Reciprocal of the CPT theorem

Luis Álvarez-Gaumé [a,b], **Moshe M. Chaichian** [c,d], **Markku A. Oksanen** [c,d], **and Anca Tureanu** [c,d]

[a] *Simons Center for Geometry and Physics, State University of New York Stony Brook, NY–11794–3636, USA*

[b] *Theory Department CERN, CH-1211 Geneva 23, Switzerland*

[c] *Department of Physics, University of Helsinki, P.O.Box 64, FI-00014 University of Helsinki, Finland*

[d] *Helsinki Institute of Physics, P.O.Box 64, FI-00014 University of Helsinki, Finland*

Abstract

The CPT theorem originally proven by Lüders and Pauli ensures the equality of masses, lifetimes, magnetic moments and cross sections of any particle and its antiparticle. We show that in a Lorentz invariant quantum field theory described by its Lagrangian, CPT-violating interaction alone does not split the masses of an elementary particle and its antiparticle but breaks only the equality of lifetimes, magnetic moments and cross sections. However, CPT violation in the mass term of a field in the Lagrangian, which can be attributed to be due to the size of the particle described by a form factor, breaks only the equality of masses. Also it is shown that the two separate effects of CPT violation in the interaction terms or in the mass term do not mix due to higher quantum corrections and remain distinguishable. Thus, we urge the experimentalists to search for such observable effects concerning differences in the masses, magnetic moments, lifetimes and cross sections between the elementary or bound state particles and their antiparticles. In the case of CPT violation only in the mass term, besides the difference in the masses of elementary bound state particles and their antiparticles, there will be also an extremely tiny difference in the lifetimes of bound states due to the difference in their phase spaces. From the details of calculations, it appears that the separate effects of the CPT violation described above are quite general, neither depending on how the nonlocality is achieved, nor depending on what this violation is due to: due to T violation, as considered in the present work, which can be attributed to a cosmological direction of time; to CP or to both T and CP violations. The latter two cases satisfy the Sakharov's conditions for explaining the baryon asymmetry in the Universe.

The Lorentz invariance, CPT and spin-statistics theorems are key properties of any relativistic quantum field theory (QFT) and understanding their relations remains a fundamental issue. We seek to answer the question what does violation of CPT invariance lead to in a Lorentz invariant QFT?

According to the CPT theorem, any local Lorentz invariant QFT is CPT invariant [1, 2]. Furthermore, the CPT theorem and the spin-statistics theorem are among the few general results which can be proven in axiomatic quantum field theory, without reference to a particular Lagrangian or Hamiltonian model [3–9]. Crucial consequences of the CPT theorem include the equality of the masses, the decay widths (lifetimes) and the magnetic moments of a particle and its antiparticle, which hold for both elementary and composite particles. No violation of the CPT invariance has been observed in experiments so far. Considering the fundamental role of CPT invariance in QFT, it is no wonder that there has been a common belief that relativistic quantum field theories are necessarily CPT invariant, and further on that violation of CPT invariance would necessarily imply violation of Lorentz invariance, although neither of the said beliefs have been proven. There is no proof that the Wightman axioms can be satisfied for every interacting theory in four dimensions. Indeed the axiomatic framework does not yet cover (non-Abelian) gauge theories or interacting theories in general [4–7]. Hence the standing of CPT invariance in relativistic QFT is a pertinent problem [10, 11].

Since the possibility of CPT invariance violation has mainly been studied in theoretical frameworks where Lorentz invariance is also violated, it is important to study the consequences of CPT violation in a Lorentz invariant theory. That is necessary for uncovering what CPT violation alone entails.

In this letter, we consider relativistic QFT where CPT invariance is broken but Lorentz invariance is valid. It has been shown that CPT violation does not lead to violation of Lorentz invariance and vice versa [10]. That was achieved by introducing nonlocal interactions and/or nonlocal mass terms, which are Lorentz invariant but violate CPT invariance [10, 12, 13]. Such theories can be considered as effective theories, which can be used to explore the consequences of CPT violation that may arise in some fundamental theory.

The other direction of the relation between Lorentz invariance and CPT invariance, i.e. that violation of Lorentz invariance does not lead to CPT violation, has been known longer, since QFT on noncommutative spacetime preserves CPT invariance [14–17], while the Lorentz invariance is broken or more precisely deformed as a part of the twisted Poincaré symmetry [18–20]. We should also note that the spin-statistics relation remains intact in the noncommutative QFT [15] based on the Groenewold-Moyal star product, $f(x) \star g(x) = f(x) \exp\left(\frac{i}{2} \overleftarrow{\partial}_\mu \theta^{\mu\nu} \overrightarrow{\partial}_\nu\right) g(x)$. A general proof has been given in [21] for the validity of spin-statistics relation in noncommutative QFT, based on twisted Poincaré symmetry discovered in [18]. Note that in a deformed QFT based on the representations of κ-deformed Poincaré algebra, which is Lorentz violating, however, the spin-statistics relation is lost, and the notion of symmetrized or antisymmetrized multiparticle states ceases to have meaning [22].

The main conclusion of the Letter is that CPT violation has two separate consequences, depending on whether Lorentz invariant CPT violation is implemented in an interaction term or in a mass term:

1. If only a CPT-violating interaction term is included, the masses of an elementary particle and its antiparticle remain equal but the equalities of their lifetimes and cross sections are broken. Furthermore, the equalities of the masses and the lifetimes of bound states and their anti-states can be broken.

2. If CPT violation is implemented only in a mass term of the field, what implies that the particle is not pointlike, the equality of the masses is broken, while equalities of cross sections and magnetic moments remain intact, but with a tiny difference in lifetimes due to slight effect of phase spaces.

If both CPT noninvariant interactions and mass terms are included, their effects are combined: the equalities of the masses, the decay widths and the magnetic moments of an elementary particle and its antiparticle can all be broken. The situation with composite particles is more involved. If a difference in the masses of a composite particle and its antiparticle is observed, it could be due to either CPT-violating mass terms or CPT-violating interactions or both, in its constituent particles. All these consequences have to be taken into account in any experimental test of CPT invariance, e.g. in tests involving possible differences in the properties of matter and antimatter. On top of that, we emphasize that the aforementioned consequences do not imply violation of Lorentz invariance.

It is also important to understand the relation of the CPT invariance and the spin-statistics theorem. The spin-statistics relation [23–25], and the Pauli exclusion principle as its manifestation, are essential for the structure and stability of matter [26, 27]. In axiomatic QFT the requirements for the spin-statistics theorem are stricter than the requirements for the CPT invariance: the proof of the spin-statistics theorem requires that the Wightman functions are invariant under cyclic permutations of fields at Jost spacetime points, while the proof of the spin-statistic theorem requires that the Wightman functions are invariant under any permutation of fields at Jost spacetime points [3–7]. Therefore, we could expect that the violation of CPT invariance must imply violation of the spin-statistics relation. However, we must keep in mind that the axiomatic framework might not be fully applicable here, since the present theory involves nonlocal terms, and also since even a local interacting theory might not satisfy the axioms, as remarked above. We show that the spin-statistics theorem remains valid in the theory of Lorentz invariant CPT violation.

Lorentz invariant CPT violation Lorentz invariant CPT violation can be realized with a nonlocal (interaction) term that includes the factor $\theta(x^0 - y^0)$, where θ is the Heaviside step function and x^0, y^0 are the time coordinates of two points in spacetime. Another factor is included to ensure Lorentz invariance, which also defines the scope of the nonlocal interaction: e.g. $\theta((x-y)^2)$ for all causally connected points [10] or $\delta((x-y)^2 - l^2)$ for all points with a certain spacetime interval [12], where l is a real constant parameter. Another possible choice for the second factor is $\theta((x-y)^2)e^{-(x-y)^2/l^2}$ [10], where the real parameter l is the scale of the nonlocal interaction. The product of those two factors, e.g. $\theta(x^0-y^0)\delta((x-y)^2-l^2)$, ensures the invariance under proper orthochronous Lorentz transformations, since the order of the times x^0 and y^0 is unchanged for timelike intervals and the second factor ensures that spacelike intervals do not contribute to the interaction.

Quantization of theories which are nonlocal in time is challenging, since canonical quantization cannot be relied on due to the lack of a reliable definition of canonical momenta. Therefore, we do not yet have a perfectly satisfying technique for the quantization of theories that are nonlocal in time. The path integral quantization based on Schwinger's action principle has been applied to the nonlocal CPT-violating interactions with interesting results [12]. This approach follows the same lines as the successful path integral quantization of space-time noncommutative theories [28]. Hence we quantize the theory with the path integral formulation based on Schwinger's action principle.

CPT-violating interactions Various types of CPT noninvariant interactions can be considered by introducing the aforementioned nonlocal factor into an interaction term. Some examples include self-interactions like a nonlocal modification of $\lambda\phi^4$ theory and interactions between fields of different types. Since all the elementary interactions of matter are described with interactions between fermions and bosons, we show here an example of such an interaction. As a nonlocal CPT-violating interaction between a spin-1/2 field ψ and a real scalar field ϕ we consider a Yukawa-type interaction. An interaction vertex is introduced into the Lagrangian as in [12] (see also [10]):

$$\mathcal{L}_Y = g\bar{\psi}(x)\psi(x)\phi(x) + g_1\bar{\psi}(x)\psi(x)\int d^4y\theta(x^0-y^0)\delta((x-y)^2-l^2)\phi(y), \qquad (1)$$

where the nonlocal term with the coupling constant g_1 is odd under T and CPT but retains CP invariance. The effect of the CPT-violating interaction in momentum space can be represented with the following two *form factors*

$$f_\pm(k) = \int d^4z\, e^{\pm ik\cdot z}\theta(z^0)\delta(z^2-l^2), \qquad (2)$$

which are related as $f_\pm(-k) = f_\mp(k)$. For a timelike momentum k we may choose a Lorentz frame such that $\vec{k}=0$, and then the form factors can be written as

$$f_\pm(k^0) = 2\pi\int_0^\infty dz\frac{z^2e^{\pm ik^0\sqrt{z^2+l^2}}}{\sqrt{z^2+l^2}} \overset{(k^0\geq 0)}{=} \frac{2\pi}{(k^0)^2}\int_0^\infty dz\frac{z^2e^{\pm i\sqrt{z^2+(k^0l)^2}}}{\sqrt{z^2+(k^0l)^2}}, \qquad (3)$$

where $k^0>0$ is assumed in the second equality. Form factors are related to the size of particles (and in the case of a composite particle also to the internal structure of the particles).

Consider the consequences of the inclusion of the CPT-violating interaction in the tree level processes (compared to the local Yukawa interaction, i.e. just the first term in (1)). The amplitude of the fermion pair creation process $\phi\to\bar{\psi}\psi$ is changed so that the Yukawa coupling constant g is replaced by $g+g_1f_+(k)$, where k is the four-momentum of the scalar particle. The amplitude of the fermion annihilation process $\bar{\psi}\psi\to\phi$ is changed so that g is replaced by $g+g_1f_-(k)$. Consequently, the amplitudes for these two processes, which are related via time reversal, contain different phases due to the two form factors (2). There is a similar result in the amplitude of the process $\phi\phi\to\bar{\psi}\psi$, since g^2 is replaced by $(g+g_1f_+(k_1))(g+g_1f_+(k_2))$, where k_1 and k_2 are the momenta of the incoming scalar particles, while in the amplitude of the process $\bar{\psi}\psi\to\phi\phi$, g^2 is replaced by $(g+g_1f_-(k_1))(g+g_1f_-(k_2))$. Such differences in the phases result to differences in decay and scattering amplitudes (as well as in induced dipole moments). The absolute squares of the tree level amplitudes do not feature the T-violating phases, since $(f_\pm(k))^* = f_\mp(k)$. Hence for example the aforementioned tree level amplitudes give $|g+g_1f_+(k)|^2 = |g+g_1f_-(k)|^2$ and $|(g+g_1f_+(k_1))(g+$

$g_1 f_+(k_2))|^2 = |(g + g_1 f_-(k_1))(g + g_1 f_-(k_2))|^2$. However, when loop corrections are included, the T-violating phases of the tree level contributions interfere with the phases of the loop corrections, which results into detectable differences in the probabilities of the two processes that are related via time reversal. For example, the one-loop $O(g^3)$ contributions to the amplitudes of the processes $\phi \to \bar{\psi}\psi$ and $\bar{\psi}\psi \to \phi$ have the same phase, say $e^{-i\alpha}$, while the tree-level $O(g_1)$ contributions have different phases, $e^{\pm i\alpha_{\rm CPT}}$. The interference of such phases leads to the violation of time reversal and CPT invariances in the squared decay and scattering amplitudes (for detailed discussion, see [12]).

Note that some scattering amplitudes for the interaction (1) do not exhibit CPT violation effect at tree level. E.g., consider the amplitude of elastic scattering of fermion and antifermion, $\bar{\psi}\psi \to \bar{\psi}\psi$. The s-channel contribution of the Yukawa interaction is changed so that g^2 is replaced by $(g + g_1 f_+(p_1 + p_2))(g + g_1 f_-(p_1 + p_2))$, while in the t-channel contribution g^2 is replaced by $(g + g_1 f_+(p_1 - p_3))(g + g_1 f_-(p_1 - p_3))$, which contain only symmetric combinations of the form factors (2). For this process one needs to look into loop corrections with higher powers of g_1.

The fermion self-energy corrections do not introduce splitting of the fermion and antifermion masses. The theory has residual symmetries under C and CP, which is sufficient to maintain the equality of the fermion and antifermion masses. The one-loop fermion self-energy correction for the interaction (1) is obtained as

$$\Sigma(p) = (2\pi)^4 \delta^{(4)}(p' - p) \int \frac{d^4k}{(2\pi)^4} \frac{1}{\not{p} - \not{k} - m} \frac{1}{k^2 - M^2} \times \left[g^2 + g g_1 \left(f_+(k) + f_-(k) \right) + g_1^2 f_+(k) f_-(k) \right], \quad (4)$$

where m and M are the fermion and scalar particle masses, respectively. The self-energy corrections contain symmetric (and Hermitian) combinations of the form factors (2) such as $f_+(k)f_-(k)$ and $f_+(k) + f_-(k)$, which are symmetric under $k \to -k$ and do not break the symmetry between positive and negative fermion energy p_0. Thus, there is no splitting in fermion and antifermion masses. This result is retained even in the presence of CP violation [12], when the local Yukawa interaction is replaced by $g\bar{\psi}(x)(1 + i\varepsilon\gamma_5)\psi(x)\phi(x)$ with a small real ε, which indicates that the invariance under C is sufficient to maintain the equality of the masses.

While we have considered a CPT-violating Yukawa interaction in the leading order, the main conclusion remains valid in general and including all higher order corrections, as shown below by involving the gauge invariance arguments.

In gauge theory, gauge invariance together with Lorentz invariance ensures that the mass of a fermion (m) is equal to the mass of its antiparticle ($\bar{m}$). For simplicity, consider quantum electrodynamics. Due to the gauge invariance, replacing the photon A_μ in the fermion-photon vertex $e\bar{\psi}\gamma^\mu A_\mu \psi$, with ∂_μ (in x-space) or with k_μ in momentum space does not change the vertex, i.e. it gives zero: $\bar{\psi}\gamma^\mu k_\mu \psi = 0 \Rightarrow m = \bar{m}$; $k = p_\psi - p_{\bar{\psi}}$. The electromagnetic current is conserved, as depicted by the Ward identity.

Now consider a theory with nonlocal interactions, where the action is still invariant under the gauge transformation [13]: $\psi(x) \to e^{i\alpha(x)}\psi(x)$, $A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu \alpha(x)$, with the gauge field coupled to all charged fields. The free Lagrangian is taken to be the local one. Consider merely for simplicity that only a Yukawa interaction is taken to be nonlocal (1). Corrections to the fermion-photon vertex are depicted in Fig. 1, where the dashed circles denote the corrections of all orders to the propagators of A_μ, ψ and $\bar{\psi}$, and to the vertex. The gauge invariance still ensures that the renormalized masses are equal, $m = \bar{m}$.

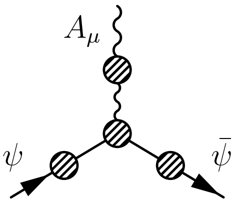

Figure 1: The hashed circles stand for all the quantum corrections to the fermion-photon interaction vertex.

Splitting of the masses of an elementary particle and its antiparticle If one starts with the general Hermitian CPT-invariant Lagrangian of a free fermion and modifies any of its terms to become nonlocal by introducing a nonlocal factor with the aforementioned method, no splitting between the masses and the widths of the particle and its anti-particle appears [12]. However, while in a local theory the term $i\mu\bar{\psi}(x)\psi(x)$ with real μ cancels with its Hermitian conjugate, a nonlocal term like $i\mu F(x,y)\bar{\psi}(x)\psi(y)$, where $F(x,y)$ is the nonlocal factor, e.g. $\theta(x^0-y^0)\delta((x-y)^2-l^2)$, gives a nonvanishing Hermitian contribution to the action. Thus, we consider the action [29]

$$S = \int d^4x \, \{\bar{\psi}(x) i\gamma^\mu \partial_\mu \psi(x) - m\bar{\psi}(x)\psi(x) \\ - \int d^4y \, \left[\theta(x^0-y^0) - \theta(y^0-x^0)\right] \delta((x-y)^2 - l^2)[i\mu\bar{\psi}(x)\psi(y)]\} \, , \tag{5}$$

where for a real parameter μ the nonlocal term is odd under charge conjugation and even under parity and time reversal. Hence the action (5) is odd under C, CP and CPT.

Dirac equation in the momentum space, with the Ansatz $\psi(x) = e^{ip\cdot x}\chi(p)$, is

$$\not{p}\chi(p) = m\chi(p) + i\mu \left[f_+(p) - f_-(p)\right] \chi(p), \tag{6}$$

where the two form factors $f_\pm(p)$ are defined in (2). The Lorentz covariant off-shell propagator is defined by

$$\int d^4x \, d^4y \, e^{ip\cdot(x-y)} \, \langle T^\star \psi(x)\bar{\psi}(y)\rangle = \frac{i}{\not{p} - m + i\epsilon - i\mu[f_+(p) - f_-(p)]}, \tag{7}$$

where the poles are shifted due to the form factors. The poles occur only for timelike momentum. The eigenvalue equation in the rest frame ($\vec{p} = 0$) is

$$p_0 = \gamma_0 \left(m - 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[p_0\sqrt{z^2+l^2}]}{\sqrt{z^2+l^2}} \right). \tag{8}$$

The CPT transformed eigenvalue equation, which is obtained by $p_0 \to -p_0$ and sandwiching with γ_5, has a similar form but the sign of the second term on the right-hand side is opposite,

$$p_0 = \gamma_0 \left(m + 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[p_0\sqrt{z^2+l^2}]}{\sqrt{z^2+l^2}} \right). \tag{9}$$

For $\mu \gg m$, we can estimate the masses of the particle and its antiparticle by solving the eigenvalue equations iteratively to the first order in μ: (8) and (9) give the mass eigenvalues as

$$m_\mp \approx m \mp 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[m\sqrt{z^2+l^2}]}{\sqrt{z^2+l^2}}. \tag{10}$$

Thus, the difference of the form factors (3) in (8), i.e. $f_+(p_0) - f_-(p_0)$, results into the splitting of the masses of the particle and its antiparticle.

The additional mass term in the action (5) contributes to the free propagator (7) of the field but does not contribute to any interaction vertices. Therefore, the equality of the widths and cross sections for a particle and its antiparticle is not altered by the CPT noninvariant mass term.

Spin-statistics theorem The classic form of the spin-statistics theorem was established by Pauli [23] and his proof involved the CPT invariance as one of the assumptions. The spin-statistics theorem was proven without assuming the CPT theorem by Lüders and Zumino [24] and Burgoyne [25]. Those traditional proofs of the spin-statistics relation rely on the operator formalism. The spin-statistics relation can be proven also in the path integral formalism [30].

We present key parts of the proof in the theory with Lorentz invariant CPT violation. Consider the CPT-violating interaction (1). The free fields are unaltered and hence the proofs of [30] apply for them. The off-shell propagators for the interacting fields are obtained from the path integral as

$$\langle T^\star \psi(x)\bar\psi(y)\rangle = \frac{i}{i\partial\!\!\!/_x - m + g\phi(x) + g_1 \int d^4z F(x,z)\phi(z) + i\epsilon}\delta^4(x-y), \tag{11}$$

where $F(x,y) = \theta(x^0 - y^0)\delta((x-y)^2 - l^2)$, and

$$\langle T^\star \phi(x)\phi(y)\rangle = \frac{i}{\Box_x + M^2 + g\bar\psi(x)\psi(x) + g_1 \int d^4z F(z,x)\bar\psi(z)\psi(z) - i\epsilon}\delta^4(x-y). \tag{12}$$

The Bjorken–Johnson–Low (BJL) method [31, 32] enables the definition of the T-product in terms of the $T^\star$-product, defined for the propagator (11) as

$$\begin{aligned} \int d^4x\, e^{ip\cdot(x-y)} \langle T\psi(x)\bar\psi(y)\rangle &= \int d^4x\, e^{ip\cdot(x-y)} \langle T^\star\psi(x)\bar\psi(y)\rangle \\ &\quad - \lim_{p_0\to 0} \int d^4x\, e^{ip\cdot(x-y)} \langle T^\star\psi(x)\bar\psi(y)\rangle . \end{aligned} \tag{13}$$

From such two-point correlation functions the equal time commutation and anticommutation relations for the fields are derived [30]. Assuming that the path integral measure for the field ψ is valued in Grassmann variables, we obtain with the BJL method from (11) and (13) that

$$\delta(x^0 - y^0)\,\langle\{\psi(x), \bar{\psi}(y)\}\rangle = \gamma^0\delta^4(x - y), \tag{14}$$

which is the basic anticommutation relation for the spin-1/2 field, as well as that $\int d^4x\, e^{ip\cdot(x-y)}\, \langle T\left(i\partial\!\!\!/_x - m + g\phi(x) + g_1\int d^4zF(x,z)\phi(z)\right)\psi(x)\bar{\psi}(y)\rangle = 0$, which is consistent with the field equation for ψ. Both ψ and $\bar{\psi}$ anticommute with themselves, i.e. $\delta(x^0 - y^0)\,\langle\{\psi(x), \psi(y)\}\rangle = 0$ and $\delta(x^0 - y^0)\,\langle\{\bar{\psi}(x), \bar{\psi}(y)\}\rangle = 0$, since the corresponding propagators vanish, $\langle T^\star\psi(x)\psi(y)\rangle = 0$ and $\langle T^\star\bar{\psi}(x)\bar{\psi}(y)\rangle = 0$. The positive energy condition is satisfied with the Feynman prescription $m - i\epsilon$. The norm in the Hilbert space is positive definite only when the spin-1/2 field in the path integral is Grassmann variable. The scalar field ϕ is considered with the same method using the propagator (12).

Similar derivation is performed for the case of the CPT-violating theory with particle-antiparticle mass splitting (5) using the propagator (7). In this case the derivation is similar to the usual free field case, since the form factors in the propagator vanish in the equal-time limit, $[f_+(p) - f_-(p)] \to 0$ when $p_0 \to \infty$. Therefore, the spin-statistics theorem remains valid when CPT-violating interactions and mass terms are present. It is interesting that a similar result occurs in the case of infinite-component QFT, where CPT can be violated, while spin-statistics theorem remains intact [33]. Seemingly, infinite-component QFT theories in a way mimic the effect of nonlocality as is the case in string theories.

As a side remark to reach still another picture of quantization for CPT-violating but Lorentz invariant QFT, it could be beneficial to mention a quantization method introduced by Umezawa and Takahashi [34], which was based on Umezawa's unique approach to the quantization of local relativistic field theories [35–37]. This method does not require canonical formulation and it might be possible to generalize it for nonlocal theories like the present one.

As final remarks, we would like to mention a few important results, which emphasize general aspects of CPT violation:

- Quantum corrections due to interaction terms always shift the values of the masses. However, the changes in the mass are equal for the particle and its antiparticle in all Lorentz invariant theories, where the mass term is not nonlocal. Therefore, only a nonlocal and CPT-violating mass term, such as the last term in (5), which corresponds to the form-factor, i.e. to a non-point-like particle, can give different masses for the particle and its antiparticle.

- From the details and performing the calculations in this work, it has appeared to us within a certain degree of assurance that the results and the effects of different CPT-violating terms obtained here are quite general ones and do not depend on how the nonlocality is achieved, and as well do not depend on how the CPT violation has appeared:

  - Nonlocal QFT can be constructed in several ways, namely by a) the Schwinger point like prescription; b) by smearing of the field operators, as presented in the present work; c) by using the higher derivative theories, such as the ones already used for many years and highlighted in some works, such as in [38] and [39], and therein for references to previous works on the subject in the literature.

– CPT violation in the nonlocal QFT can be achieved in a few ways: i) as in the present work by breaking the time reversal, what could be attributed to the cosmological arrow of time, where time evolves only towards future, on which many works have appeared. Examples can be found, e.g. in [40–43] and references therein; ii) by breaking the CP invariance, introducing a complex phase, originating e.g. due to the experimentally verified Kobayashi–Maskawa quark mixing, while keeping the time invariance; also iii) by breaking both the CP invariance and independently also breaking the time invariance. The two cases ii) and iii), whereby both CP and C violation are present is one of the conditions for the Sakharov mechanism [44] to explain the matter-antimatter asymmetry in Universe.

We see that there are still several further possibilities to be considered together with their consequent implications on observable effects, some of which are under study and will be presented in a future communication.

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References

[1] G. Lüders, *On the Equivalence of Invariance under Time Reversal and under Particle-Antiparticle Conjugation for Relativistic Field Theories*, Kong. Dan. Vid. Sel. Mat. Fys. Med. **28N5**, no.5, (1954) 1–17.

[2] W. Pauli, *Exclusion principle, Lorentz group and reflexion of space-time and charge*, in *Niels Bohr and the Development of Physics*, ed. W. Pauli, Pergamon Press, London, 1955.

[3] R. Jost, *Eine Bemerkung zum CTP theorem*, Helv. Phys. Acta **30** (1957) 409–416.

[4] R. Streater and A. S. Wightman, *PCT, Spin & Statistics, and All That*, Benjamin, New York, 1964.

[5] R. Jost, *The General Theory of Quantized Fields*, American Mathematical Society, Providence, 1965.

[6] N. N. Bogoliubov, A. A. Logunov and I. T. Todorov, *Axiomatic Quantum Field Theorem*, W.A. Benjamin, Reading, Massachusetts, 1973.

[7] N. N. Bogoliubov, A. A. Logunov, A. I. Oksak and I. T. Todorov, *General Principles of Quantum Field Theory*, Kluwer, Dordrecht, 1990.

[8] L. Álvarez-Gaumé and M. A. Vázquez-Mozo, *An Invitation to Quantum Field Theory*, Lecture Notes in Physics **839**, Springer, 2012.

[9] K. Nishijima, *Quantum Field Theory*, (translated from the 8th Japanese edition), eds. M. Chaichian and A. Tureanu, Springer, 2023.

[10] M. Chaichian, A. D. Dolgov, V. A. Novikov and A. Tureanu, *CPT violation does not lead to violation of Lorentz invariance and vice versa*, Phys. Lett. B **699** (2011) 177–180 [arXiv:1103.0168 [hep-th]].

[11] M. Duetsch and J. M. Gracia-Bondia, *On the assertion that PCT violation implies Lorentz non-invariance*, Phys. Lett. B **711** (2012) 428–433 [arXiv:1204.2654 [hep-th]].

[12] M. Chaichian, K. Fujikawa and A. Tureanu, *Lorentz invariant CPT violation*, Eur. Phys. J. C **73** (2013) 2349 [arXiv:1205.0152 [hep-th]].

[13] M. Chaichian, K. Fujikawa and A. Tureanu, *Electromagnetic interaction in theory with Lorentz invariant CPT violation*, Phys. Lett. B **718** (2013) 1500–1504 [arXiv:1210.0208 [hep-th]].

[14] M. M. Sheikh-Jabbari, *C, P, and T invariance of noncommutative gauge theories*, Phys. Rev. Lett. **84** (2000) 5265–5268 [arXiv:hep-th/0001167].

[15] M. Chaichian, K. Nishijima and A. Tureanu, *Spin statistics and CPT theorems in noncommutative field theory*, Phys. Lett. B **568** (2003) 146–152 [arXiv:hep-th/0209008].

[16] L. Álvarez-Gaumé and M. A. Vázquez-Mozo, *General properties of noncommutative field theories*, Nucl. Phys. B **668** (2003) 293–321 [arXiv:hep-th/0305093].

[17] M. Chaichian, M. N. Mnatsakanova, K. Nishijima, A. Tureanu and Yu. S. Vernov, *Towards an axiomatic formulation of noncommutative quantum field theory*, J. Math. Phys. **52** (2011) 032303 [arXiv:hep-th/0402212].

[18] M. Chaichian, P. P. Kulish, K. Nishijima and A. Tureanu, *On a Lorentz-invariant interpretation of noncommutative space-time and its implications on noncommutative QFT*, Phys. Lett. B **604** (2004) 98–102 [arXiv:hep-th/0408069].

[19] J. Wess, *Deformed coordinate spaces: Derivatives*, Proceedings of BW2003, 122–128 [arXiv:hep-th/0408080].

[20] M. Chaichian, P. Presnajder and A. Tureanu, *New concept of relativistic invariance in NC space-time: Twisted Poincaré symmetry and its implications*, Phys. Rev. Lett. **94** (2005) 151602 [arXiv:hep-th/0409096].

[21] A. Tureanu, *Twist and spin-statistics relation in noncommutative quantum field theory*, Phys. Lett. B **638** (2006) 296–301 [arXiv:hep-th/0603219].

[22] M. Arzano and J. Kowalski-Glikman, *Quantum particles in noncommutative spacetime: An identity crisis*, Phys. Rev. D **107** (2023) 065001 [arXiv:2212.03703 [hep-th]].

[23] W. Pauli, *The Connection Between Spin and Statistics*, Phys. Rev. **58** (1940) 716–722.

[24] G. Lüders and B. Zumino, *Connection between Spin and Statistics*, Phys. Rev. **110** (1958) 1450–1453.

[25] N. Burgoyne, *On the connection of spin with statistics*, Nuovo Cim. **8** (1958) 607–609.

[26] E. H. Lieb and W. E. Thirring, *Bound for the Kinetic Energy of Fermions Which Proves the Stability of Matter*, Phys. Rev. Lett. **35** (1975) 687–689.

[27] E. H. Lieb, *The Stability of Matter: From Atoms to Stars*, ed. W. Thirring, Springer-Verlag, Berlin Heidelberg, 2005.

[28] K. Fujikawa, *Path integral for space-time noncommutative field theory*, Phys. Rev. D **70** (2004) 085006 `arXiv:hep-th/0406128`.

[29] M. Chaichian, K. Fujikawa and A. Tureanu, *Lorentz invariant CPT violation: Particle and antiparticle mass splitting*, Phys. Lett. B **712** (2012) 115–118 [`arXiv:1203.0267 [hep-th]`].

[30] K. Fujikawa, *Spin statistics theorem in path integral formulation*, Int. J. Mod. Phys. A **16** (2001) 4025–4044 `arXiv:hep-th/0107076`.

[31] J. D. Bjorken, *Applications of the Chiral $U(6) \otimes U(6)$ Algebra of Current Densities*, Phys. Rev. **148** (1966) 1467–1478.

[32] K. Johnson and F. E. Low, *Current algebras in a simple model*, Prog. Theor. Phys. Suppl. **37** (1966) 74–93.

[33] A. I. Oksak and I. T. Todorov, *Invalidity of TCP-theorem for infinite-component fields*, Commun. Math. Phys. **11** (1968) 125–130

[34] Y. Takahashi and H. Umezawa, *Relativistic quantization of fields*, Nucl. Phys. **51** (1964) 193.

[35] H. Umezawa, *Quantum Field Theory*, North-Holland Publishing Co., 1956.

[36] Y. Takahashi, *An Introduction to Field Quantization*, Pergamon Press, 1969.

[37] H. Umezawa, *Advanced Field Theory: Micro, Macro, and Thermal Physics*, American Institute of Physics, 1993.

[38] T. Biswas and N. Okada, *Towards LHC physics with nonlocal Standard Model*, Nucl. Phys. B **898** (2015) 113–131 [`arXiv:1407.3331 [hep-ph]`].

[39] F. Abu-Ajamieh and S. K. Vempati, *A proposed renormalization scheme for non-local QFTs and application to the hierarchy problem*, Eur. Phys. J. C **83** (2023) 1070 [`arXiv:2304.07965 [hep-th]`].

[40] S. W. Hawking, *The Direction of Time*, New Sci. **115** (1987) 46.

[41] A. D. Sakharov, *Cosmological models of the Universe with rotation of time's arrow*, Sov. Phys. JETP **52** (1980) 349, Sov. Phys. Usp. **34** (1991) 401.

[42] A. Barnaveli and M. Gogberashvili, *Cosmological 'arrow of time' and baryon asymmetry of the universe*, Phys. Lett. B **316** (1993) 57–60.

[43] J. D. Bowman and V. Gudkov, *Search for time reversal invariance violation in neutron transmission*, Phys. Rev. C **90** (2014) 065503 [`arXiv:1407.7004 [hep-ph]`].

[44] A. D. Sakharov, *Violation of CP Invariance, C asymmetry, and baryon asymmetry of the universe*, JETP Lett. **5** (1967) 24, Sov. Phys. Usp. **34** (1991) 392.