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LONGITUDINAL BEAM DYNAMICS AND RF REQUIREMENTS FOR A CHAIN OF MUON RCSs

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Abstract

A facility for the collision of muons offers a unique path to a compact lepton collider with an energy reach in the multi-TeV regime, well beyond the possibilities of conventional electron accelerators. However, due to the short lifetime of muons, the constraints for acceleration and collisions are very different. An extremely fast energy increase in combination with intense and ultra-short bunches is essential for a high muon survival rate and luminosity. A chain of rapid cycling synchrotrons (RCS) for acceleration from around 60 GeV to several TeV is proposed by the International Muon Collider Collaboration. We study the longitudinal beam dynamics and radio-frequency (RF) requirements for these RCSs with respect to induced voltages from intensity effects. A high synchrotron tune due to the large RF voltages is a particular challenge. We present simulation results of the longitudinal bunch distribution to determine the number of RF stations distributed over the RCS to mitigate that large tune. The impact of the induced voltages from short-range wakefields and beam loading is analyzed, for both fundamental and higher-order modes.

INTRODUCTION

The work of the International Muon Collider Collaboration (IMCC) aims at a design proposal for a multi-TeV collision muon facility. In the current design [1] which is based on the US Muon Acceleration Program (MAP) [2, 3], one μ^+ and one μ^- bunch are produced by depositing protons on a target, followed by 6-dimensional cooling and initial acceleration to an energy of ≈ 60 GeV. Two counter-rotating bunches will accelerated to a target energy of 3 TeV or 10 TeV in a chain of rapid cycling synchrotrons (RCS) with 5 Hz repetition rate. Collisions will take place in a separate ring.

Due to the decay of muons, these processes must take place during the relativistically dilated muon lifetime of $\gamma \cdot \tau_\mu$, with $\tau_\mu = 2.197$ µs as the muon lifetime at rest and γ the relativistic Lorentz factor. Unique requirements with respect to the longitudinal beam dynamics and the radio-frequency (RF) system evolve. High-intensity bunches with a population of up to $2.5 \cdot 10^{12}$ muons must be accelerated on a millisecond timescale while respecting challenging target parameters for the longitudinal (4σ) emittance of $\varepsilon_\mathrm{L} = 0.31$ eVs and for the bunch lengths of $1\sigma = 5$ mm in the collider. Gigavolts RF voltages per turn and only tens of turns per accelerator are the consequence. The high bunch charges lead to large bunch currents $I_\mathrm{b} \leq 20.4$ mA per beam and strong transient beam loading.

We present simulation-based studies of the longitudinal bunch distribution with regards to these specific requirements using the longitudinal macro-particle tracking code BLonD [4]. The high synchrotron tune resulting from the ultra-fast acceleration is a particular challenge, which will be mitigated by a larger number of RF stations. The intensity effects of short-range wakefields and fundamental beam loading due to the high bunch population are studied. We present calculations for the power induced from higher-order modes (HOM).

ACCELERATION AND RF PARAMETERS

The layout for the high-energy acceleration stage of the 3 TeV collider consists of three RCSs with intermediate energy levels of ≈ 300 GeV and 750 GeV to accelerate the muons to 1.5 TeV per bunch, as sketched in Fig. 1. A fourth

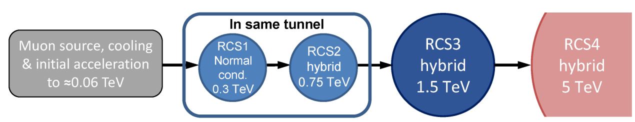

Figure 1: Schematic of the chain of rapid cycling-synchrotrons for the high-energy acceleration complex.

RCS will later accelerate the muon bunches from 1.5 TeV to 5 TeV for the 10 TeV collider, which is however not part of the work presented here. A selection of example parameters of the first three RCSs is summarized in Table 1.

Table 1: Example parameters for the first three muon RCSs for a 90% survival rate and 1.3 GHz cavities.

	RCS1	**RCS2**	**RCS3**
Circumference [m]	5990	5990	10700
Injection energy [TeV]	0.06	0.30	0.75
Ejection energy [TeV]	0.30	0.75	1.50
Injection γ	597	2971	7099
Ejection γ	2971	7099	14198
Acceleration time [ms]	0.34	1.10	2.37
Number of turns	17	55	66
Energy gain per turn [GeV]	14.8	7.9	11.4
Harmonic number	26k	26k	46k
Bunch intensity [10^{12}]	2.54	2.34	2.2
Bunch current [mA]	20.4	18.8	10.0
Target 4σ emittance [eVs]	0.31	0.31	0.31

The RCS1 and RCS2 are supposed to share the same tunnel and layout [5]. While the first RCS is foreseen to

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be equipped with normal conducting, i.e., warm magnets, RCS2 and RCS3 are planned as hybrid RCSs. In such a RCS, normal conducting magnets cycling from $-B_{nc}$ to $+B_{nc}$ are interleaved with fixed-field, superconducting magnets to achieve a large energy swing combined with a high average bending field.

RF System Parameters

The design of the RF system is strongly driven by the strict gradient requirements dictated by the muon decay, despite the relativistic time dilation. For the Lorentz factors at injection and ejection γ_{inj} and γ_{ej} (Table 1), and for a fixed survival rate of $N_{ej}/N_{inj} = 0.9$, the acceleration times τ_{acc} and average accelerating gradients G_{acc} (also listed in Table 1) are fully determined [6]:

$$\tau_{acc} = -\tau_\mu(\gamma_{ej} - \gamma_{inj}) \ln\left(\frac{N_{ej}}{N_{inj}}\right) / \ln\left(\frac{\gamma_{ej}}{\gamma_{inj}}\right) , \tag{1}$$

$$G_{acc} = -\frac{1}{\tau_\mu} m_\mu c \ln\left(\frac{\gamma_{ej}}{\gamma_{inj}}\right) / \ln\left(\frac{N_{ej}}{N_{inj}}\right) . \tag{2}$$

Here, $m_\mu = 105.658\,\mathrm{MeV}/c^2$ denotes the muon rest mass. The resulting average gradients in each RCS of respectively $G_{acc} = 2.4\,\mathrm{MV/m}$, $1.3\,\mathrm{MV/m}$ and $1.1\,\mathrm{MV/m}$ together with strong beam loading call for a superconducting and high-gradient cavity structure. A TESLA-like 1.3 GHz cavity [7] seems to fulfill these requirements and has therefore been chosen as a starting point. This cavity consists of 9 cells, $R/Q = 518\,\Omega$, and the loaded quality factor $Q_L \approx 2.3 \cdot 10^6$ (for a beam loading compensation [8] of $\Delta f \approx 0.3\,\mathrm{kHz}$ depending on the RCS). Assuming a maximum gradient of 30 MV/m, the energy gains per turn ΔE listed in Table 1, and a synchronous phase of $\phi_s = 45°$, the total RF voltages $V_{RF} = \Delta E/\cos(\phi_s)$ required for RCS1, RCS2, and RCS3 are 20.9 GV, 11.2 GV, and 16.1 GV, respectively. This translates to about 700, 380 and 540 cavities to be installed in the equivalent RCSs.

SYNCHROTRON TUNE

As a consequence of the large V_{RF}, the synchrotron tune $Q_s \propto \sqrt{V_{RF}}$ is extremely high. Values range between 1.52 at injection in RCS1 and 0.24 at ejection in RCS3, which by far exceeds the limit for stable synchrotron oscillations and phase focusing of $Q_s \ll 1/\pi$ [9]. These cavities must therefore be distributed over a large number of RF stations n_{RF} to assure a low synchrotron tune between two RF stations, i.e., a small $Q'_s = Q_s/n_{RF}$. We note that a very large n_{RF} would be advantageous for the longitudinal beam dynamics by providing a more continuous acceleration due to smaller energy increments between RF stations. However, a larger n_{RF} will cause a significantly greater constructional complexity.

To determine the minimal required n_{RF}, we simulated the evolution of the bunch emittance for various n_{RF}. For each RCS, we keep all acceleration and RF parameters unchanged, and only vary n_{RF}, i.e., Q'_s. Intensity effects are not included in the simulations to only observe the zero-order effect of Q_s on the bunch parameters. We measure the growth in 4σ emittance as main bunch quality criterion.

Figure 2 (a) depicts the longitudinal phase space in RCS1 for $n_{RF} = 4$ stations after half a turn, i.e. after two kick and drifts in the tracking code, and $Q'_s = 1.52/4 = 0.38 > 1/\pi$. Fast de-bunching is observed and major parts of the bunch,

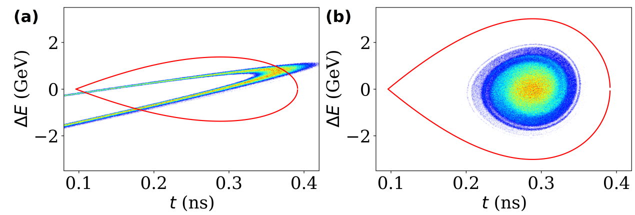

Figure 2: Longitudinal phase space for a μ^+ bunch in RCS1 for (a) $n_{RF} = 4$ after half a turn in the ring and (b) $n_{RF} = 32$ after 17 turns at ejection. The red line indicates the separatrix (with continuous energy approximation) for $f_{RF} = 1.3\,\mathrm{GHz}$, $V_{RF} = 20.9\,\mathrm{GV}$, and $\phi_s = 45°$.

82% of the macro-particles, are lost from the RF bucket at this early stage of the acceleration cycle. The phase space for $n_{RF} = 32$ in RCS1 after all 17 turns and $Q'_s = 0.05$ is shown in Fig. 2 (b). Despite minor dilution, the bunch shape and emittance are preserved. The emittance grows by $\Delta\varepsilon_L/\varepsilon_L \approx 4\%$ and no particles are lost from the bucket in this case.

Figure 3 summarizes the simulation results for various numbers of RF stations between $n_{RF} = 2$ and $n_{RF} = 96$ for all three RCSs. Here, the emittance growth is plotted versus n_{RF}. The black circles represent RCS1 for tracking without intensity effects, as mentioned before. For $n_{RF} = 2$ and $n_{RF} = 4$, the bunches are lost completely. For increasing values of n_{RF}, the bunch is preserved and the emittance growth decreases continuously. For $n_{RF} \geq 32$ or $Q'_s \leq 0.05$, a value of $\Delta\varepsilon_L/\varepsilon_L \approx 4\%$ is reached. Larger n_{RF} do not provide any further improvements with respect to $\Delta\varepsilon_L/\varepsilon_L$. The small fluctuations at the percent level for $n_{RF} \geq 32$ are caused by uncertainties in determining $\Delta\varepsilon_L/\varepsilon_L$. To verify that intensity effects have indeed no influence on the results of these studies, the same set of simulations for RCS1 with intensity effects were repeated and are also represented in Fig. 3 by the red triangles. Both curves follow the same trend. For RCS2 (blue squares) and RCS3 (green diamonds), results are similar for $n_{RF} \geq 32$ with $\Delta\varepsilon_L/\varepsilon_L \approx 4\%$, but

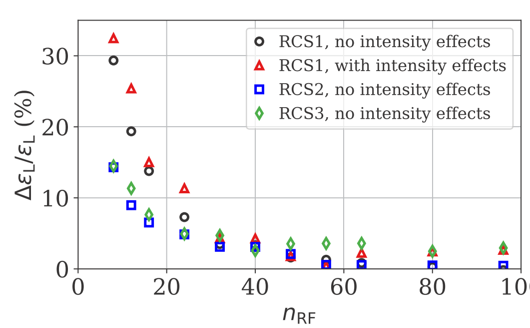

Figure 3: Relative emittance growth at the end of the cycle with respect to the emittance at injection versus n_{RF} for RCS1 without (black circle) and with intensity effects (red triangle), and for RCS2 and RCS3 without intensity effects.

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differ slightly for lower n_{RF}. For $n_{RF} = 2$, the bunches are not completely lost, but the emittance growth would be large, around 100%, with similar results for $n_{RF} = 4$ (for a better visibility, these data points are omitted from the plot). An emittance growth of $\Delta\varepsilon_L/\varepsilon_L = 5\%$ is reached for $n_{RF} = 24$. Due to the smaller Q_s of 0.50 and 0.34 at injection, this behavior is also expected.

Based on these results, the number of required RF stations should be be around $n_{RF} \approx 32$ for RCS1 and around $n_{RF} \approx 24$ for RCS2 and RCS3.

BEAM-INDUCED VOLTAGES

To include intensity effects, short-range and fundamental beam loading contributions are treated separately. The short-range wakefields are computed following the theory of K. Bane [10]. Their longitudinal wake function $W_{L,SR}$ can be approximated for bunch lengths small with respect to the cavity cell length by

$$W_{L,SR} = \frac{Z_0 c}{\pi a^2} \exp\left(-\sqrt{s/s_0}\right) , \quad (3)$$

with $Z_0 = 377\,\Omega$, c the speed of light in vacuum, $a = 35$ mm the iris radius in the cavity, s the longitudinal coordinate, and s_0 a geometry-depended constant. With the corresponding parameters for the TESLA cavity, W_L reaches 30 V/pC/m. The resulting induced voltage per cavity V_{ind} for RCS1, $n_{RF} = 32$, and a bunch length at injection of $1\sigma = 13.1$ mm is plotted versus time in Fig. 4, together with the charge line density $\lambda(t)$ (in arb. units, light-blue dotted line). Time $t = 0$

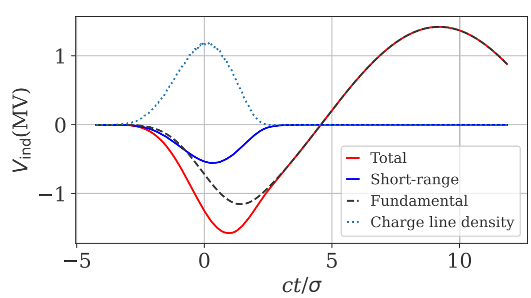

Figure 4: Normalized bunch charge density (light-blue dotted) and induced voltages per cavity from single-turn effects in RCS1 versus time normalized to $1\sigma/c$. The voltages from short-range wakefields (dark blue) and beam loading at the fundamental frequency (black dashed) are added up (red).

marks the bunch center. The short-range induced voltage shows a maximum amplitude of 0.55 MV per cavity. Since the length of the 9-cell cavity is 1.0 m, the induced potential per cavity is also 0.55 MV/m.

For the contribution from the fundamental mode beam loading, the induced voltage is directly computed from f_{RF} by a resonator model [4] that receives as input the total shunt impedance calculated from R/Q and Q_L, with their values from above. The induced voltage reaches 1.15 MV/m (Fig. 4, black dashed line).

The total induced voltage (red line), i.e., the sum of both, reaches up to 1.56 MV/m fo a single bunch passage. However, due to the large V_{RF}, this corresponds to only 6%, and has negligible impact on the bucket area and beam stability as shown by Fig. 3 (red triangles). The longitudinal particle distribution looks very similar to that in Fig. 2 (b).

HOM POWER LOSSES

Induced voltages on the MV scale are expected to also induce large power into the HOM of the cavity. The product of $I_b \cdot V_{ind} \approx 11.2$ kW can serve as an upper limit estimate. We calculate more precise values from the short-range wake $W_{L,SR}$ that contains information about all modes. The average HOM power per bunch during acceleration is calculated in the simulation as [11]

$$P_{HOM} = -k_{||,SR} \cdot \frac{q^2}{T_{rev}} , \; k_{||,SR} = \int W_{L,SR}(t) \cdot \lambda(t) . \quad (4)$$

Here, $k_{||,SR}$ represents the longitudinal loss factor, q the bunch charge and T_{rev} the revolution period which is identical to the bunch spacing for a single bunch. For RCS1 and the previously mentioned parameters, we obtain $P_{HOM} = 7.6$ kW, which is consistent with the previous upper limit estimate. For RCS2 and RCS3, values are $P_{HOM} = 10.4$ kW and $P_{HOM} = 6.0$ kW, respectively. Due to the q^2 scaling, direct bunch crossings inside cavities must be avoided as twice the total charge could lead to four times larger power losses. To handle these peak power losses, high-power HOM couplers are required. Designs with up to five HOM couplers per cavity, each with a CW capacity of 3 kW to 4 kW, are under development [12].

CONCLUSION

We presented the RF parameters for the ultra-fast acceleration of muons in a chain of rapid-cycling synchrotrons. Energy gains up to 15 GeV per turn, only tens of turns and high bunch charges lead to unique RF requirements and longitudinal beam dynamics. High synchrotron tunes up to $Q_s = 1.5$ require around 30 distributed RF stations per ring to assure stability of the longitudinal synchrotron motion. The induced voltages due to the high bunch charge were calculated for single-turn effects and are on the order of MV per cavity, but do not harm beam transport. As a consequence of the large induced voltages, also large (tens of kW) HOM power losses are expected. Future studies will be devoted to the impact of this strong beam loading on the choice of the RF frequency.

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