm osc} = \left(\frac{1}{\sqrt{d+n-2}}, \sqrt{\frac{n}{(d+n-2)(d-2)}}\right), \qquad (14)$$

while $\vec{\zeta}_{{\rm KK},n}$ and $\vec{\mathcal{Z}}_{{\rm KK},n}$ are computed as in (12). Recall that the leading tower (species) becomes that with maximal projection along a given normalized tangent vector $\hat{T}$. Therefore, for intermediate directions [i.e., not aligned with any ζ-vector in (14) above], $\Lambda_{\rm sp}$ will be given by the string scale, while having the KK tower as the leading one. However, even in such case the pattern is still fulfilled, since

$$\vec{\zeta}_{{\rm KK},n} \cdot \vec{\mathcal{Z}}_{\rm osc} = \frac{1}{d-2}. \qquad (15)$$

Let us mention that all the previous considerations can be easily understood in a geometric way, upon depicting the different charge-to-mass and species vectors that enter in the game as illustrated below. Interestingly, despite the apparent simplicity of the previous "toy models" it turns out that all the different asymptotic corners of the moduli spaces arising from string theory constructions fit into one of these two sub-cases [44]. In fact, essentially the same type of pictures are always drawn, as analyzed in detail in the companion paper [34], differing only in how the diagrams are glued together in a way that respects the pattern, which puts non-trivial constraints on how different perturbative dual descriptions of the theory can fit together. This will also be further explored in [45].

Derived bounds on the exponential rates.—We would like to stress that a sharp relation like (7) becomes rather constraining. In fact, several bounds in the Swampland literature immediately follow from imposing the pattern, as we now explain. First, (7) implies a *lower* bound for the scalar charge-to-mass ratio of the leading tower of states. This is a direct consequence of the consistency condition $m_{\rm t} \leq \Lambda_{\rm sp}$—assuming an exponential behavior for both scales [35], from where one deduces that $|\vec{\zeta}_{\rm t} \cdot \vec{\mathcal{Z}}_{\rm sp}| \leq |\vec{\zeta}_{\rm t}|^2$ (by Cauchy-Schwarz) and therefore

$$|\vec{\zeta}_{\rm t}|^2 \geq \frac{1}{d-2}. \qquad (16)$$

This leads precisely to the following lower bound for the exponential rate of the lightest tower

$$\lambda_{\rm t} = |\vec{\zeta}_{\rm t}| \geq \frac{1}{\sqrt{d-2}}, \qquad (17)$$

which was recently proposed in [36]. Relatedly, the fact that any infinite tower always satisfies the pattern with its own species scale, i.e., $\vec{\zeta}_{\rm t} \cdot \vec{\mathcal{Z}}_{\rm sp} = |\vec{\zeta}_{\rm t}||\vec{\mathcal{Z}}_{\rm sp}| = [1/(d-2)]$, implies via the same argument, that there is an *upper* bound for the decay rate of $\Lambda_{\rm sp}$:

$$\lambda_{\rm sp} = |\vec{\mathcal{Z}}_{\rm sp}| \leq \frac{1}{\sqrt{d-2}}. \qquad (18)$$

This coincides with another recent proposal in [10] based both in string theory examples and consistency of the EFT description [46].

Moreover, the pattern (7) constrains the structure of the possible towers of states and how they can fit together as we move in moduli space. It is clearly related to the emergent string conjecture (ESC) [44], which proposes that any infinite distance limit is either a decompactification or a perturbative string limit, since these are the obvious cases that fulfill the pattern. However, it is important that, when having several KK towers becoming light and signaling different decompactification limits, they can all be interpreted as KK towers in the *same* dual frame. Otherwise, the pattern will not hold, as we further discuss later on.

Computing the quantum gravity cutoff.—Let us now use the pattern to compute the scaling of the QG cutoff from the behavior of the mass of the tower, but without assuming anything else about its microscopic origin nor the associated density of states. This does not fix by itself the overall magnitude of $\Lambda_{\rm sp}$, but provides enough information to identify the QG resolution of the limit taken; see Ref. [34] and references therein.

Consider first a single tower of states with a scalar charge-to-mass vector (4), denoted by $\vec{\zeta}_I$. The species scale associated to this tower has $\vec{\mathcal{Z}}_I$ parallel to $\vec{\zeta}_I$, satisfying moreover (7). This scale sets the QG cutoff if we move along an asymptotic direction parallel to $\vec{\zeta}_I$, so that the exponential rates are related as

$$\lambda_{\rm sp} = \lambda_{\rm t}^{-1}/(d-2). \qquad (19)$$

Notice that the structure and density of states of the tower determine the relation between $\Lambda_{\rm sp}$ and $m_{\rm t}$, while the pattern forces this relation to be fully fixed by the variation of the mass in the field space, so that $\Lambda_{\rm sp} \sim m_{\rm t}^{1/[(d-2)\lambda_{\rm t}^2]}$ in this particular case.

As we start moving along other asymptotic directions, there might be additional towers becoming light, thus contributing to the species scale. This is illustrated in Fig. 1, where we have another tower with vector $\vec{\zeta}_J$. Hence, the species scale along any other intermediate asymptotic direction $\hat{T}$ will not be given by $\vec{\mathcal{Z}}_I$ but rather determined by another vector $\vec{\mathcal{Z}}_{\rm sp}$ that receives contributions from both towers. Interestingly, the pattern (1) determines *uniquely* the species scale $\vec{\mathcal{Z}}_{\rm sp}$ once the mass of the leading towers is known. First, notice that satisfying the pattern for both towers, i.e., $\vec{\zeta}_I \cdot \vec{\mathcal{Z}}_{\rm sp} = \vec{\zeta}_J \cdot \vec{\mathcal{Z}}_{\rm sp} = [1/(d-2)]$, implies that

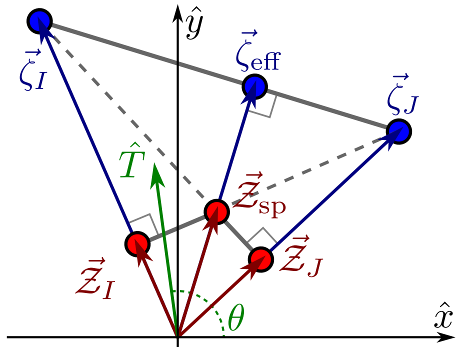

FIG. 1. Sketch on how the pattern (1) and the ζ vectors associated to the leading towers m_I and m_J, depending on some scalars $\hat{x}$ and $\hat{y}$, uniquely determine the *multiplicative* species scale $\vec{\mathcal{Z}}_{\rm eff}$.

$\vec{\mathcal{Z}}_{\rm sp}$ must be perpendicular to $\vec{\zeta}_J - \vec{\zeta}_I$. Second, the projection of $\vec{\mathcal{Z}}_{\rm sp}$ over $\vec{\zeta}_I$ must be precisely $\vec{\mathcal{Z}}_I$ since $\vec{\zeta}_I \cdot \vec{\mathcal{Z}}_I = \vec{\zeta}_I \cdot \vec{\mathcal{Z}}_{\rm sp}$. Finally, $\vec{\mathcal{Z}}_{\rm sp}$ sets the value of the species scale as soon as we cease to move parallel to $\vec{\zeta}_I$, which is only consistent if the second tower $\vec{\zeta}_J$ starts contributing (i.e., its mass gets below Λ_I) at the same moment, implying that the projection of $\vec{\zeta}_J$ over $\vec{\zeta}_I$ must also be $\vec{\mathcal{Z}}_I$. All this determines the magnitude of $\vec{\mathcal{Z}}_{\rm sp}$ and forces the vectors to be geometrically related as illustrated in Fig. 1.

In summary, the exponential decay rate of the QG cutoff for any direction $\hat{T} = (\cos\theta, \sin\theta)$ within the cone spanned by $\vec{\zeta}_I$ and $\vec{\zeta}_J$ reads

$$\lambda_{\rm sp}(\theta) = \hat{T}\cdot\vec{\mathcal{Z}}_{\rm sp} = \frac{1}{d-2}\frac{\left(\vec{\nabla}\log\frac{m_I}{m_J}\right)^{\intercal}\varepsilon\hat{T}}{(\vec{\nabla}\log m_I)^{\intercal}\varepsilon\vec{\nabla}\log m_J}, \qquad (20)$$

where for simplicity we work in a local basis of flat coordinates in the tangent bundle of the moduli space such that $\mathsf{G}_{ij} = \delta_{ij}$, and $\varepsilon = \left(\begin{smallmatrix}0 & -1\\ 1 & 0\end{smallmatrix}\right)$. This gets reduced to (19) in the particular case that $\hat{T}$ is parallel to $\vec{\zeta}_I$.

Therefore, the maximum (geodesic) variation of the scalar fields that can be consistently described by an EFT coupled to gravity in some perturbative corner reads

$$\Delta\phi_{\max} = \frac{1}{\lambda_{\rm sp}}\log\frac{M_{\rm Pl}}{\Lambda_{\rm sp}}, \qquad (21)$$

where we have used that the QG cutoff $\Lambda_{\rm sp}$ decays exponentially with the field distance, and the exponential rate $\lambda_{\rm sp}$ can be either computed or bounded as explained above. If replacing $\Lambda_{\rm sp} \leftrightarrow m_t$ and $\lambda_{\rm sp} \leftrightarrow \lambda_t$, we get the maximum scalar field range before we encounter the first state of the tower.

Towards a bottom-up rationale.—The pattern introduced in this note has been tested for a wide range of string theory compactifications, with different amounts of supersymmetry and diverse internal manifolds [34]. It is natural to wonder whether this relation is a general feature of quantum gravity or just a lamp-post effect of the string landscape. While we do not have yet a purely bottom-up argument (e.g., based on black hole physics), we are still able to identify and motivate some sufficient conditions that allow the pattern to hold in a general way.

The distance conjecture [35] ensures that the mass of the leading tower—and consequently the species scale—decreases *exponentially* with the moduli space distance (3). This can be further motivated from a bottom-up perspective by the emergence proposal [14,15], by which all the IR dynamics emerges from integrating out the dual massive degrees of freedom. This guarantees that $\vec{\zeta}_{\rm t} \cdot \vec{\mathcal{Z}}_{\rm sp}$ approaches to some constant asymptotically, but not necessarily the same one for all infinite distance limits. To argue for this, we propose three *sufficient* conditions which together ensure that (1) is fulfilled along any asymptotic direction.

Condition 1: The exponential rates λ_I of the different towers m_I are continuous over the asymptotic regions where they are defined. Furthermore, $\vec{\zeta}_{\rm t} \cdot \vec{\mathcal{Z}}_{\rm sp}$ must be well defined along any asymptotic direction.

This means that the exponential rate $\lambda_{\rm t} = \hat{T}\cdot\vec{\zeta}_{\rm t}$ of the leading tower is purely determined by the asymptotic direction $\hat{T}$, regardless of the particular geodesic we follow towards it. This does not require $\vec{\zeta}_{\rm t}$ to remain constant along parallel trajectories, being allowed to change or *slide* in the components perpendicular to $\hat{T}$ [47]. It implies, though, that the change in $\vec{\zeta}_{\rm t}$ has to be seen as a discrete *jump* in terms of the asymptotic direction. This can occur either because (i) the microscopic interpretation of the leading tower changes as a different tower starts dominating, in whose case the decay rate for both towers automatically coincide in the transition and $\lambda_{\rm t}$ is continuous, or (ii) because a complicated moduli dependence of the mass makes $\vec{\zeta}_{\rm t}$ to *jump* when crossing some *sliding loci* (see Ref. [38] for a detailed example when decompactifying to running solutions). In this latter case, we further need to require that $\vec{\zeta}_{\rm t} \cdot \vec{\mathcal{Z}}_{\rm sp}$ remains well defined, otherwise the product will depend on the trajectory taken. The consequence of this condition is that we can divide the set of infinite distance limits into regions over which the $\vec{\zeta}_{\rm t}$ and $\vec{\mathcal{Z}}_{\rm sp}$ take fixed expressions, and thus their product is constant.

Condition 2: For every infinite distance limit along which several towers decay at the same rate, there must exist bound states involving all of them, such that the species scale is given by the associated multiplicative species.

Consider several towers $\{m_1, \ldots, m_k\}$ becoming light at the same rate along some trajectory (or interface) with unit tangent vector $\hat{T}$, so that $\lambda_{\rm t} = \hat{T}\cdot\vec{\zeta}_1 = \ldots = \hat{T}\cdot\vec{\zeta}_k$. These towers span a lattice of "charges" $(n_1, \ldots, n_k)$ given by the tower levels $m_{n_i,i} \sim n_i^{1/p_i} m_i$ (with $p_i > 0$ depending on the nature and multiplicities of the tower [42]). If a (sub-)lattice of these charges is populated by particle states, then the total number of species is $N \sim \prod_{i=1}^{k} N_i$ (i.e., *multiplicative*) rather than $N \sim \sum_{i=1}^{k} N_i$ (i.e., *additive*). It can be shown [40] that in the former case the resulting $\vec{\mathcal{Z}}_{\rm sp}$ is orthogonal to the hull spanned by the ζ vectors, and moreover dominates over the individual species scales. This implies that $\vec{\zeta}_1 \cdot \vec{\mathcal{Z}}_{\rm sp} = \ldots = \vec{\zeta}_k \cdot \vec{\mathcal{Z}}_{\rm sp}$, such that the product (7) takes the same value in the different adjacent regions (as well as in the interface). For additive species, though, we do not obtain any additional species vector, and $\vec{\zeta}_{\rm t} \cdot \vec{\mathcal{Z}}_{\rm sp}$ would generically change upon crossing the interfaces. This is why Condition 2 requires the existence of the (sub-)lattice of bound states yielding a multiplicative number of species, which can be further motivated by Swampland considerations [48].

Condition 3: For every connected component of the space of infinite distance limits, there exists at least one direction associated to an emergent string limit or the homogeneous decompactification of an internal cycle to a higher dimensional vacuum.

With the previous two conditions, we have divided the moduli space into different regions and shown that $\vec{\zeta}_{\rm t} \cdot \vec{\mathcal{Z}}_{\rm sp}$ remains constant across those. The only thing left is to set this constant to $[1/(d-2)]$, which occurs if there exists *at least one* asymptotic direction resulting in a string perturbative limit or a decompactification to a higher dimensional vacuum. This resembles but it is a weaker condition than the emergent string conjecture [44].

Conclusions.—We propose a very concrete relation (1) between the quantum gravity cutoff $\Lambda_{\rm sp}$ in an EFT consistently coupled to quantum gravity, and the mass scale $m_{\rm t}$ of the lightest tower, which holds asymptotically in moduli space. At the moment, it is a common thread underlying all known string theory examples that have been explored so far. Finding a purely bottom-up rationale would have profound consequences for the consistency of EFTs coupled to gravity, since it constrains the possible towers of states predicted by the Swampland distance conjecture and implies a precise lower bound on how fast they can become light. It also provides a clear recipe to determine the species cutoff upon knowledge of the leading tower of states, which puts further constraints on how different perturbative limits can fit together in the field space of a QG theory.

The authors are supported by the Spanish Agencia Estatal de Investigación through Grant "IFT Centro de Excelencia Severo Ochoa" No. CEX2020-001007-S and Grant No. PID2021-123017NB-I00, funded by MCIN/AEI/10.13039/501100011033 and by ERDF A way of making Europe. The work of A. C. is supported by Spanish FPI Grant No. PRE2019-089790 and by the Spanish Science and Innovation Ministry through a grant for postgraduate students in the Residencia de Estudiantes del CSIC. The work of I. R. is supported by the Spanish FPI Grant No. PRE2020-094163. The work of I. V. is also partly supported by Grant No. RYC2019-028512-I from the MCI (Spain) and the ERC Starting Grant No. QGuide-101042568—StG 2021.

*alberto.castellano@csic.es
†ignacio.ruiz@uam.es
‡irene.valenzuela@cern.ch

[1] G. Dvali and M. Redi, Phys. Rev. D **77**, 045027 (2008).
[2] G. Dvali, Fortschr. Phys. **58**, 528 (2010).
[3] J. F. Donoghue, Phys. Rev. D **50**, 3874 (1994).
[4] U. Aydemir, M. M. Anber, and J. F. Donoghue, Phys. Rev. D **86**, 014025 (2012).
[5] M. M. Anber and J. F. Donoghue, Phys. Rev. D **85**, 104016 (2012).
[6] X. Calmet, R. Casadio, A. Y. Kamenshchik, and O. V. Teryaev, Phys. Lett. B **774**, 332 (2017).
[7] T. Han and S. Willenbrock, Phys. Lett. B **616**, 215 (2005).
[8] S. Caron-Huot, Y.-Z. Li, J. Parra-Martinez, and D. Simmons-Duffin, J. High Energy Phys. 05 (2023) 122.
[9] D. van de Heisteeg, C. Vafa, M. Wiesner, and D. H. Wu, arXiv:2212.06841.
[10] D. van de Heisteeg, C. Vafa, and M. Wiesner, Fortschr. Phys. **71**, 2300143 (2023).
[11] D. van de Heisteeg, C. Vafa, M. Wiesner, and D. H. Wu, arXiv:2310.07213.
[12] A. Castellano, A. Herráez, and L. E. Ibáñez, arXiv:2310.07708.
[13] T. D. Brennan, F. Carta, and C. Vafa, Proc. Sci. TASI2017 (**2017**) 015 [arXiv:1711.00864].
[14] E. Palti, Fortschr. Phys. **67**, 1900037 (2019).
[15] M. van Beest, J. Calderón-Infante, D. Mirfendereski, and I. Valenzuela, Phys. Rep. **989**, 1 (2022).
[16] M. Graña and A. Herráez, Universe **7**, 273 (2021).
[17] D. Harlow, B. Heidenreich, M. Reece, and T. Rudelius, Rev. Mod. Phys. **95**, 035003 (2023).
[18] N. B. Agmon, A. Bedroya, M. J. Kang, and C. Vafa, arXiv:2212.06187.
[19] T. Van Riet and G. Zoccarato, Phys. Rep. **1049**, 1 (2024).
[20] T. W. Grimm, E. Palti, and I. Valenzuela, J. High Energy Phys. 08 (2018) 143.
[21] N. Gendler and I. Valenzuela, J. High Energy Phys. 01 (2021) 176.
[22] P. Corvilain, T. W. Grimm, and I. Valenzuela, J. High Energy Phys. 08 (2019) 075.
[23] B. Heidenreich and T. Rudelius, Phys. Rev. D **104**, 106016 (2021).
[24] S. Lanza, F. Marchesano, L. Martucci, and I. Valenzuela, J. High Energy Phys. 02 (2021) 006.
[25] S. Lanza, F. Marchesano, L. Martucci, and I. Valenzuela, J. High Energy Phys. 09 (2021) 197.
[26] D. Harlow and H. Ooguri, Phys. Rev. Lett. **122**, 191601 (2019).
[27] D. Harlow and H. Ooguri, Commun. Math. Phys. **383**, 1669 (2021).
[28] D. Harlow, Proc. Sci. TASI2017 (**2018**) 002 [arXiv:1802.01040].

[29] T. Banks and N. Seiberg, Phys. Rev. D **83**, 084019 (2011).
[30] L. Susskind, arXiv:hep-th/9501106.
[31] W. Israel, Commun. Math. Phys. **8**, 245 (1968).
[32] M. Scalisi and I. Valenzuela, J. High Energy Phys. 08 (2019) 160.
[33] P. W. Graham, D. E. Kaplan, and S. Rajendran, Phys. Rev. Lett. **115**, 221801 (2015).
[34] A. Castellano, I. Ruiz, and I. Valenzuela, arXiv:2311.01536.
[35] H. Ooguri and C. Vafa, Nucl. Phys. **B766**, 21 (2007).
[36] M. Etheredge, B. Heidenreich, S. Kaya, Y. Qiu, and T. Rudelius, J. High Energy Phys. 12 (2022) 114.
[37] J. Calderón-Infante, A. M. Uranga, and I. Valenzuela, J. High Energy Phys. 03 (2021) 299.
[38] M. Etheredge, B. Heidenreich, J. McNamara, T. Rudelius, I. Ruiz, and I. Valenzuela, J. High Energy Phys. 12 (2023) 182.
[39] A. Castellano, A. Herráez, and L. E. Ibáñez, J. High Energy Phys. 06 (2023) 047.
[40] J. Calderón-Infante, A. Castellano, A. Herráez, and L. E. Ibáñez, J. High Energy Phys. 01 (2024) 039.
[41] In certain supersymetric setups, (7) holds at any point of $\mathcal{M}_\phi$ [34]. It is still unclear whether there exists a generalization of the pattern valid for any theory outside of the strict asymptotic regime as well.
[42] A. Castellano, A. Herráez, and L. E. Ibáñez, J. High Energy Phys. 08 (2022) 217.
[43] The vectors in (9) arise when considering decompactifications to a higher dimensional *vacuum*. In the presence of warping, these are generically modified, see Ref. [38].
[44] S.-J. Lee, W. Lerche, and T. Weigand, J. High Energy Phys. 02 (2022) 190.
[45] M. Etheredge, B. Heidenreich, T. Rudelius, I. Ruiz, and I. Valenzuela (to be published).
[46] One can analogously obtain from (7) the *lower* bound for $\lambda_{\rm sp} \geq [1/\sqrt{(d-1)(d-2)}]$ recently proposed in [40] upon assuming that the maximum value for the exponential rate of the leading tower corresponds to that of a one-dimensional (unwarped) KK tower.
[47] Note that, given two parallel trajectories along which the tower becomes light with different exponential rates, as we move towards infinite distance $m_{\rm t}$ would take *parametrically* distinct values between points separated a finite distance, rendering $\vec{\zeta}_{\rm t}$ with infinite norm. To avoid this, $\vec{\zeta}_{\rm t}$ should remain constant along parallel trajectories.
[48] Notice that the scenario of additive species would result in independent towers of states becoming light at the same rate, naively implying different massless gravitons asymptotically, which goes against Swampland considerations [49,50]. Furthermore, the case of multiplicative species is guaranteed by the emergent string conjecture [44] if assuming that all KK tower becoming light along some direction can be indeed microscopically interpreted as KK towers in the same dual frame.
[49] A. Bedroya, S. Raman, and H.-C. Tarazi, arXiv:2303.13585.
[50] H.-C. Kim, H.-C. Tarazi, and C. Vafa, Phys. Rev. D **102**, 026003 (2020).