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DESIGN AND OPTIMISATION OF AN 800 MHz 5-CELL ELLIPTICAL SRF CAVITY FOR $t\bar{t}$ WORKING POINT OF THE FUTURE CIRCULAR ELECTRON-POSITRON COLLIDER

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Abstract

The Future Circular electron-positron Collider (FCC-ee) is planned to operate with beam energies from 45.6 to 182.5 GeV and beam currents from 5 to 1400 mA [1]. The purpose is to study the properties of the Z, W and Higgs bosons and the top and anti-top quarks. This study presents a novel 5-cell 800 MHz elliptical SRF cavity design optimised for minimising higher-order mode impedance. The design differs from the existing 5-cell cavity proposed for FCC-ee, which prioritises minimising peak surface fields. In addition to $t\bar{t}$ working point, the proposed design may also have advantages for use in the Z machine booster and H working point due to its ability to handle high beam currents.

INTRODUCTION

The FCC-ee aims to operate at four distinct operating points characterised by different beam currents and RF voltages. To study the properties of the top and anti-top quarks, beams of 5 mA current are to be accelerated to an energy level of 182.5 GeV before collision. 5-cell elliptical accelerating cavities made of bulk niobium, operating at 2K with a fundamental mode (FM) frequency of 801.58 MHz, is being considered for use in $t\bar{t}$ [2]. In [3], a 5-cell SRF elliptical cavity (FCCUROS5) was designed mainly to minimise peak surface fields, enabling the achievement of high E_{acc} and low surface losses for $t\bar{t}$ operation.

This contribution explores the potential for enhancing the performance of the cavity design for the $t\bar{t}$ operating point, with a specific focus on addressing the impacts of higher-order modes (HOMs). Improving the HOM characteristics of the 5-cell 800 MHz cavity may make it feasible to use this cavity at the H working point. Moreover, with the Z booster operating at a high beam current of 140 mA, enhancing the HOM characteristics of the 5-cell cavities would allow the use of the same type of 5-cell 801.58 MHz cavities for the boosters of all four operating points. This study also compares the properties of the designed cavity geometry to the FCCUROS5 cavity geometry and the TESLA cavity geometry [4] scaled to 801.58 MHz with its number of cells reduced from 9 to 5.

SRF CAVITY DESIGN GOAL

Several figures of merit quantify the performance of accelerator cavities. The FM peak electric and magnetic fields on the surface of the cavity normalised to the accelerating gradient, i.e. $E_{\text{pk}}/E_{\text{acc}}$ and $B_{\text{pk}}/E_{\text{acc}}$, geometric shunt impedance R/Q are the figures of merit associated with the FM. The longitudinal loss factor $k_{\parallel}$ and the transversal kick factor $k_{\perp}$ quantify the HOM effects. These figures of merit are strongly dependent on the cavity shape.

The geometric factor G, cell-to-cell coupling factor k_{cc}, and fundamental mode loss factor k_{FM} are additional figures of merit included in the presentation and discussion of results.

Figure 1 shows the parameterised surface profile of an elliptical cavity's mid-cell and end-cell geometry. Seven geometric variables - $A, B, a, b, R_{\text{i}}, L$, and R_{eq} - define the half-cell profile for the illustrated parameterisation. Subscripts i and e distinguish between mid-cell and end-cell dimensions. The mid-cell half-length L_{i} is set to a quarter wavelength, and the equator radius R_{eq} is selected for frequency tuning due to its strong influence on the FM frequency.

Accelerator cavities are optimised for FM operation by minimising $E_{\text{pk}}/E_{\text{acc}}$ and $B_{\text{pk}}/E_{\text{acc}}$ and maximising R/Q which is equivalent to minimising $-R/Q$. Low peak surface fields allow for an increase in the cavity's accelerating gradient without reaching the surface field emission or thermal breakdown limits.

To quantify the effects of the HOMs, the maximum HOM impedance in defined frequency intervals is used in this study to assess the worst-case scenario rather than relying on the loss and kick factors which represent the average effect of all HOMs. Minimising the maximum impedances is essential to optimise the cavity, which in turn helps to decrease $k_{\parallel}$ and $k_{\perp}$.

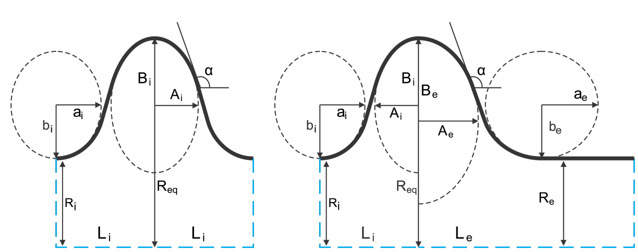

Figure 1: Parameterised mid-cell (left) and end-cell (right) of an elliptical cavity.

The optimisation problem is stated as follows:

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$$\begin{aligned}
\min_{\mathbf{x}_i, \mathbf{x}_e \in \mathbf{X}} \quad & \left(\frac{E_{\mathrm{pk}}}{E_{\mathrm{acc}}}, \frac{B_{\mathrm{pk}}}{E_{\mathrm{acc}}}, -R/Q, |Z_{\parallel,p}|, |Z_{\perp,q}|\right) \\
\text{s.t.} \quad & R_{\mathrm{eq}}/\mathrm{mm} = \underset{R_{\mathrm{eq}}}{\arg\min} \quad |f(R_{\mathrm{eq}}) - f_{\mathrm{FM}}|, \\
& L_{\mathrm{e}}/\mathrm{mm} = \underset{L_{\mathrm{e}}}{\arg\min} \quad |f(L_{\mathrm{e}}) - f_{\mathrm{FM}}|, \\
& A_{\mathrm{i,e}}, B_{\mathrm{i,e}}/\mathrm{mm} \in [20, 80], \\
& a_{\mathrm{i,e}}, b_{\mathrm{i,e}}/\mathrm{mm} \in [10, 60], \\
& R_{\mathrm{i,i,e}}/\mathrm{mm} \in [60, 85], \\
& L_{\mathrm{i}}/\mathrm{mm} = 93.5, \\
& \alpha_{\mathrm{i}} \geq 90^\circ, \\
& \frac{E_{\mathrm{pk}}}{E_{\mathrm{acc}}} \leq 3, \\
& \frac{B_{\mathrm{pk}}}{E_{\mathrm{acc}}} / \frac{\mathrm{mT}}{\mathrm{MV/m}} \leq 5,
\end{aligned} \tag{1}$$

where $\mathbf{x}_i = [A_{\mathrm{i}}, B_{\mathrm{i}}, a_{\mathrm{i}}, b_{\mathrm{i}}, R_{\mathrm{i,i}}]$, $\mathbf{x}_e = [A_{\mathrm{e}}, B_{\mathrm{e}}, a_{\mathrm{e}}, b_{\mathrm{e}}, R_{\mathrm{i,e}}]$ are design points, and $\mathbf{X} \subset \mathbb{R}^5$ is the feasible set of geometric variables, $f_{\mathrm{FM}} = 801.58$ MHz, and $p, q \in \{1, 2, 3\}$. The maximum interval impedances are defined mathematically as follows:

$$\begin{aligned}
|Z_{\parallel,1}| &= \max_{f/\mathrm{GHz} \in [1,2]} |Z_{\parallel}(f)|, \\
|Z_{\parallel,2}| &= \max_{f/\mathrm{GHz} \in [2,4]} |Z_{\parallel}(f)|, \\
|Z_{\perp,1}| &= \max_{f/\mathrm{GHz} \in [0.7,1.3]} |Z_{\perp}(f)|, \\
|Z_{\perp,2}| &= \max_{f/\mathrm{GHz} \in [1.3,2.25]} |Z_{\perp}(f)|, \\
|Z_{\perp,3}| &= \max_{f/\mathrm{GHz} \in [2.25,4]} |Z_{\perp}(f)|.
\end{aligned} \tag{2}$$

The critical surface magnetic field of niobium at 2 K, approximately 200 mT [3, p. 55], poses a stringent limit for this operating point. As a result, the optimisation process is constrained by the requirement that $B_{\mathrm{pk}}/E_{\mathrm{acc}} \leq 5$ mT/MV/m considering a conservative peak magnetic field value of 120 mT at E_{acc}=24 MV/m. Additionally, a constraint is placed on the maximum surface electric field by imposing the condition $E_{\mathrm{pk}}/E_{\mathrm{acc}} \leq 3$.

OPTIMISATION METHOD

A Pareto-based genetic algorithm is used to obtain a family of optimal solutions in a Pareto sense. A half-cell geometry is considered Pareto optimal if there is no way to enhance one objective without causing a deterioration in at least one other objective. The pseudo-code is given in Algorithm 1. The SLANS [5] code, a 2D code that calculates the monopole modes of RF cavities, is used for cavity frequency tuning and calculating $E_{\mathrm{pk}}/E_{\mathrm{acc}}$, $B_{\mathrm{pk}}/E_{\mathrm{acc}}$, and R/Q. $|Z_{\parallel,p}|$ and $|Z_{\perp,q}|$ were obtained from wakefield simulations using ABCI [6]. The optimisation procedure is divided into optimising the mid-cell first and then optimising the end-cell to reduce the computational cost.

Algorithm 1 Pareto-based Genetic Algorithm

```
Initialisation: X = (x_1, ..., x_Q), n = 0, v_t;
    ▹ First generation, v_t : hypervolume threshold value.
while n < N do                                  ▹ Generation loop
    for x = x_1 ..., x_Q do                     ▹ Half-cell geometry loop
        run SLANS, ABCI
            ▹ for freq. tuning, E_pk/E_acc, B_pk/E_acc, R/Q, |Z_∥,p|, |Z_⊥,q|
    P_n ← Evaluate and interpolate Pareto hypersurface
    if n > 0 then
        Evaluate hypervolume v between P_n and P_{n-1}
        if v < v_t then
            break
    do Ranking and Selection, Crossover, Mutation
    do Introduce random half-cell geometries
    X ← newly created geometries
return P_n
```

RESULTS AND DISCUSSION

The geometric properties of the cavity selected from the Pareto family of cavities are given in Table 1. The optimised cavity geometry is hereafter referred to as the C3795. Table 2 gives a result summary for the C3795, FCCUROS5 and TESLA cavities. The bar plot in Fig. 4 displays the normalised figures of merit for the three cavities, allowing for easy comparison of their differences. C3795 has a 16% lower R/Q compared to the FCCUROS5 and a 24% lower R/Q compared to the TESLA cavity. Additionally, the geometric factor of C3795 is around 4% lower than that of the FCCUROS5 and TESLA cavities, which implies a 4% reduction in Q_0 for the same material surface resistance R_{s}. In terms of peak field ratios, C3795 has a 16% greater $E_{\mathrm{pk}}/E_{\mathrm{acc}}$ and an 11% greater $B_{\mathrm{pk}}/E_{\mathrm{acc}}$ than the FCCUROS5. Compared to the TESLA cavity, there is a 7% increase in peak electric field ratio and a 12% increase in peak magnetic field ratio.

The designed cavity performs worse than the FCCUROS5 and TESLA cavities considering the FM figures of merit except in k_{cc}. This is expected since the C3795 has a larger mid-cell iris radius which is strongly negatively correlated to the FM figures of merit, except in k_{cc}, which is positively correlated.

Figures 2 and 3 show the longitudinal and transverse impedance plots of C3795, FCCUROS5 and TESLA cavities. The plots show the reduced longitudinal and transversal HOM impedance peaks of C3795 compared to the FCCUROS5 and TESLA cavities. The decrease in impedance peaks is also apparent in the decrease in the loss and kick factors. For the $\mathrm{t\bar{t}}$ synchrotron radiation(SR) bunch length ($\mathrm{t\bar{t}}_\sigma = 1.67$ mm), the loss factor of the FCCUROS5 cavity geometry is about 40% more than that of C3795. Its HOM power is 50% more than the HOM power of the designed cavity for the $\mathrm{t\bar{t}}$ operating point. The kick factor of the FCCUROS5 is 76% greater than that of C3795. In general, there is a notable enhancement in the HOM performance of the C3795 compared to the FCCUROS5 cavity geometry.

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Table 1: Optimised Geometric Parameters for the C3795 Cavity Geometry

C3795 - Geometric properties							
A_i/A_e [mm]	B_i/B_e [mm]	a_i/a_e [mm]	b_i/b_e [mm]	$R_{i,i}/R_{i,e}$ [mm]	L_i/L_e [mm]	R_{eq} [mm]	α_i/α_e [°]
62.22/62.58	66.13/57.54	30.22/17.21	23.11/12.00	72.00/80.00	93.50/93.795	171.20	94.50/112.40

Table 2: Summary of Some Figures of Merit of the C3795, FCCUROS5 and TESLA Cavities

	FM figures of merit						HOM figures of merit ($t\bar{t}_\sigma$ = 1.67 mm)		
	R/Q_{FM} [Ω]	G [Ω]	k_{cc}* [%]	E_{pk}/E_{acc} [·]	B_{pk}/E_{acc} [mT/(MV/m)]	$\lvert k_{FM}\rvert$ [V/pC]	$\lvert k_\parallel\rvert$ [V/pC]	$k_\perp$ [V/pC/m]	P_{HOM}/cav [W]
C3795	448.46	263.61	2.64	2.43	4.88	0.564	2.639	2.014	748.64
FCCUROS5	521.06	272.93	2.04	2.05	4.33	0.656	3.784	3.539	1132.66
TESLA**	560.21	271.65	1.71	2.00	4.20	0.703	3.641	4.323	1063.05

*$k_{cc} = 2 \times \frac{f_\pi - f_0}{f_\pi + f_0}$ %; **5-Cell cavity scaled to operate at 801.58 MHz FM constructed using the TESLA-cavity geometric parameters.

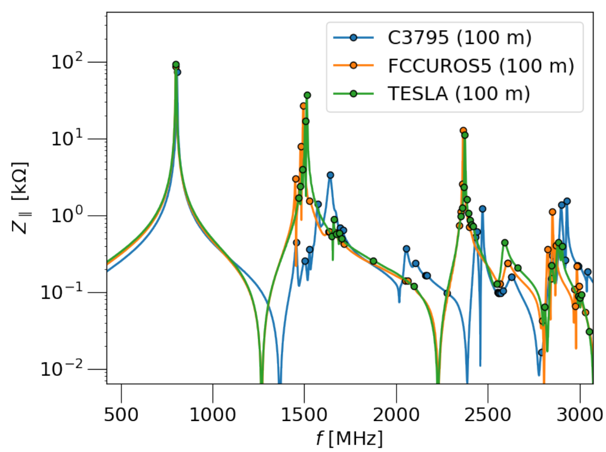

Figure 2: Longitudinal wakefield impedance of the C3795, FCCUROS5 and TESLA cavity geometries.

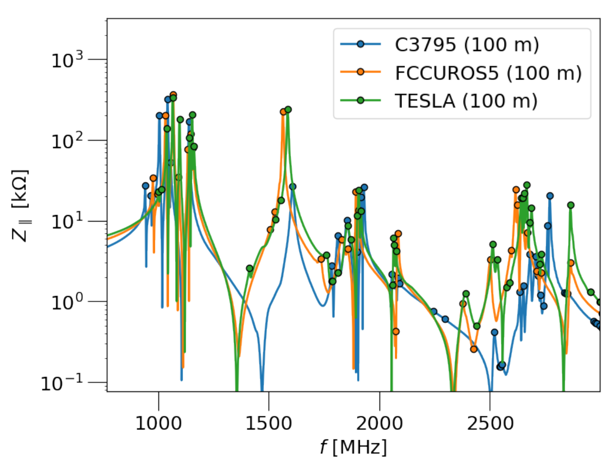

Figure 3: Transversal wakefield impedance of the C3795, FCCUROS5 and TESLA cavity geometries.

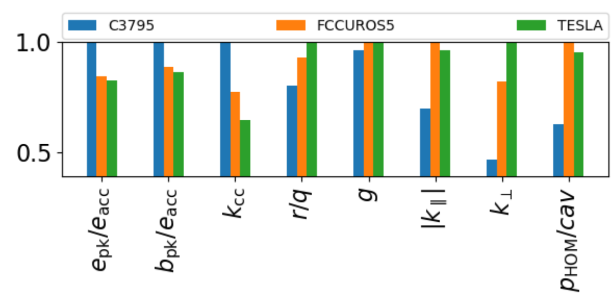

Figure 4: Normalised bar plot of figures of merit for the C3795, FCCUROS5 and TESLA cavity geometries.

CONCLUSION

An optimised 5-cell elliptical cavity geometry (C3795) was developed specifically for the FCC-ee $t\bar{t}$ operating point and compared against the FCCUROS5 and TESLA cavity geometries. In conclusion, our study demonstrates that the C3795 cavity design outperforms the FCCUROS5 cavity in terms of HOM power, kick factor, and HOM impedance peaks. While the FCCUROS5 cavity remains a viable option for the $t\bar{t}$ operating point and for boosters where high E_{acc} is favoured, the C3795 cavity is better suited for machines where higher beam current limits using 5-cell cavities, such as the booster of Z working point and possible operation of the H machine with 5-cell 800 MHz cavities.

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