

COMPARISON OF TRACKING CODES FOR BEAM-MATTER INTERACTION *

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Abstract

The interaction of particle beams with materials is important for muon colliders, as it causes particle scattering, energy loss and energy-straggling processes. Such interactions are also relevant in high-precision applications such as radiation oncology treatment planning, where the beam travels through air before reaching the patient, and are also the crucial mechanism for ionization cooling processes, such as those required for generating high-brightness beams for muon colliders. Few particle tracking codes integrate such effects in an environment suitable for lattice design. This work presents the simulation of these effects in the beam tracking program RF-Track (v2.1), compares the beam-matter interactions with the tracking programs ICOOL (v331.1) and G4Beamline (v3.08) and discusses the results.

INTRODUCTION

So far, computational studies and simulations of muon ionization cooling have been executed using two codes: ICOOL [1] and G4beamline [2]. Recently, the International Muon Collider Collaboration (IMCC) [3] started to use the program RF-Track [4] as a third option to simulate ionization cooling since the tracking dynamics of muons through matter has been implemented.

ICOOL

ICOOL is a tracking code expressly created for ionization cooling studies. The legacy code was written by R. C. Fernow and most physics which describes the interactions of charged particles with matter was implemented based on version 3.21 of Geant4.

G4beamline

G4beamline's code is based on the Geant4 toolkit and was developed for beamline simulation studies [5]. It was explicitly created to answer questions regarding the design of muon colliders. One big advantage is the visualization of the beamline geometry.

RF-Track

RF-Track is developed at CERN by A. Latina, and specializes in optimizations of low energy linacs with space-charge and other collective effects. It enables simulation of trajectories of particles with arbitrary charge and mass transported

through conventional elements and field maps. RF-Track is written in C++ and uses the languages Octave and Python as user interfaces. Since CERN is strongly involved in ionization cooling studies through the IMCC, we took the opportunity to add the implementation of beam-matter interactions in RF-Track to suit muon cooling simulations. This paper presents a step-by-step overview of the physical processes implemented as follows: energy loss, energy straggling, and particle scattering. Finally, a benchmark of the corresponding results against ICOOL and G4beamline is presented.

ENERGY LOSS OF CHARGED PARTICLES

The energy loss $\langle \partial E / \partial s \rangle$ of charged particles in matter is described by the Bethe-Bloch equation [6, 7]. This energy loss depends on the charge, the mass and the particle momentum, as well as material-specific properties such as the atomic mass and the mean excitation energy. The term that describes the density effect was not considered since density effects only contribute to energies in the GeV range, while muon cooling uses muons in the MeV region. Materials with atomic numbers ranging from 1 to 4 are presented since these are interesting for muon cooling. Helium is not included because of its full electron shell, the fact that it is hard to ionize, and its inefficiency in ionization cooling. Figure 1 illustrates the energy loss of a mono energetic muon beam in lithium without any straggling effects.

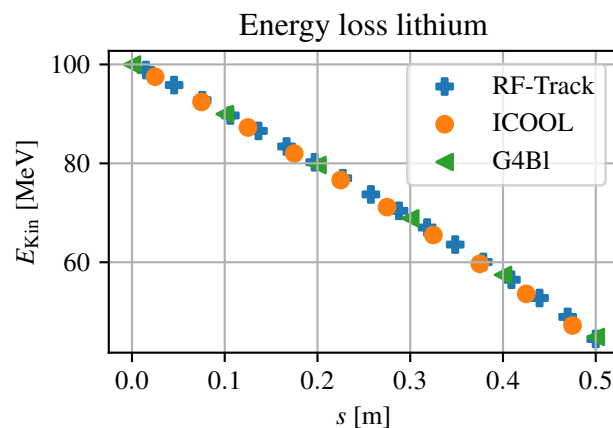


Figure 1: All three programs have similar values for energy loss through material.

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IMPACTS ON THE ENERGY SPREAD

This section discusses the impact on the energy spread due to the energy loss and the physics of stochastic energy straggling and benchmarks the results of RF-track with ICOOL and G4Beamline.

Systematic Effects

Each momentum of a muon is different compared to the other muons in the bunch. Given the Bethe-Bloch law, each muon releases a different amount of energy in the material. This causes a change of the bunch energy spread σ_E , which can be described as

$$\frac{d\sigma_E^2}{ds} \approx -2 \frac{d\langle \partial E / \partial s \rangle}{ds} \sigma_E^2. \quad (1)$$

Equation (1) shows that the energy spread depends on the slope of the Bethe-Bloch equation at the reference momentum of the beam bunch. Since a final cooling for muon colliders operates only on the negative slope of $\langle \partial E / \partial s \rangle$, the σ_E of a muon bunch always grows after passing an energy-absorbing material. Figure 2 illustrates such a energy spread increase at low energies without including stochastic effects.

Stochastic Effects

Heavy charged particles (such as muons) can directly collide with the electrons in material [8]. This statistical straggling in a target material can be approximated by

$$\frac{d\sigma_E^2}{ds} = kz^2 \frac{Z}{A} \rho \gamma^2 \left(1 - \frac{\beta^2}{2} \right). \quad (2)$$

In Eq. (2), $k = 0.153 \text{ MeV cm}^2 \text{ mol}^{-1}$ and comes from the properties of the electron; A is the target atomic weight in g mol^{-1} ; Z the atomic number; ρ is the density of the material in g cm^{-3} ; γ and β are the relativistic Lorentz factors of the traveling bunch; and z is the charge of the impacting particles.

The total energy spread of a bunch penetrating a material can be summarized by combining Eq. (1) with Eq. (2). The right plot in Fig. 2 shows the statistical fluctuation of the energy spread when stochastic straggling is considered.

Straggling Benchmarking

The stochastic straggling has been implemented in the RF-Track code. Figure 3 shows a test run of RF-Track with an initial kinetic energy of 100 MeV and energy spread of $\sigma_E = 5 \text{ MeV}$ through a 10 cm long liquid hydrogen absorber. The results of RF-Track are in good agreement with the ICOOL and G4Beamline simulations.

COULOMB SCATTERING

When a charged particle passes through matter, it is deflected by the Coulomb potentials of the atomic nuclei in the material. The standard deviation of the angular distribution

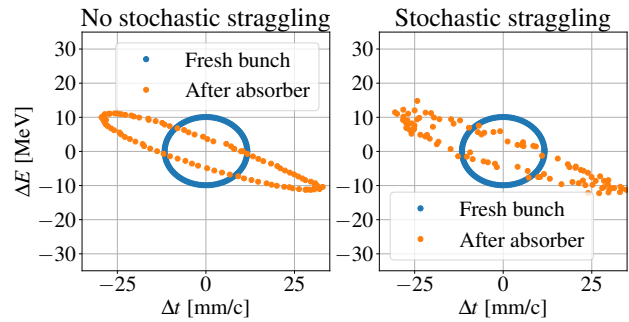


Figure 2: A fresh bunch (blue dots) penetrates a liquid hydrogen target with a reference kinetic energy of 100 MeV. After the absorber, the faster particles in the initial momentum distribution lose less energy than the slower particles (orange dots), causing a systematic energy spread increase (left plot). Collisions of muons with electrons cause an additional contribution to the straggling (right plot).

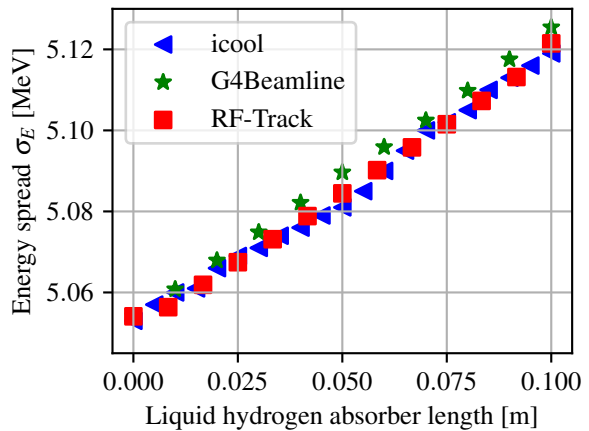


Figure 3: RF-Track simulation of a 100 MeV bunch in liquid hydrogen is compared with ICOOL and G4Beamline. Each simulation was performed with 10^5 muons.

due to the scattering can be approximated by

$$\theta = \frac{13.6[\text{MeV}]}{\beta pc} z \sqrt{\frac{s}{L_R}} \left[1 + 0.038 \ln \left(\frac{s}{L_R} \right) \right]. \quad (3)$$

In Eq. (3), the variable s describes the path length of the particle inside the absorber; z is the charge of the travelling particle; L_R is the radiation length of the material; c is the speed of light; β the relativistic Lorentz factor; and p is the momentum of the impacting particle in MeV c^{-1} . The literature states that the scattering angle distribution approximates a Gaussian distribution when the deflection angles are small [9]. An implementation of the particle scattering, according to the equation, can only follow without the logarithmic term. Otherwise, discrepancies of the scattering angle would appear when the simulation step size changes.

Scattering Benchmarking

Figure 4 compares the scattering angles after different absorber paths carried out with the mentioned simulation programs. Each simulation was performed by tracking muons

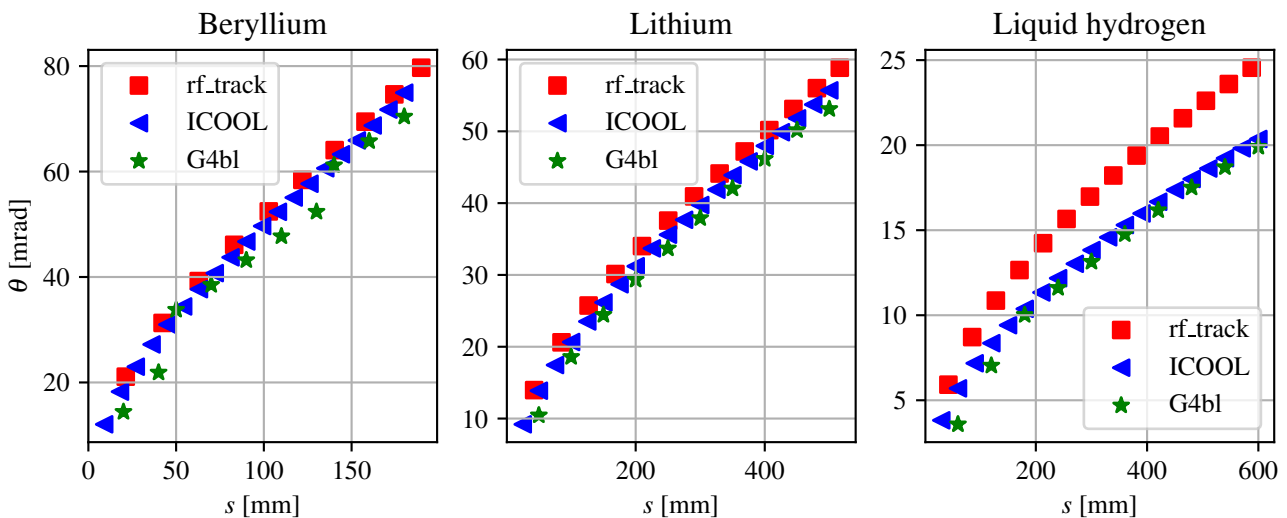


Figure 4: The comparison of scattering angles of muons obtained using the simulation codes ICOOL, RF-Track and G4Beamline. Beryllium, lithium and liquid hydrogen are used as test materials since they are relevant for the IMCC studies. For liquid hydrogen, the scattering angles in RF-Track simulations significantly disagree with the deflections of the other programs.

through beryllium, lithium, and liquid hydrogen. In Fig. 4, the RF-Track results for beryllium and lithium agree with the equivalent ICOOL and G4Beamline simulations. However, the scattering angles in the RF-Track simulation with liquid hydrogen differ from the other simulations' results. In Fig. 5, the transverse particle distributions of ICOOL and RF-Track are compared. The histogram obtained with ICOOL shows hard scatterings of muons on atomic nuclei, and the muon displacements no longer follow a Gaussian distribution. In RF-Track, since hard-scattering effects are not included, the number of particles in the core might be overestimated. In comparison to ICOOL and G4Beamline, RF-Track needs improvements for the presently used scattering algorithm, which leads to differences in the particle deflections on very low-Z materials.

CONCLUSION

CERN uses RF-Track for low energy acceleration simulations, like recently in ionization cooling studies for a muon collider. This paper presented how the physical interactions between charged particles and material were implemented in this tracking code. The results from RF-Track were compared and discussed in comparison with ICOOL and G4Beamline simulations. All three programs show similar results comparing the energy loss and the energy straggling. Nevertheless, inconsistencies appear in the scattering angles with very low-Z materials, such as liquid hydrogen.

OUTLOOK

The next step is implementing the hard scattering of charged particles with atomic nuclei in RF-Track. After that, it will be possible to simulate ionization cooling processes with RF-Track with high accuracy, which is essential for the design of a future muon collider.

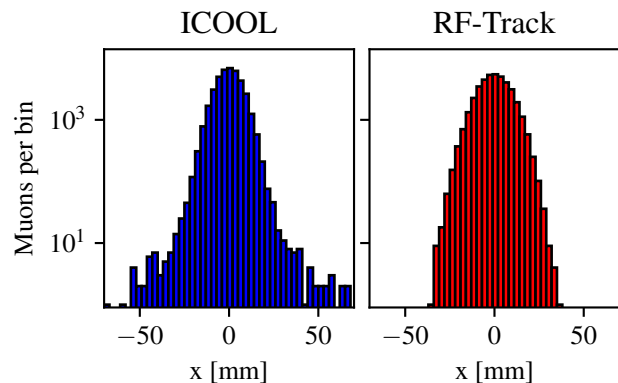


Figure 5: Histogram comparison of a mono energetic 100 MeV pencil beam with $4 \cdot 10^4$ muons impacting on liquid hydrogen. The right-hand-side plot with a vertical logarithmic scale demonstrates that the particle positions in RF-Track is Gaussian distributed. In the left diagram, a few large angle deflections are included in the ICOOL histogram, whose distribution no longer follows a Gaussian shape.

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