PS/AA/BA/afm 14 October 1982

AA LONG TERM NOTE No. 19

Summary of the meeting of October 14, 1982

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Topic : Betatron cooling in the Antiproton Collector, by B. Autin

It is assumed that a beam of 10^8 particles could be injected into the antiproton collector within an acceptance of 200 π mm.mrad. The aim of the betatron cooling is to reduce the transverse emittances, say by a factor 10 in 2 seconds. This goal seems to be achievable with a system having the following characteristics :

Pick-up or kicker structure length	2 m
Total length for horizontal and vertical systems	8 m
Frequency range	1-2 GHz
Signal to noise ratio	2
Output power	330 W
Electronic gain	140 dB
Temperature of terminating resistors	40°K
Noise temperature of the preamplifiers	40°K

As the gain of the system is far from the optimum gain in order to keep the output power to a reasonable level, the performances of the system can be improved by varying the gaps of the pick-up electrodes and of the kickers in such a way that they fit the beam size during the cooling. Under these conditions, the limit emittance of 2.5 π mm.mrad can practically be reached within 2 seconds (Figure 2).

B. Autin

Distribution

PS/2 list

Fast betatron cooling

12-10-82

1. Equation of cooling

$$\frac{d\epsilon}{dt} = \sum_{n=-\infty}^{+\infty} \sum_{\pm Q} \frac{-2G\epsilon + b_n}{(1+S_n)^2}$$

t, time

ϵ, emittance

n, harmonic number of a longitudinal Schottky band

G, relative change of particle amplitude a_n in a given betatron band or "transfer function" (see § 2)

b_n, emittance increase per unit of time in a given band for a beam of zero initial emittance excited by thermal noise (see § 3)

S_n, "beam feedback" term which can be expressed

$$S_n = \frac{NG}{f_0 \eta \frac{\Delta p}{p}} \cdot \frac{1}{n}$$

with: N, number of particles

f_0, revolution frequency

η, dispersion in revolution frequencies $\left(\frac{\Delta f}{f_0} / \left(\frac{\Delta p}{p}\right)\right)$

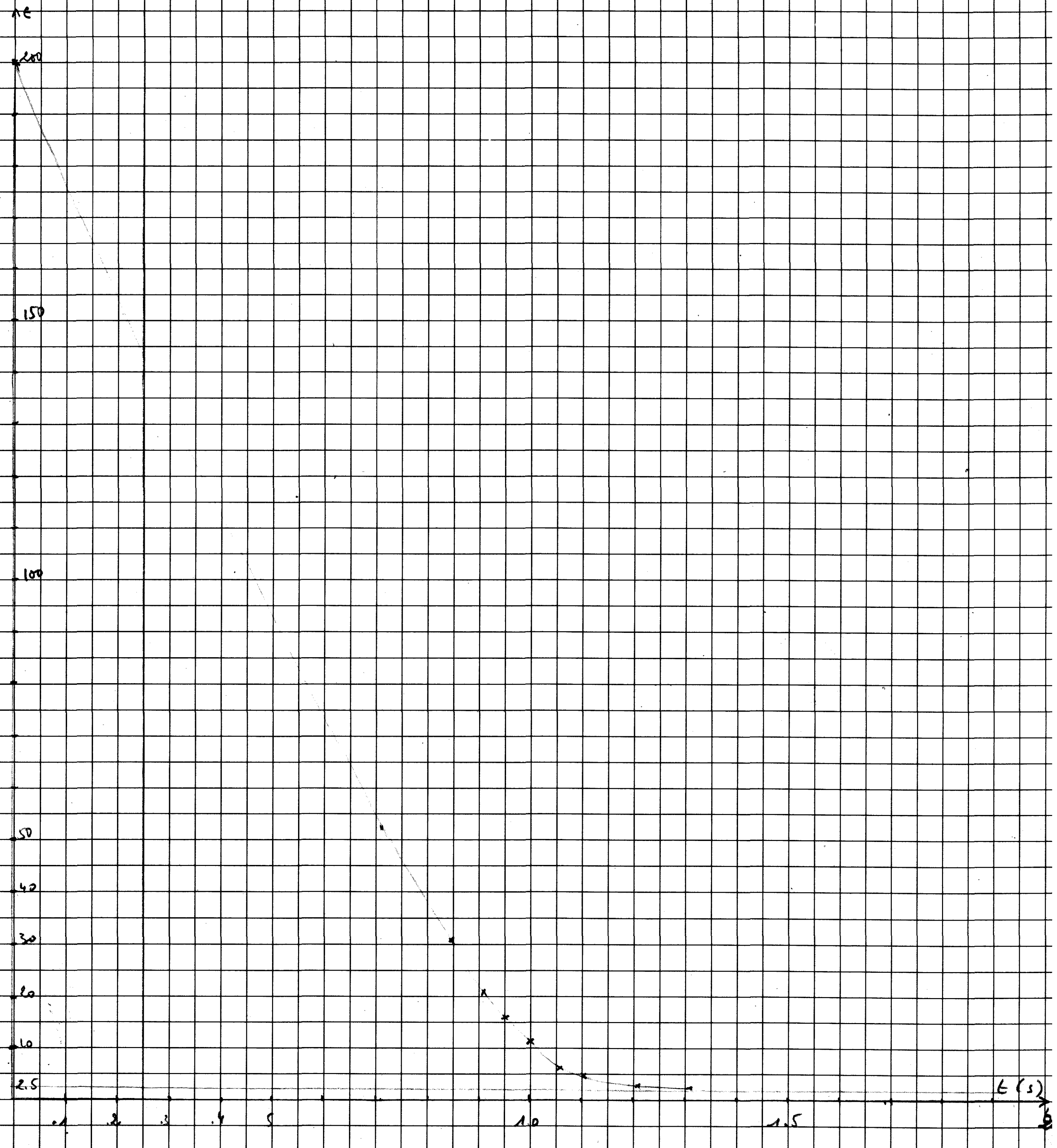
$\frac{\Delta p}{p}$, dispersion in momenta

As n is large, the summation over n can be replaced by an integration over n and:

$$\frac{d\epsilon}{dt} = -8n\,Y(x)\,G\,\epsilon + 4n\,Y(x)\,b$$

with: $x = \frac{NG}{W \eta \frac{\Delta p}{p}}$, W: bandwidth $n f_0$ over one octave

$$Y(x) = 1 - 2x \ln\frac{x+2}{x+1} + \frac{x^2}{(x+1)(x+2)}$$



c
200
150
100
50
40
30
20
10
2.5
t (s)
.1
.2
.3
.4
.5
1.0
1.5

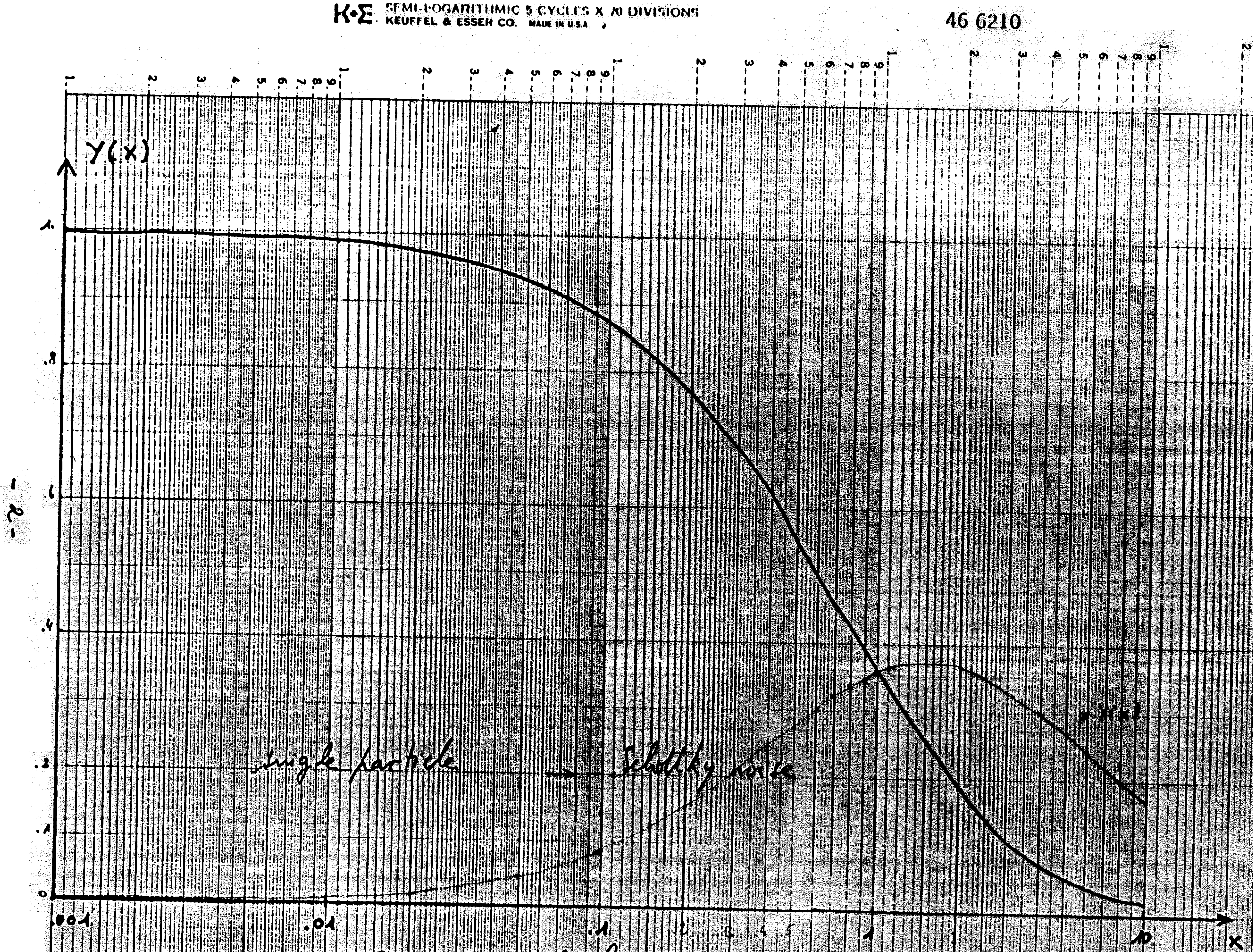


Figure 2 Cooling regimes

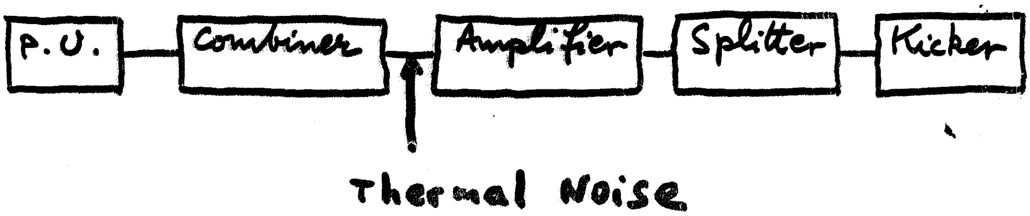
2. Transfer function "G"

By definition:

$$G = \frac{1}{a_n} \frac{d a_n}{d t}$$

or, interms of the correction per turn Δa_n:

$$G = f_0 \frac{\Delta a_n}{a_n}$$



Correction applied by the kicker is such that:

$$\Delta a_n = \sqrt{\beta_P \beta_K} \sin \Delta\psi_{P,K} \; \Delta x'_K$$

β : β-function

$\Delta\psi_{P,K}$: betatron phase shift between pick-up and kicker $\sim$ an odd multiple of $\frac{\pi}{2}$

$\Delta x'_K$: angular deflection related to the voltage difference $2 V_K$ between the kicker plates through the expression

$$\Delta x'_K = \sqrt{n_K} \cdot \frac{1+\beta}{\beta (pc/e)} \cdot \frac{\sigma_K}{g_K} \cdot \frac{\sin(k_n \ell)}{k_n \ell} \cdot V_K \cdot \ell$$

with : n_K , number of kickers

β , relativistic factor : v/c

c , light velocity

σ_K , kicker sensitivity

g_K , kicker half-gap : $g_K = \sqrt{\epsilon \beta_K}$

$k_n \ell$, phase of the electromagnetic wave of frequency Ω : $k_n \ell = \frac{\Omega \ell}{c}$; over one octave: $k_n \ell \in (\frac{\pi}{3}, \frac{2\pi}{3})$

ℓ , length of a kicker module

The splitter (or the combiner) matches the kicker (or pick-up) impedance Z_K (or Z_P) to the line impedance Z_L and the voltage at the output of the amplifier is V_{AK} such that:

$$\frac{V_{AK}^2}{Z_l} = 2\,\frac{V_K^2}{Z_K}$$

As $Z_L = 2 Z_K$:

$$V_{AK} = V_K$$

Between the input and the output of the amplifier:

$$V_{AK} = g_a\, V_{AP}$$

with : g_a , electronic gain.

At the exit of the combiner, the signal voltage is:

$$V_{AP} = \frac{e f_0}{2}\, Z_L\, \frac{\sigma_P}{g_P}\, \sqrt{n_P}\, \sin(k_n l)\, a_n$$

with

- e , electron charge
- σ_P , pick-up sensitivity
- g_P , pick-up half-gap : $\sqrt{\epsilon\, \beta_P}$
- n_P , number of pick-up modules

By performing all the multiplications and replacing $\frac{\sin^2(k_n l)}{k_n l}$ by its mean value $2/\pi$, we get

$$G = \frac{\sqrt{n_P n_K}}{\pi}\, e\, f_0^2\, Z_L\, \frac{\sigma_P \sigma_K}{\epsilon}\, \frac{1+\beta}{\beta(pc/e)}\, l\, g_a$$

3. Thermal Noise

The increase of the squared amplitude per unit of time due to random kicks produced by the thermal noise is:

$$\frac{d(a_n^2)}{dt} = f_0 \, \rho_K \, (\Delta x'_K)^2$$

As we consider a beam emittance which contains 95% of the particles:

$$\frac{d\epsilon}{dt} = 3 \, \frac{d(a_n^2)}{dt}$$

The noise voltage V_B at the input of the amplifier is such that:

$$V_B^2 = P_B \, Z_L$$

where P_B is the noise power whose spectral density is:

$$\frac{dP_B}{df} = k(T_A + T_S)$$

with: k , Boltzmann's constant

T_A , noise temperature of the amplifier related to the noise factor v through:

$$T_A = 290\,(10^{v/10} - 1)$$

T_S , temperature of the pick-up terminating resistor

In term of n :

$$\frac{dP_B}{dn} = \frac{1}{2} f_0 \, k \, (T_A + T_S)$$

Using the expression of $\Delta x'_K$ and the definition of b :

$$b = .81 \, n_K \, Z_\ell \, k \, \frac{T_A + T_S}{E} \left[g_a \, \frac{1+\beta}{\beta(pc/e)} \, \sigma_K \, \ell \, f_0 \right]^2$$

where $\frac{\sin^2(k_n \ell)}{k_n \ell}$ has been replaced by its mean value .54.

4. Signal to Noise Ratio

The Schottky power S_i generated by one particle in a one octave-bandwidth is :

$$S_i = \int_n^{2n} dn \left(\frac{1}{T} \int_0^T \frac{V_{AP}^2(t)}{Z_L} dt \right)$$

for N particles with an amplitude distribution $f(a)$ normalized to unity: $f(a) = \frac{2a}{\hat{a}^2}$, the total Schottky power is:

$$S = N \int_0^{\hat{a}} S_i f(a)\, da$$

$$= .45\, N\, n_p\, Z_\ell \left(e f_0\, \tau_p \frac{\hat{a}}{g_r} \right)^2 . n$$

The noise power is :

$$B = k (T_A + T_S)\, n\, f_0$$

and the signal-to-noise ratio is :

$$\frac{S}{B} = \frac{.45\, N\, n_p\, Z_\ell\, f_0}{k (T_A + T_S)} \left(e\, \tau_p \frac{\hat{a}}{g_r} \right)^2 ,$$

independent of the bandwidth.

5. Cooling with constant transfer function

In this case : $\quad G = G_0 \,, \quad x = x_0 \,, \quad b = b_0$

the equation of cooling is linear in ϵ and its solution is

$$\epsilon(t) = (\epsilon_0 - \epsilon_{\infty_0})\, e^{-t/\tau} + \epsilon_{\infty_0}$$

with: $\quad \epsilon_0$, initial emittance

$$\frac{1}{\tau} = 8\, n\, G_0\, Y(x_0)$$

$$\epsilon_{\infty_0} = \frac{b_0}{2 G_0}$$

6. Cooling with variable transfer function

In this mode of operation, the output power of the amplifier is maintained constant during all the cooling process in order to improve, if possible, the cooling rate calculated in the previous section.

The equation of cooling is no longer linear in ϵ and, because of the complicated expression of Y, x instead of ϵ will be used as a variable.

The various possible conditions of operation are summarized in the following table

	$\frac{G}{G_0}$ or $\frac{x}{x_0}$	$\frac{k}{k_0}$	t $\epsilon_0 \to \frac{\epsilon_0}{m}$	$\frac{\epsilon_\infty}{\epsilon_0}$
Constant transfer function	1	1	$\frac{1}{8 n G_0 Y(x_0)} \ln \frac{1 - \epsilon_\infty/\epsilon_0}{1/m - \epsilon_\infty/\epsilon_0} = t_0$	$\frac{\epsilon_{\infty_0}}{\epsilon_0}$
Variable pick-up	$\sqrt{\frac{\epsilon_0}{\epsilon}}$	1	$\frac{x_0}{4 n G_0} \int_{x_0}^{\sqrt{m}\, x_0} \frac{dx}{x^2 Y(x)(1 - x/x_\infty)} = t_p$	$\left(\frac{\epsilon_{\infty_0}}{\epsilon_0}\right)^2$
Variable electronic gain	$\sqrt{\frac{\epsilon_0}{\epsilon}}$	$\frac{\epsilon_0}{\epsilon}$	$\frac{x_0}{4 n G_0} \int_{x_0}^{\sqrt{m}\, x_0} \frac{dx}{x^2 Y(x)\left[1 - \left(\frac{x}{x_\infty}\right)^3\right]} = t_a$	$\left(\frac{\epsilon_{\infty_0}}{\epsilon_0}\right)^{\frac{2}{3}}$
Variable pick-up + kicker	$\frac{\epsilon_0}{\epsilon}$	$\frac{\epsilon_0}{\epsilon}$	$\frac{x_0}{8 n G_0} \int_{x_0}^{m x_0} \frac{dx}{x^2 Y(x)\left(1 - \frac{x}{x_\infty}\right)} = t_{p,k}$	$\frac{\epsilon_{\infty_0}}{\epsilon_0}$

In order to have a guide line in the choice of x_0, let us neglect, for a moment, the thermal noise and consider the ratios:

$$R_p = \frac{\ln m}{2 Y(x_0) \int_{x_0}^{\sqrt{m}\, x_0} \frac{dx}{x^2 Y(x)}} \quad ; \quad R_{p+k} = \frac{\ln m}{Y(x_0) \int_{x_0}^{m x_0} \frac{dx}{x^2 Y(x)}}$$

They indicate the reduction in time needed to reach a final emittance when the gap of the pick-up electrodes or both gaps of electrodes and kickers are varied. See Figure 1 for $m = 10$.

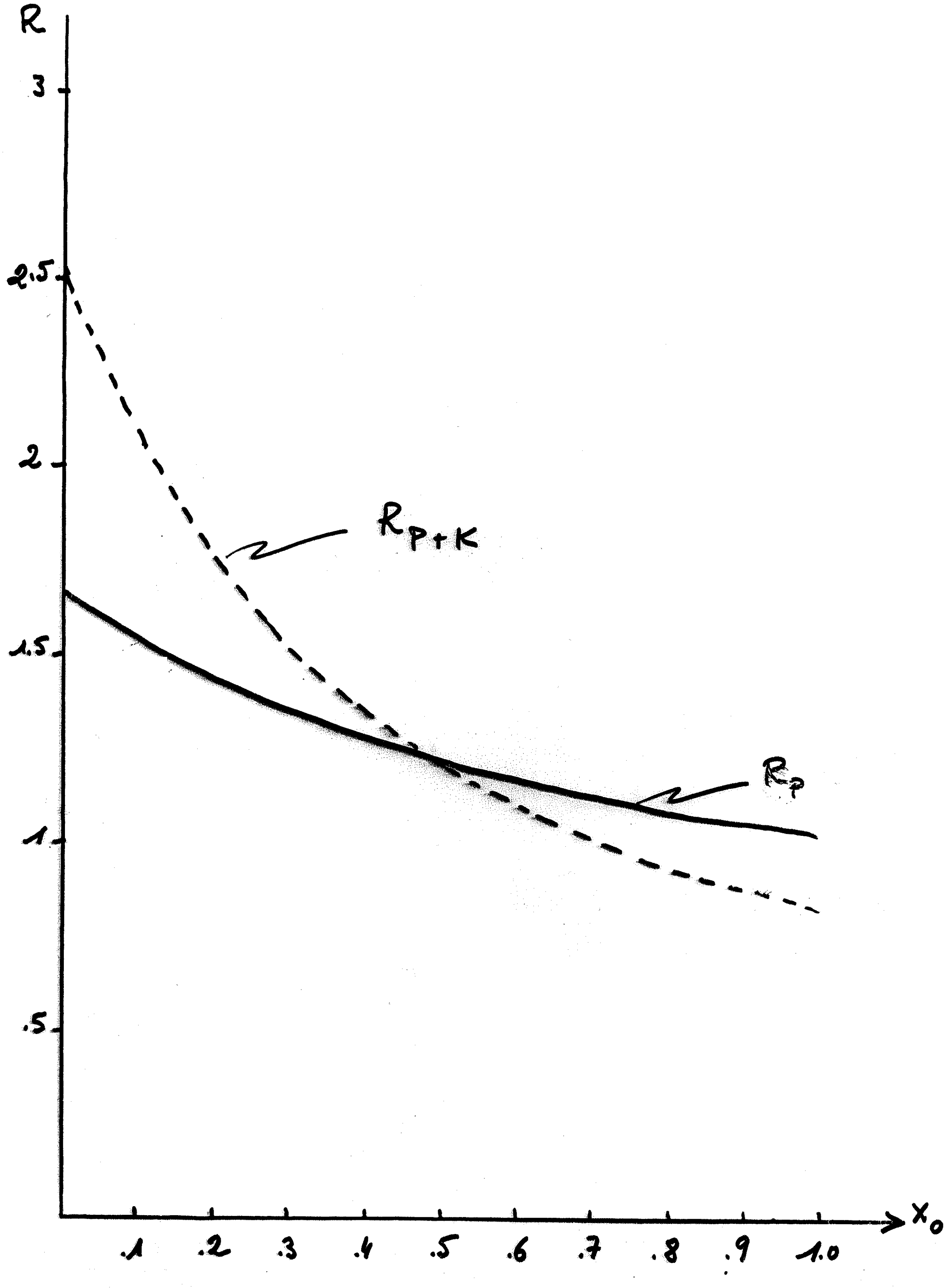


Figure 1 R_p, $R_{p+K} = f(x_0)$

7. Application to A.C.

7.1 Signal to noise ratio for constant transfer function

$$\frac{S}{B} = .45 \; \frac{N n_p Z_L f_0 (e \sigma \frac{\hat{a}}{g_p})^2}{k(T_A + T_S)}$$

$N = 10^8$

$Z_L = 50\ \Omega$

$f_0 = 1.856 \times 10^6$

$\sigma_p = .8$

$T_A = T_S = 40\,°K$

$n_p = 32 \quad , \quad \hat{a}/g_p = 1$

$$\frac{S}{B} = 2$$

7.2 Choice of x_0

Let us aim at an emittance reduction of a factor 10 in 2 seconds:

Neglecting the noise term:

$$\frac{2}{\tau} = \ln 10 \rightarrow \frac{1}{\tau} = 1.15$$

$$\frac{1}{\tau} = 8 n G_0 Y(x_0)$$

Let us assume that x_0 is small enough so that $Y(x_0) \sim 1$

$$n = \frac{W}{f_0} \quad , \quad W = 1 \times 10^9$$

$$G_0 = 2.67 \times 10^{-4}$$

$$x_0 = \frac{N G_0}{W \eta \frac{\Delta p}{p}}$$

$\eta = .05 \quad , \quad \frac{\Delta p}{p} = 1.5 \times 10^{-2} \rightarrow x_0 = .0356$

$Y(x_0) = .95$, the assumption about $Y(x_0)$ is thus checked

7.3 Electronic gain and power

$$G_0 = \frac{\sqrt{n_p n_k}}{\pi} e f_0^2 Z_L \frac{\sigma_p \sigma_k}{\varepsilon_0} \frac{1+\beta}{\beta(p c/e)} \ell g_a$$

$n_K = n_P = 32$; $\sigma_P = \sigma_K = .8$; $\epsilon_0 = 200 \times 10^{-6}$; $\beta \sim 1$

$pc/e = 3.5 \times 10^9$; $\ell = 5 \times 10^{-2}$

$$g_a = 10^7 \quad \text{or} \quad 140\ \text{dB}$$

Output power (neglecting feedback):

$$P = g_a^2 \,(S + B)$$

$$S = 2.19 \times 10^{-12}$$

$$P = 330\ W$$

7.4 Asymptotic emittance

$$\epsilon_{\infty_0} = \frac{b_0}{2 g_0}$$

$$b_0 = .81\, n_K Z_L\, k \frac{T_A + T_S}{\epsilon} \left[g_a \frac{1+\beta}{\beta (pc/e)} \sigma_K\, \ell f_0 \right]^2$$

$$= 1.35 \times 10^{-9}$$

$$\epsilon_{\infty_0} = 2.5 \times 10^{-6}$$

7.5 Final emittance

After 2 seconds:

$$\epsilon = (\epsilon_0 - \epsilon_{\infty_0})\, e^{-\frac{t}{\tau}} + \epsilon_{\infty_0}$$

$$= 22.3 \times 10^{-6}$$

7.6 Variable gain

From Figure 1, it is clear that the combination of variable kicker and pick-up is attractive since the time to reach the final emittance could be reduced by a factor 2.5

In order to have the variations $\epsilon(t)$ we have integrated:

$$t = \frac{x_0}{8 g_0} \int_{x_0}^{x} \frac{dx}{x^2 Y(x) \left(1 - \frac{x}{x_\infty}\right)}$$

for various values of $x = x_0 \frac{\epsilon_0}{\epsilon}$ and with $x_\infty = x_0 \frac{\epsilon_{\infty}}{\epsilon_{\infty_0}} = 2.85$

See Figure 2.

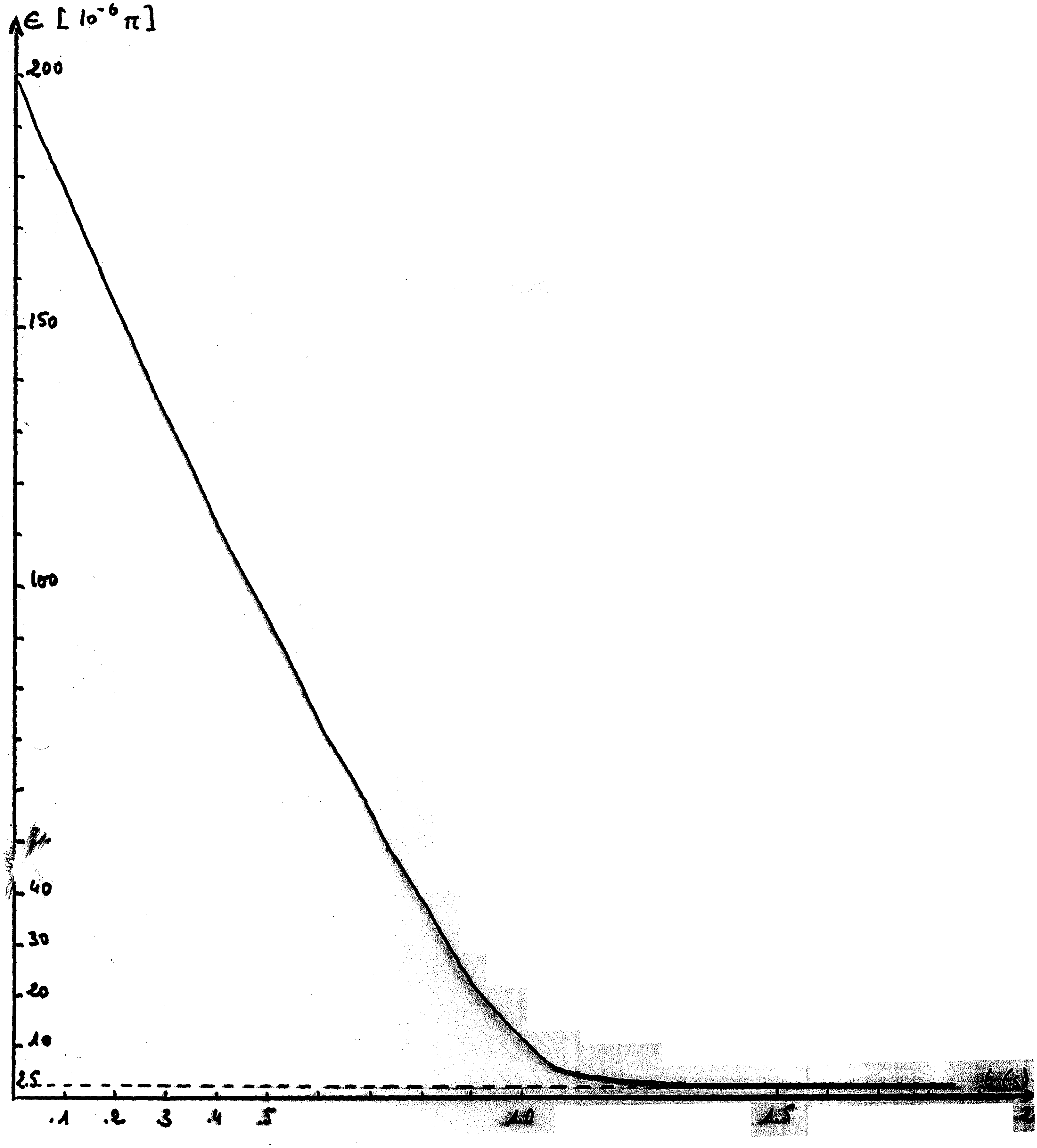


Figure 2 : ε = f(t)