MPS/SM/Note 74-3
3 January 1974

# DESIGN OF THE COMPACT QUADRUPOLE ($\gamma$ TRANSITION) POLE PROFILE

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## 1. INTRODUCTION

The pole profile of the compact quadrupole has been designed with the help of two computer programs, namely, MAGNET[1)] and POISSON[2)]. The stored energy in the magnet, computed by POISSON and approximate formulae, has a good agreement.

## 2. DESIGN OF MAGNETIC CIRCUIT

### 2.1 Required magnetic field

The required strength of the compact quadrupole is:

$$\int g \, d\ell = 0.3 \text{ T} . \tag{1}$$

To accommodate the normal PS ceramic chamber, the pole aperture of 61 mm is sufficient. Keeping in view that the stray field effect is more pronounced with shorter yoke length, a reasonable value of 140 mm has been chosen for the yoke length (effective length $\simeq$ 190 mm),

$$\therefore \qquad g = \frac{0.3 \text{ T}}{0.19 \text{ m}} = 1.576 \text{ T/m} .$$

This gradient of the field can be obtained by the ampere-turn

$$NI = \frac{gR^2}{2\mu_0} = 2320 \text{ A} ,$$

where $g = 1.576$ T/m, $R = 0.061$ m.

### 2.2 Pole shaping

It has been observed that the pole width, the iron length and the aperture of the magnet are some of the guiding factors for fringing field. Instead of using a three-dimensional magnet program, the characteristics of existing quadrupoles have been considered. The pole profile has then been designed such that at 80% of the aperture, the gradient is overdone by 7% so as to compensate for the fringing field effect. The pole shape is obtained with the segment of a circle of radius 70 mm and straight lines near the coil windows (Fig. 1). Figure 2 shows the details of the magnet. The program MAGNET has been used to obtain the desired pole shape with the trial and error method. The same has been checked by POISSON. The shim (straight lines) as calculated by MIRT is shown in Fig. 3. Figures 4 and 5 show, respectively, the basic mesh for the air region and the flux lines in the magnet. The flux lines in the presence of three holes (per quadrant) in the magnet can be seen in Fig. 6. The variation of gradient of the field along the x and y axes is shown in Fig. 7, and Fig. 8 shows the measured gradient variation along the x axis for one of the quadrupoles.

2.3 Magnetic (stored) energy

In a true quadrupole, in rectangular coordinates,

$$\frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} = g_x = g_y = g$$

$$B_x = g\ y$$

$$B_y = g\ x$$

$$B^2 = g^2(x^2 + y^2)$$

$$\frac{\text{Stored energy}}{\text{Unit length}} = \frac{W_i}{\ell} = \frac{1}{2}\,Li^2 = \frac{1}{2\mu_0}\iint B\ dx\ dy\ .$$

The energy can be calculated by using Eq. (A.10) (see Appendix):

$$\frac{W_i}{\ell} = g^2\ (R\ a)^2\left[1 + \frac{1}{12}\left(\frac{R}{a}\right)^4 + \frac{1}{3}\left(\frac{R}{a}\right)^2\frac{b}{h}\right] \times 10^5 \text{ per octant.}$$

$$\frac{W}{\ell} = 63\ \mathrm{J/m}$$

$$\frac{W}{\text{lens}} = 12\ \mathrm{J}$$

$$L = \frac{2W}{i^2}$$

$$= 0.3\ \mathrm{mH\ for\ } i = 290\ \mathrm{A}\ (NI = 2320,\ N = 8)\ .$$

The stored energy of the magnet has been calculated from POISSON and is found to be equal to 63.5 J/m.

## 3. MAIN CHARACTERISTICS OF THE QUADRUPOLE

| | |
|---|---|
| $\int g\, d\ell$ | = 0.3 T |
| Nominal gradient | = 1.576 T/m |
| Useful aperture | = 120 mm |
| Inscribed circle diameter | = 122 mm |
| Iron length | = 140 mm |
| Over-all length | = 234 mm |
| Stored energy | = 12 J |
| Inductance | = 0.3 mH |
| Resistance | = 7.3 m$\Omega$ |
| Time constant | = 40 msec |
| Total ampere-turn | = 2320 A |
| Number of turns | = 8 |

## 4. OTHER CONSIDERATIONS

Though the pole profile has been designed to take into account the effect of fringing field, provision has been made for final shimming by means of adjustable iron screws on the stainless steel end plates. The tolerance of the profile dimensions is ±0.05 mm. The stampings are made of thin laminations of 1 mm thickness and are clamped together by three bolts per quadrant as shown in Fig. 2.

## 5. COMMENTS

Eight such quadrupoles have been constructed and a quick measurement on each of these magnets showed satisfactory results. Seven of these magnets have been installed in the synchrotron ring to fit in the gamma-transition jump system. The performance of these lenses satisfies the requirements of the system.

* * *

## REFERENCES

1) C. Iselin, Two-dimensional magnetic field including saturation (MAGNET), Long Write-up T600.

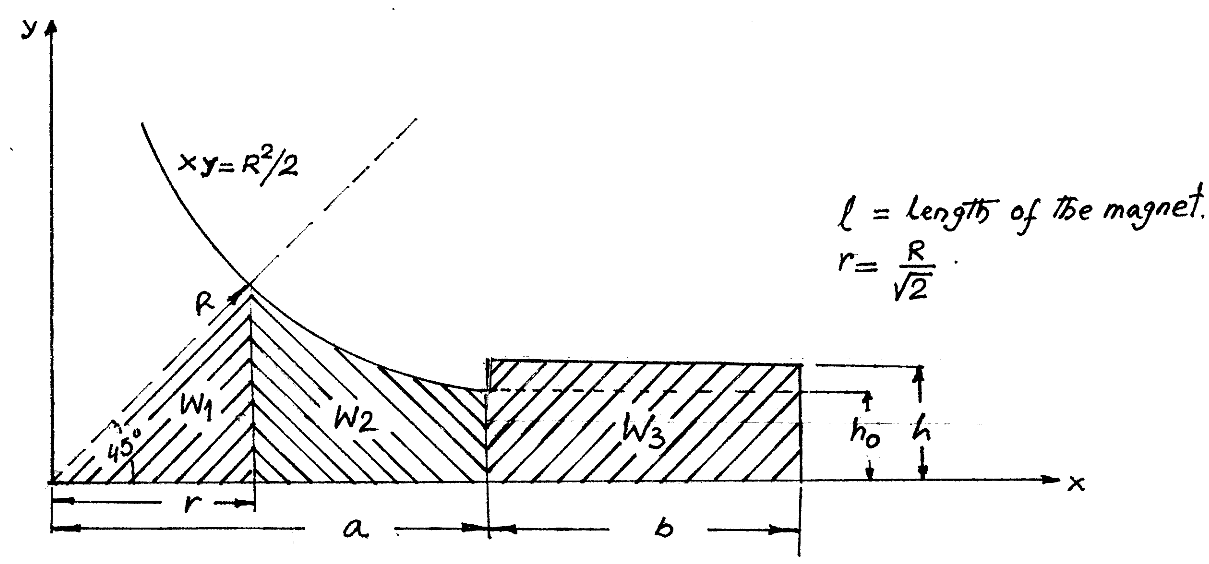
2) R.F. Holsinger, Lattice, Poisson, and Mirt computer program.

Distribution (open)

MPS/SR/AM

APPENDIX

## CALCULATION OF STORED MAGNETIC ENERGY IN A QUADRUPOLE




We shall assume a symmetric quadrupole for the analysis.

In Cartesian coordinate system:

$$B_x = g_x\, y \tag{A.1}$$

$$B_y = g_y\, x \tag{A.2}$$

Since, in a true quadrupole, $g_x = g_y = g$

$$B = g\,\sqrt{x^2 + y^2}\ . \tag{A.3}$$

The stored energy

$$\frac{W}{\ell} = \frac{1}{2\mu_0} \iint B^2\ dx\ dy$$

$$\frac{W}{\ell} = \frac{g^2}{2\mu_0} \iint (x^2 + y^2)\ dx\ dy\ . \tag{A.4}$$

From the above figure, $W/\ell = (W_1 + W_2 + W_3)/\ell$

$$\frac{W_1}{\ell} = \frac{g^2}{2\mu_0} \int_{y=0}^{x} \int_{x=0}^{r} (x^2 + y^2)\,dx\ dy = \frac{g^2}{2\mu_0}\left(\frac{r^4}{4} + \frac{r^4}{12}\right) \tag{A.5}$$

$$\frac{W_2}{\ell} = \frac{g^2}{2\mu_0} \int_{y=0}^{r^2/x} \int_{x=r}^{a} (x^2 + y^2)\ dx\ dy = \frac{g^2}{2\mu_0} \left[\frac{r^2 a^2}{2} - \frac{r^4}{2} + \frac{r^4}{6} - \frac{r^6}{6a^2}\right] \tag{A.6}$$

To find $W_3$, we shall assume a linear field distribution from $a$ to $b$, i.e.

$$B_y = B_0 \frac{b - x}{b} ,$$

where $B_0 = g\ a$.

$$\frac{W_3}{\ell} = \frac{g^2}{2\mu_0} \frac{a^2}{b^2} \int_{y=0}^{h} \int_{x=0}^{b} (b - x)^2\ dx\ dy = \frac{g^2}{12\mu_0} R^2\ a\ b \tag{A.7}$$

for $h = h_0 = R^2/2a$.

$$\frac{W_3}{\ell} = \frac{g^2}{24\mu_0} R^4 \left(\frac{b}{h}\right) , \tag{A.8}$$

when $h \neq h_0$.

From Eqs. (A.5), (A.6), and (A.8), the total energy can be expressed as

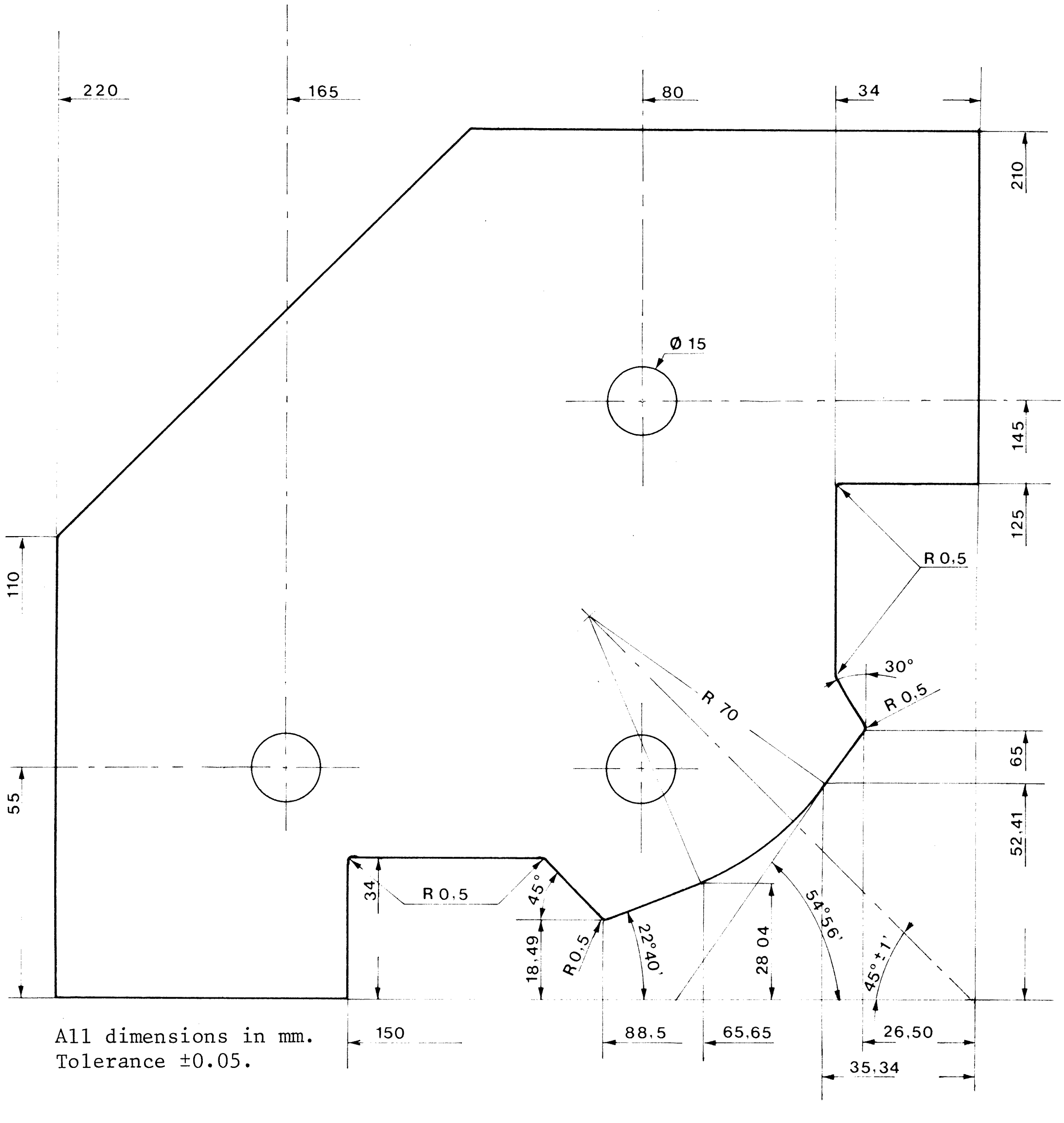
$$\frac{W}{\ell} = \frac{1}{8\mu_0} g^2\ (R\ a)^2 \left[1 - \frac{1}{12}\left(\frac{R}{a}\right)^4 + \frac{1}{3}\left(\frac{R}{a}\right)^2 \left(\frac{b}{h}\right)\right] . \tag{A.9}$$

This gives the stored energy per octant of the lens per unit length.

Substituting the value of $\mu_0 = 4\pi \times 10^{-7}$, the empirical formula for energy is given by

$$\frac{W}{\ell} \cong g^2\ (R\ a)^2 \left[1 - \frac{1}{12}\left(\frac{R}{a}\right)^4 + \frac{1}{3}\left(\frac{R}{a}\right)^2 \frac{b}{h}\right] \times 10^5\ \mathrm{J/m} , \tag{A.10}$$

where $g$ is in T/m, and $R$ and $a$ are in m.




All dimensions in mm.
Tolerance ±0.05.

Fig. 1 Compact quadrupole pole profile.

# DEMANDE DE REPRODUCTION

| Procedé : | Demandeur : | Division : | Date : | Délai |
|---|---|---|---|---|
| Offset ☐<br>Xerox ☒ | I. Winklemann | MPS | 1-9-75 | |

| Nombre d'originaux | Nombre de copies de chaque original | Description |
|---|---|---|
| 3 | 15 | RV |

Travail exécuté le : ............

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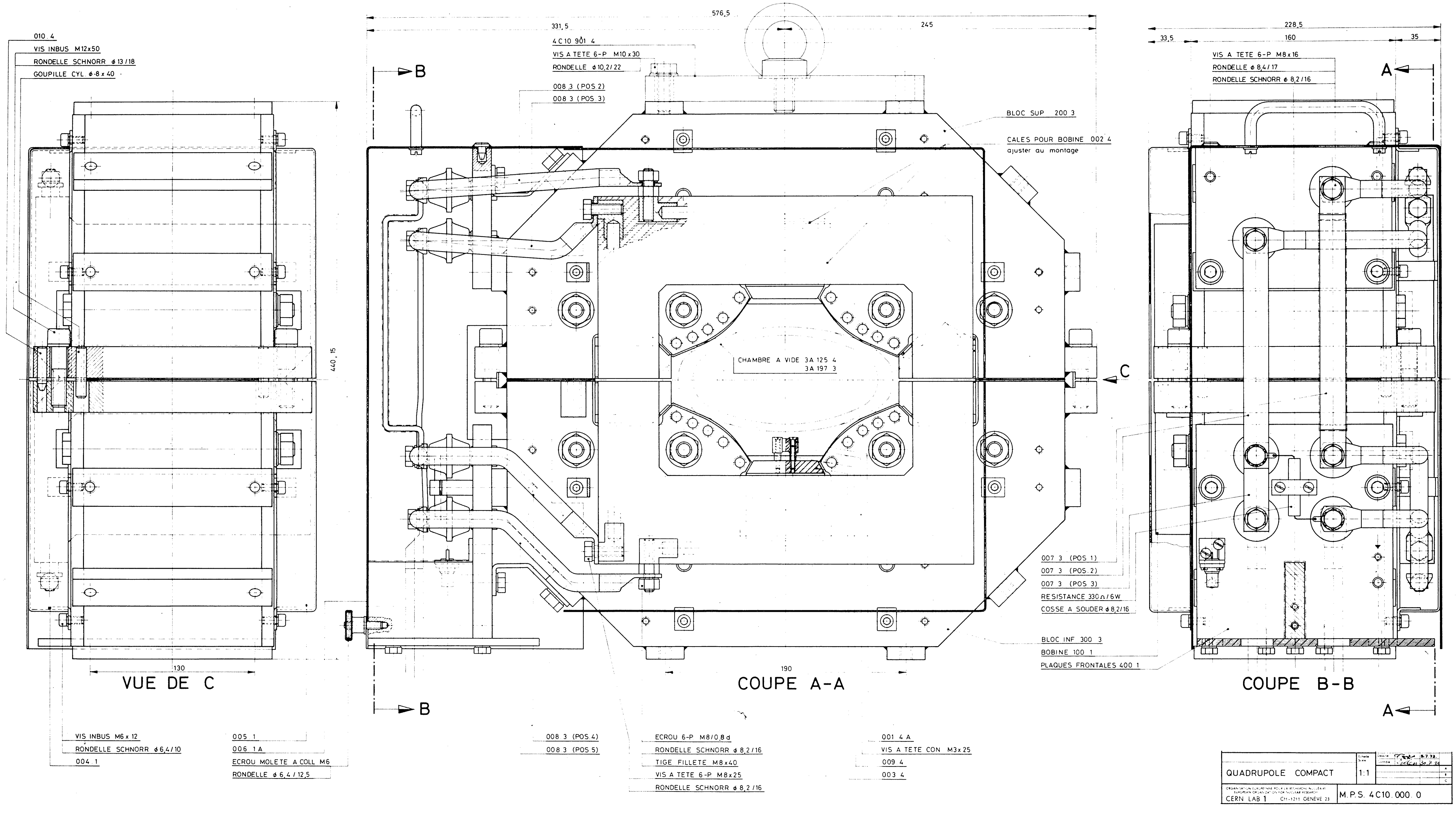


Fig. 2 Compact quadrupole

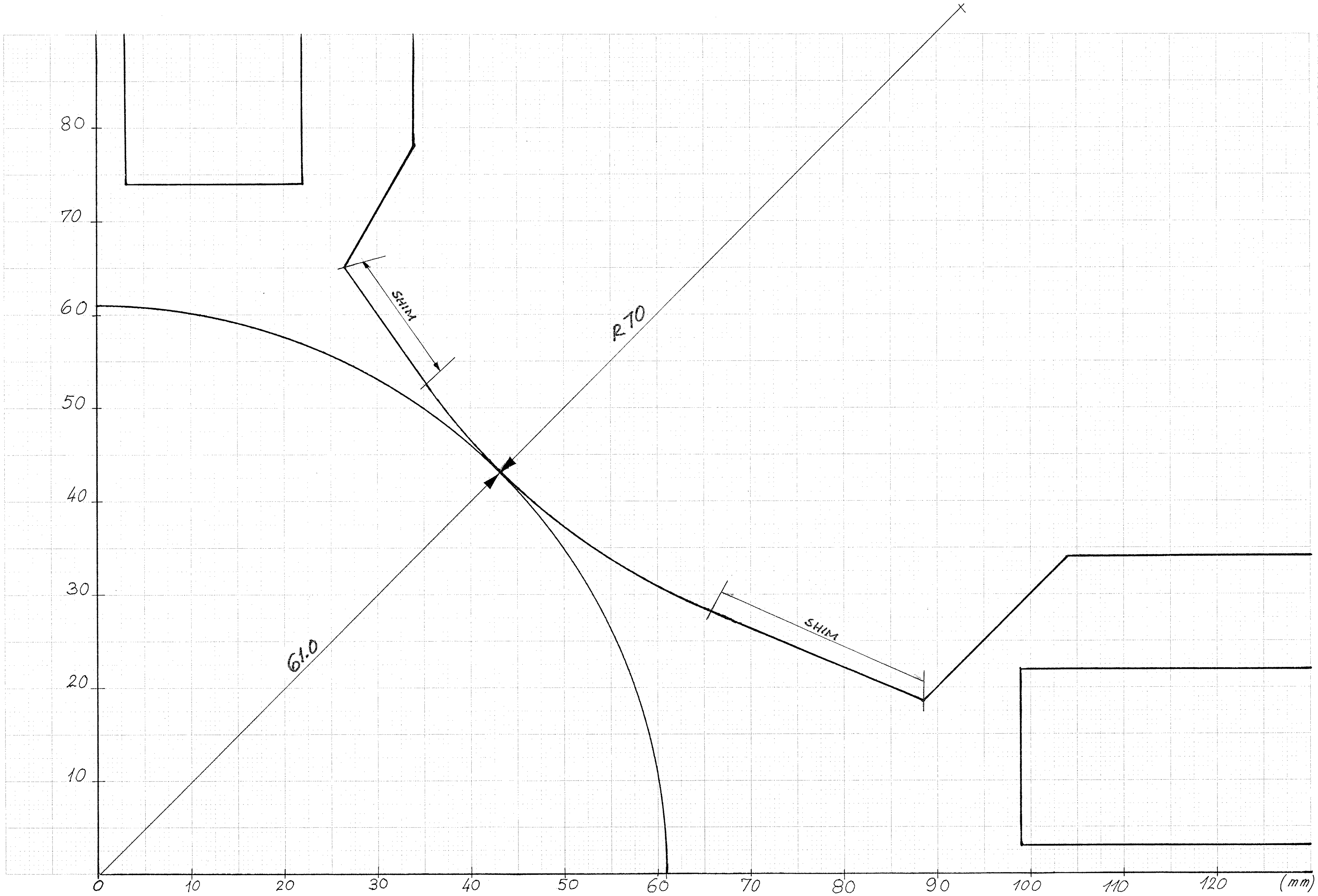


Fig. 3 Shim calculation by MIRT (optimization program of Poisson)

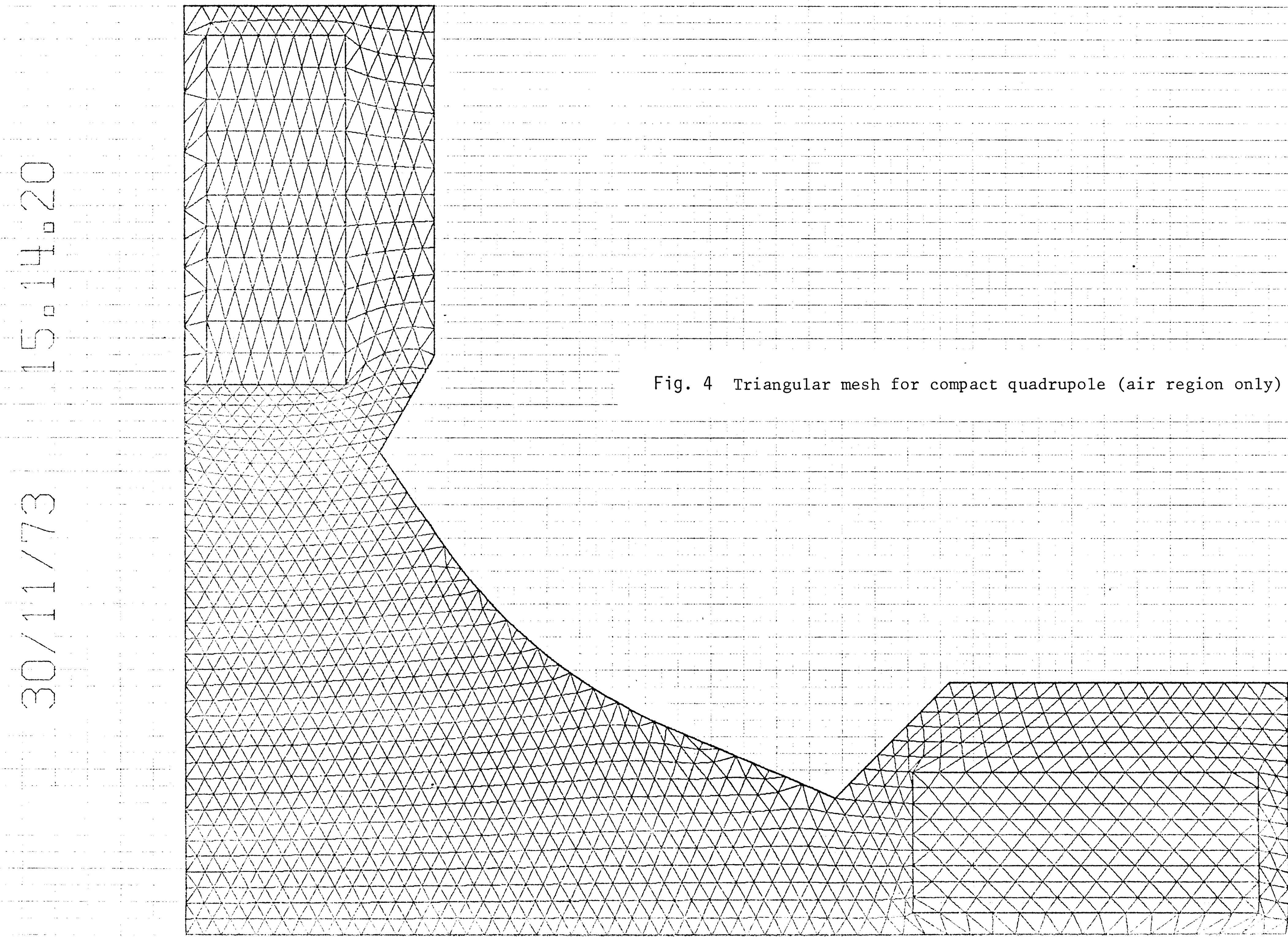


Fig. 4 Triangular mesh for compact quadrupole (air region only)

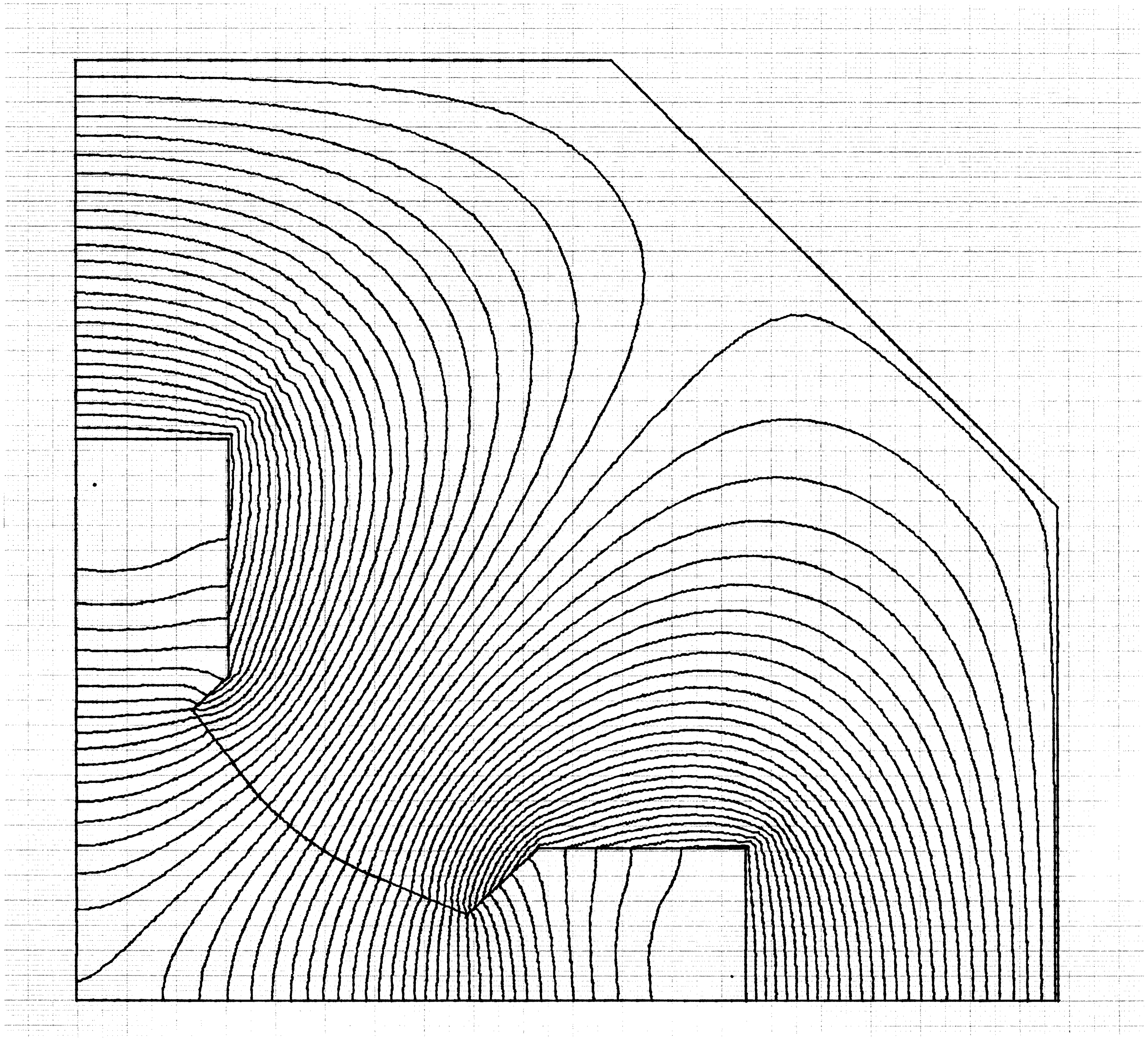

Fig. 5 Flux plot of the compact quadrupole

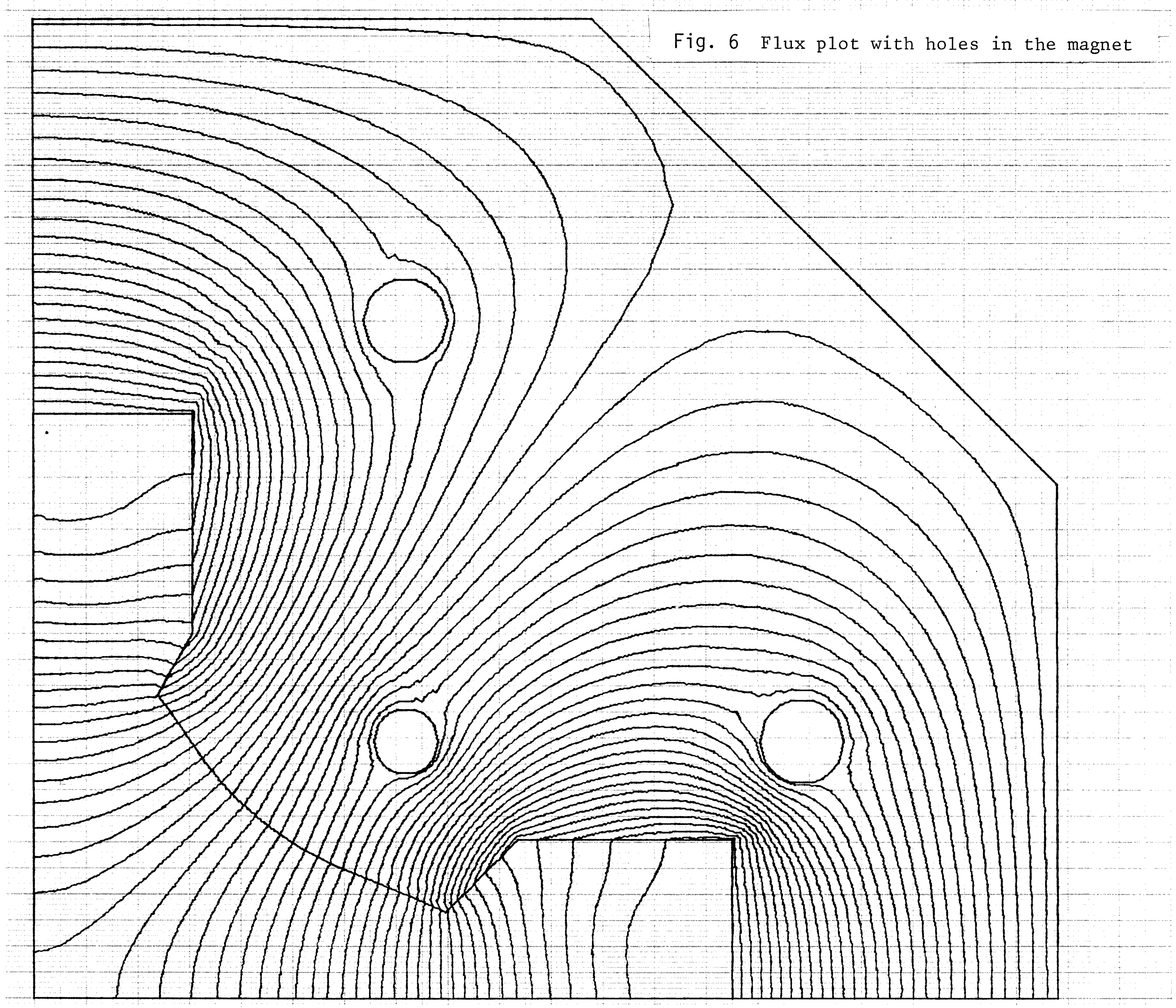

Fig. 6 Flux plot with holes in the magnet

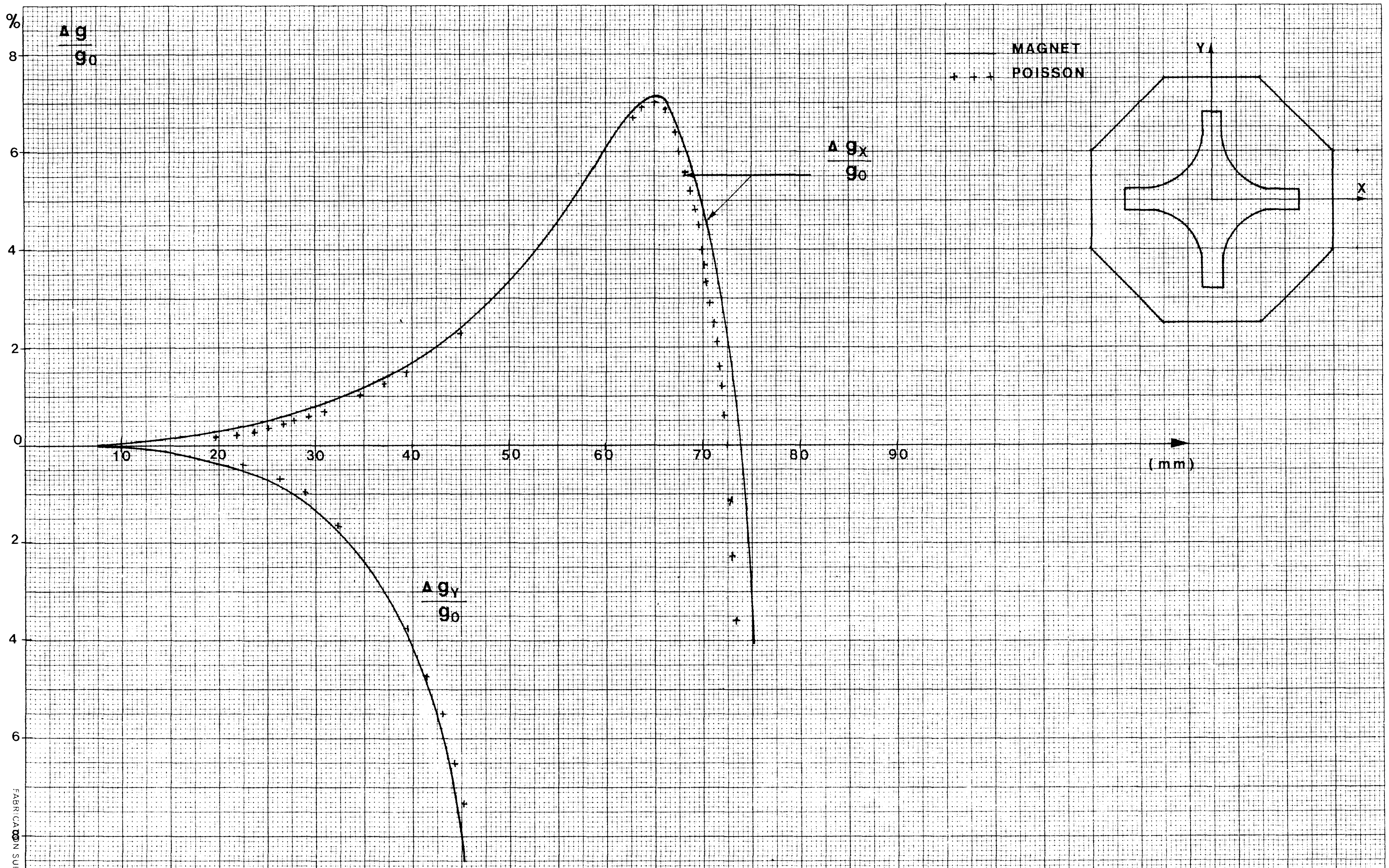


Fig.7 - Gradient error in compact quadrupole

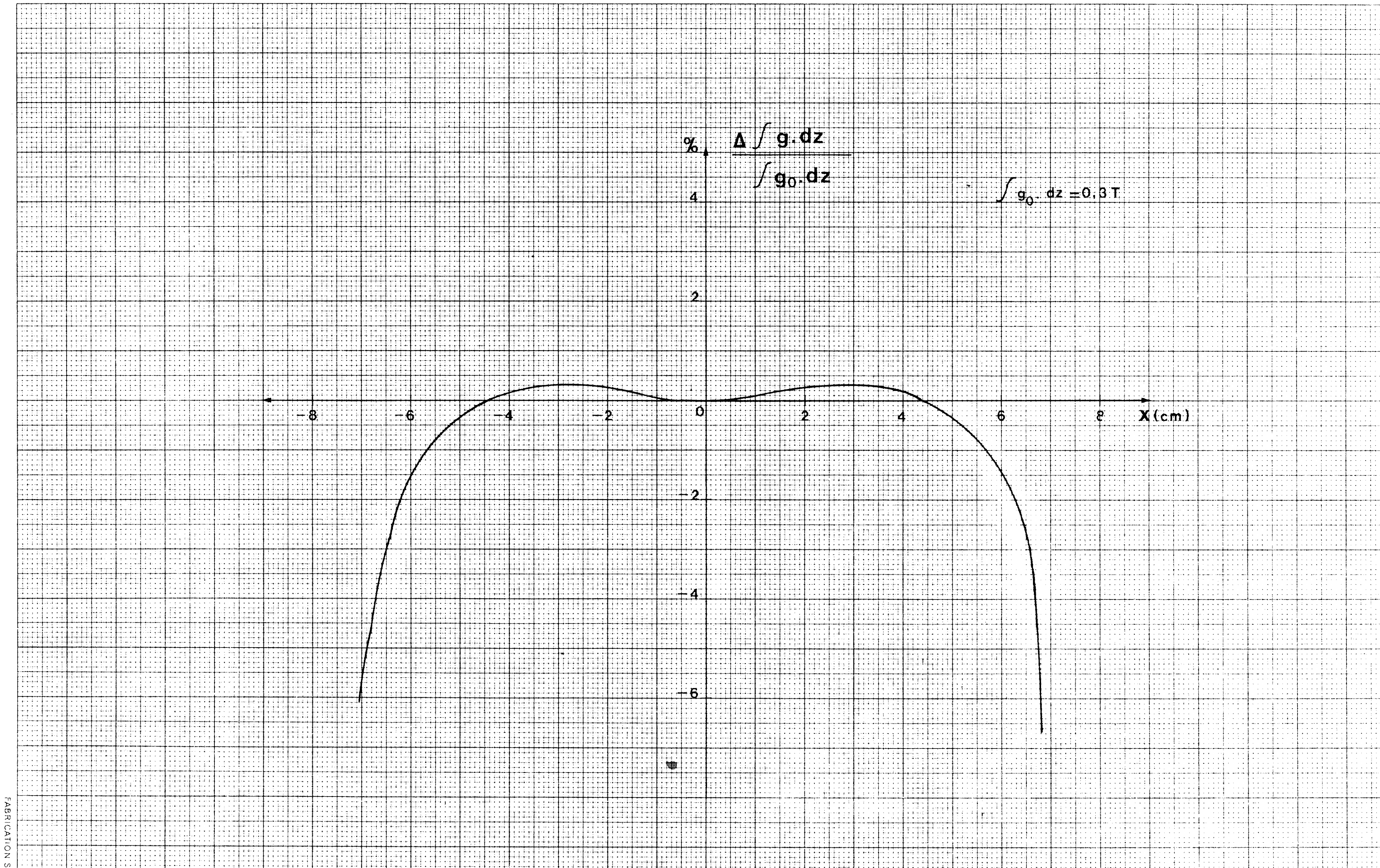


**Fig. 8 - Measured gradient error in compact quadrupole**