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AA LONG TERME NOTE N° 31

Summary of the working party on "Target and Lenses" of April 28, 1983

Present : B. Autin, K. Frank, E. Jones, H. Riege, R. Sherwood, P. Sievers, S. van der Meer

Topics : Status of Plasma Lens, H. Riege.
Quasi-linear focusing around a target, B. Autin

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1. STATUS OF THE PLASMA LENS PROTOTYPE AND FUTURE PLANS

1.1. Why a plasma lens

Provided the problems of energy dissipation and of the stability of the discharge can be solved, the plasma lens is the best device for collecting antiprotons. $\bar{p}$-absorption is small compared to other lenses. Geometrically it is very flexible and the current distribution in space (uniformity) and time (penetration) can be controlled in contrast to a metal lens. Since we are interested in a short current pulse the energy consumption can be low. There are no pressure problems due to the low gas pressure working regime (<1 Torr).

1.2. Status

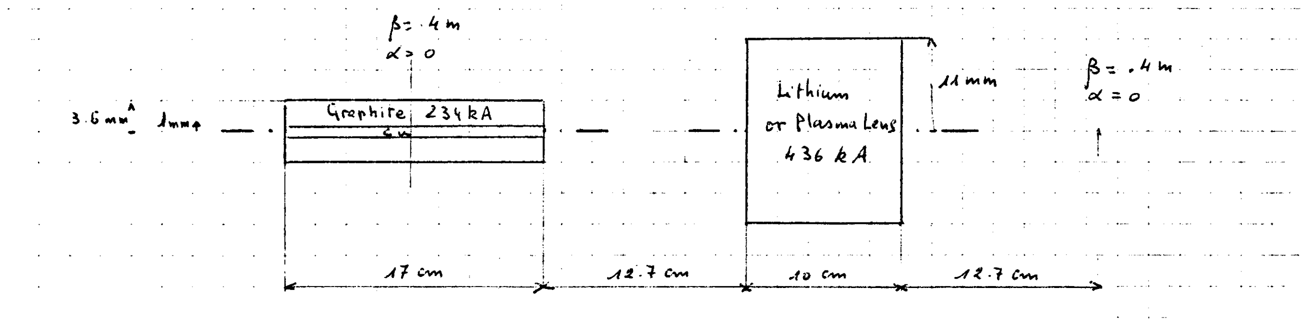
The test equipment is nearly complete. A 280 μF capacitor bank is connected via ignitions or pseudo-spark high current switches to s strip line at the end of which the plasma lens prototype will be mounted. All auxiliary equipment, like vacuum pumps, gas injection valves, measuring transformers, HV-probes, gauges and a Faraday cage, is available. The first prototype lens will have a length of 200 mm and an inner radius of 20 mm. Hollow electrodes with 12.5 mm inner diameter are used to provide an additional expansion volume for the hot plasma and to increase the Debye sheath surface at the cathode (see CERN/PS/DL/82-6 and CERN/PS/AA/83-22) in order to raise maximum current.

1.3. First steps

First we have to get the test set-up going. A proper circuit analysis will allow extrapolations to goals which may be achieved later. The tests will start with a short-circuiting metal tube mounted in the place of the plasma lens. The initial test with the plasma lens itself aims at measurements of initial resistance and energy dissipation as function of gas pressure, gas type, current and electrode geometry. Stability and reproducibility of the discharge can be measured optically.

2. QUASI-LINEAR FOCUSING AROUND A TARGET

The details of the calculation are given in the attached note and the parameters have been calculated in the spirit of simplifying the system as much as possible. Yield calculations have still to be made.



Layout and parameters of a target and lens set-up

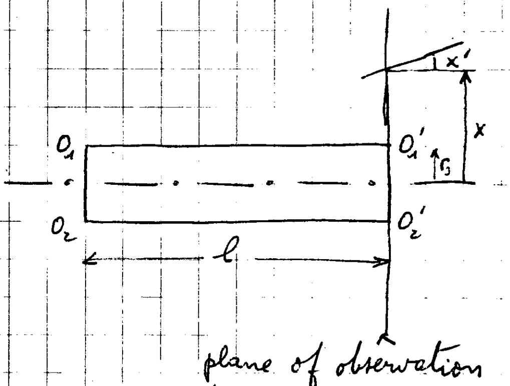
B. Autin
H. Riege

Effect of a conducting lens around the target

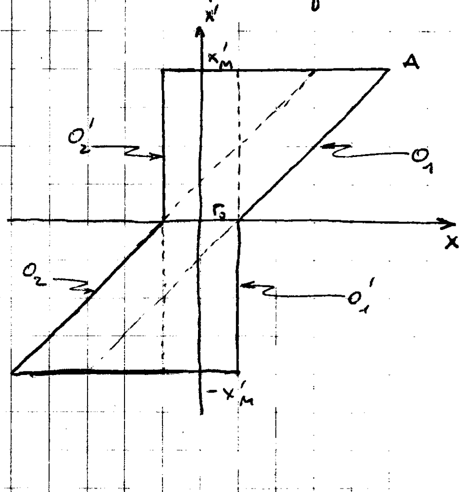
B. AUTIN 28/4/83

The focusing of the antiprotons inside a target (AA-LT) and outside a target with a hyperbolic field (AA/LT-7) have been studied. We give here envelope calculations for a quasi-linear field around the target like the one which would be given by a conducting lens.

1. No field around the target



The diagram of particle production at the end of the target has a "butterfly" shape. The contour is defined by the cut-off angles $\pm x'_M$ and the segments corresponding to the particles emitted from the ends of the target O_1, O_1', O_2, O_2'.



The coordinates of the point A are:

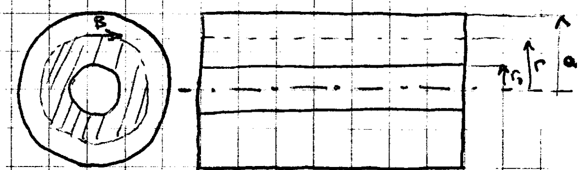
$$\begin{cases} x_A = \ell x'_M + r_0 \\ x'_{A} = x'_M \end{cases}$$

The area of the "butterfly" is:

$$\epsilon = 4 r_0 x'_M + \ell x'^2_M$$

2. Field around the target

2.1 Field in a hollow cylinder



From Ampère's theorem:

$$\int \vec{B}\, d\vec{l} = \mu_0 I$$

By symmetry, a field line is a circle

centered on the axis of the target:

$$B \times 2\pi r = \mu_0 \pi j (r^2 - r_0^2)$$

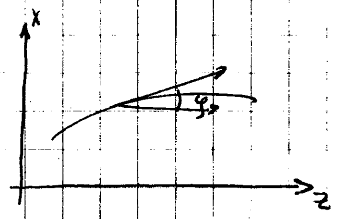
so that

$$B = \frac{\mu_0 j}{2}\left(r - \frac{r_0^2}{r}\right)$$

where j is the current density. This expression is valid for a static field. When the current is pulsed the time after which the static field distribution is reached fixes the pulse duration.

2.2 Trajectories

The general equations of motion are



$$\begin{cases} \dfrac{dx}{dz} = \tan\varphi \\ \dfrac{d(\sin\varphi)}{dz} = -\dfrac{e}{p} B \end{cases}$$

The exact trajectories can be obtained by a numerical integration of the above system. In order to derive an order of magnitude of the current, we make simplifying approximations:

$$\begin{cases} \tan\varphi \simeq \sin\varphi \sim \varphi \\ B \sim r - r_0 \end{cases}$$

The trajectories are then arcs of sinusoides:

$$\begin{cases} x = x_0 + \dfrac{x'_0}{\sqrt{K}} \sin(\sqrt{K}\, z) \\ x' = x'_0 \cos(\sqrt{K}\, z) \end{cases}$$

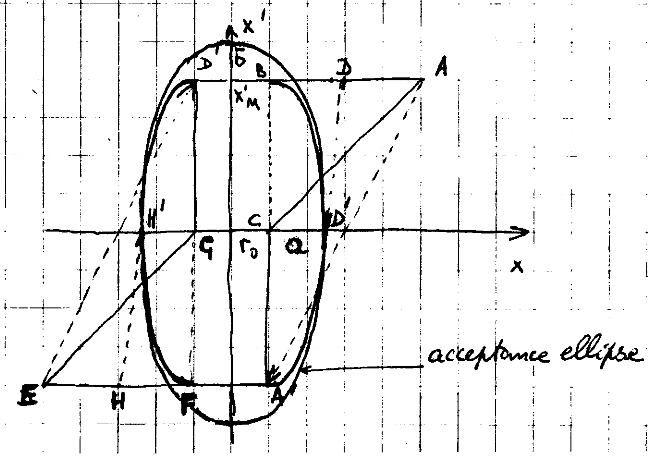
where the "o" subscript refers to initial condition and K is defined by:

$$K = \frac{1}{2}\,\frac{e}{p}\,\mu_0 j$$

Under the condition:

$$\sqrt{K}\; l = \pi$$

the mapping of the butterfly is shown on the figure. The segment BA is transformed into half an ellipse of semi-axes $(x'_M/\sqrt{K},\ x'_M)$, and CA into CA'.



The same is true for the negative x values. The acceptance of the downstream transfer channel is an ellipse centered at the origin and osculating the arc BA' in D'. In D', the radius of curvature of the ellipse $\widehat{BA'}$ is:

$$R = \frac{x'^2_M}{(x'_M/\sqrt{K})} = x'_M \sqrt{K}$$

The semi-axes of the acceptance ellipse are then:

$$\left| \begin{array}{l} a = r_0 + \dfrac{x'_M}{\sqrt{K}} \\ b = \sqrt{aR} = x'_M \sqrt{1 + \dfrac{r_0\sqrt{K}}{x'_M}} \end{array} \right.$$

so that the area is

$$\epsilon = \pi \sqrt{\frac{x'_M}{K}} \left(x'_M + r_0\sqrt{K} \right)^{3/2}$$

2.3 Numerical application

From the definition of K, the current density is

$$j = \frac{2K}{\mu_0 (e/p)}$$

For: $\sqrt{K}\, l = \pi$, $l = 17$ m, $p = 3.5$ GeV/c

$$j = 6.34 \times 10^9 \ \mathrm{A\,m^{-2}}$$

For: $\epsilon = 200\pi$ mm.mrad, $r_0 = 1$ mm (§2.2)

$$x'_M = 47.5 \ \text{mrad}$$

$$a = 3.6 \ \text{mm}$$

$$b = 56 \ \text{mrad}$$

$$I = 234 \ \text{kA}$$

The ratio of the ellipse area to the butterfly area (182π mm-mrad) indicates a 10% dilution which is small as compared to the dilution in the ellipse circumscribed to the butterfly ($2\pi\, x_A\, x'_A = 862\pi$ mm-mrad).

3. Focussing

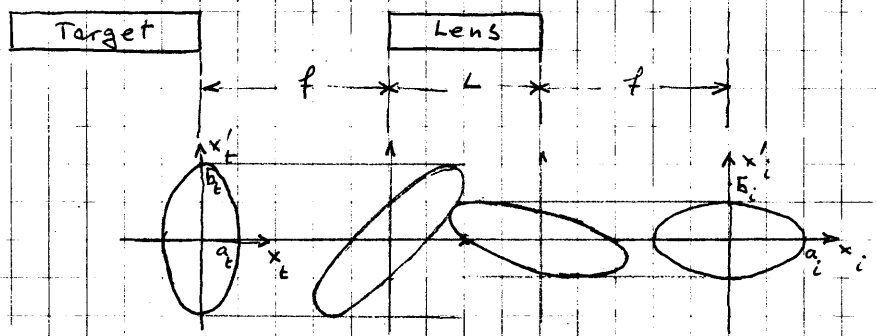
3.1 Proton focussing

At 26 GeV/c, the nominal emittance of the proton beam is $\epsilon_p = 2.5\pi$ mm-mrad. For a target radius $r_0 = 1$ mm, the value of the β-function is:

$$\beta_p = \frac{r_0^2}{(\epsilon/\pi)} = 40 \text{ cm}$$

3.2 Antiproton focussing

If we assume (with optimism!) that the β value for the antiprotons is also of the order of 40 cm, the divergence of the beam which can be accepted by the transfer channel is at the most $\sqrt{\frac{\epsilon_{\bar{p}}}{\beta_{\bar{p}}}} \sim 20$ mrad. It is therefore necessary to add a lens (lithium or plasma) between the target and the quadrupoles of the transfer channel.



For a strength K of the lens, the transfer matrices are:

$$\begin{pmatrix} 1 & f \\ 0 & 1 \end{pmatrix} \quad \text{for the drift spaces, with: } f = \frac{1}{\sqrt{K}\,\tan(\sqrt{K}L)}$$

and:

$$\begin{pmatrix} \cos(\sqrt{K}L) & \frac{1}{\sqrt{K}}\sin(\sqrt{K}L) \\ -\sqrt{K}\sin(\sqrt{K}L) & \cos(\sqrt{K}L) \end{pmatrix} \quad \text{for the lens,}$$

so that the total transfer matrix is:

$$\begin{pmatrix} 0 & \frac{1}{\sqrt{K}\sin(\sqrt{K}L)} \\ -\sqrt{K}\sin(\sqrt{K}L) & 0 \end{pmatrix}$$

The length L and the magnetic strength K of the lens have to satisfy the condition:

$$\sqrt{K}\sin(\sqrt{K}L) = \frac{\sigma_i}{a_t} = \frac{20}{3.6} = 5.56$$

For $\sqrt{K}L = \frac{\pi}{4}$,

$$\sqrt{K} = 7.86\ m^{-1}\ ,\quad L = 10\ cm\ ,$$

and the drift length is:

$$f = \frac{1}{\sqrt{K}\tan(\sqrt{K}L)} = 12.7\ cm$$

The radius of the lens is determined by the envelope of the beam. The tracing of the β-function and of $\alpha\ (= -\frac{1}{2}\frac{d\beta}{ds})$ are given by:

$$\beta_1 = \beta_0 + \frac{s^2}{\beta_0}\ ;\quad \alpha_1 = \alpha_0 - \frac{s}{\beta_0}\ ;\quad \gamma_1 = \frac{1+\alpha_1^2}{\beta}$$ in the drift space

and

$$\beta = \cos^2(\sqrt{K}s)\cdot\beta_1 - 2\frac{\sin(\sqrt{K}s)}{\sqrt{K}}\cos(\sqrt{K}s)\cdot\alpha_1 + \frac{\sin^2(\sqrt{K}s)}{K}\cdot\gamma_1$$

For $\beta_0 = \frac{a_t^2}{(\epsilon/\pi)} = \frac{(3.6)^2}{200} = .0648\ m$, $\alpha_0 = 0$

$$\beta_1 = .3137\ m,\quad \alpha_1 = -1.96\ ,\quad \gamma_1 = 15.43$$

$\beta_{max} = .533$ in the lens which leads to a radius of the lens

$$r_L \sim 11\ mm$$

The current intensity is:

$$I = j\,S = \frac{2K}{\mu_0(e/p)}\pi r_L^2 = 4.36\ kA$$