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COMPARISON BETWEEN QUADRUPOLE LENSES OF CLASSICAL DESIGN AND "PANOFSKY LENSES"

SUMMARY :

It is shown that quadrupole lenses of classical design are preferable to Panofsky lenses [1)] in all cases, except

a) if the available space is restricted,

b) if a gradient x aperture product higher than 1 Wb/m^2 is required,

c) if a very large aperture, together with a low gradient is required.

In case a and b, the Panofsky lens is an expensive solution, but the only possible one. In case c, the cost of both solutions must be compared carefully from case to case.

I. MAGNETIC FIELD IN PANOFSKY LENSES.

In fig. 1 one quadrant of a Panofsky lens is shown. For a current density j in the coils the field can be described as follows:

$$\left.\begin{array}{lll}
\text{In (1) :} & B_{x1} = \mu j \dfrac{b}{a+b} \cdot y & \\
 & B_{y1} = \mu j \dfrac{b}{a+b} \cdot x & \\
\text{In (2) :} & B_{x2} = \mu j \dfrac{a}{a+b} (c+d-y) & \\
 & B_{y2} = \mu j \dfrac{b}{a+b} \, x & \\
\text{In (3) :} & B_{x3} = \mu j \dfrac{a}{a+b} (c+d-y) & \\
 & B_{y3} = \mu j \dfrac{a}{a+b} (a+b-x) & \\
\text{In (4)} & B_{x4} = \mu j \dfrac{b}{a+b} \, y & \\
 & B_{y4} = \mu j \dfrac{a}{a+b} (a+b-x) &
\end{array}\right\} \qquad (1)$$

1) L.N. Hand and W.K.H. Panofsky, Magnetic quadrupole with rectangular aperture, HEPL - 169 (Stanford)

It can be verified that these relations satisfy the field equations

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = \begin{cases} 0 \text{ in (1) and (3)} \\ \mu j \text{ in (2)} \\ - \mu j \text{ in (4)} \end{cases}$$

and the boundary conditions

$$B_y = 0 \qquad \text{for } x = 0 \text{ and } x = a + b$$

$$B_x = 0 \qquad \text{for } y = 0 \text{ and } y = c + d$$

$$\left.\begin{aligned} B_{x1} &= B_{x4} \\ B_{y1} &= B_{y4} \\ B_{x2} &= B_{x3} \\ B_{y2} &= B_{y3} \end{aligned}\right\} \quad \text{for } x = a$$

$$\left.\begin{aligned} B_{x1} &= B_{x2} \\ B_{y1} &= B_{y2} \\ B_{x3} &= B_{x4} \\ B_{y3} &= B_{y4} \end{aligned}\right\} \quad \text{for } y = c$$

The following remarks can be made:

a) As can be seen from (1), the maximum value of B in the steel occurs at $x = a$, $y = c + d$, and it is equal to the maximum value of B_y in the useful aperture (at $x = a$, $y = 0$). This shows at once that with a Panofsky lens a higher product of gradient x aperture can be reached than with a normal quadrupole lens. In the latter case it is very difficult to obtain a value higher than 1 Wb/m^2. With a Panofsky lens, it should be possible to go up to 1.8 Wb/m^2 without saturation difficulties.

b) The number of ampere-turns per pole, divided by the gradient $\frac{\partial B_y}{\partial x}$ in (1) is

$$\frac{jbc}{\mu j \frac{b}{a+b}} = \frac{(a+b)c}{\mu}$$

The total dissipated power is proportional to the square of this value and also proportional to the resistance of the windings. The latter is inversely proportional to the cross-section of the windings:

$$R = \frac{A}{bc}$$

Therefore the total power is proportional to

$$\frac{(a+b)^2 c^2}{bc} = \frac{(a+b)^2 c}{b}$$

With a given value of a and c (aperture), this expression will have a minimum value for

$$b = a \tag{2}$$

(minimum power Panofsky lens)

c) It is clear that the exact positioning of the windings is very important for obtaining the proper field shape. The fabrication of windings being what it is, it may be said that the Panofsky lens will always be at a disadvantage in this respect. On the other hand, in a normal quadrupole lens errors are caused by the limited width of the poles.

2. COMPARISON BETWEEN CLASSICAL DESIGN AND PANOFSKY LENS.

It is clear that if either a restricted space is available, or a product of gradient x aperture higher than 1 Wb/m^2 is required, a Panofsky lens must be used.

In all other cases, economic considerations will decide for one construction or the other.

It is very difficult to make a general comparison on this basis, because many variables are involved. It can be said, however, that the power cost will in most cases decide against the use of Panofsky lenses. The reason for this is that with lenses of classical design the power consumption can be reduced by increasing the cross-section of the windings. If the pole distance is kept constant, this will not increase the number of ampere-turns required. With Panofsky lenses, however, increasing the winding cross section means that the number of ampere-turns required will increase at the same time. Therefore the power cannot be reduced below the minimum, obtained if equation (2) is satisfied.

If we now compare a minimum power Panofsky lens with a lens of classical design having the same aperture (fig. 2) we find that the number of ampere-turns in the Panofsky lens is two times that in the old fashioned quadrupole, (neglecting saturation effects). If we suppose for a moment the same winding cross-section for both, the power for a Panofsky lens will be 4 times higher.

Because for higher power the power installation cost per KW is somewhat lower, we may say that for the Panofsky lens in this example the power cost will be 3,5 times higher.

We can now make the following comparison:

	normal lens	Panofsky lens
cost of lens cost of power installation + energy for 5000 operating hours	X p. X	Y 3.5 p. X
Total	(p + 1) X	Y + 3.5 p X

Even if we would suppose that $Y = 0$, the Panofsky lens would still be at a disadvantage if $p > 0.4$. In practice, this is nearly always the case. Some examples may be given:

a) beam transport lenses for PS — $p \approx 2.2$
b) matching lenses between linac and PS — $p \approx 0.5$
c) correcting lenses PS — $p \approx 0.7$
d) μ-meson channel SC — $p \approx 2$

The disadvantage of a Panofsky lens will be still more pronounced, because:

1. Its manufacturing cost is not zero.
2. The supposition of equal winding cross-sections is not realistic. In fact, for the classical quadrupole a greater cross-section than that of the equivalent Panofsky lens will nearly always decrease the total cost of lens + power supply.
3. The assumption of 5000 operating hours is not always a good guess. For instance in examples b and c above, the number of hours would be higher, making p still greater.

A small value of p would only occur in the rare case that lenses with a very large aperture and a low gradient would be required, (particles of low momentum). Given the great number of variables, the comparison between the two solutions would have to be made from case to case.

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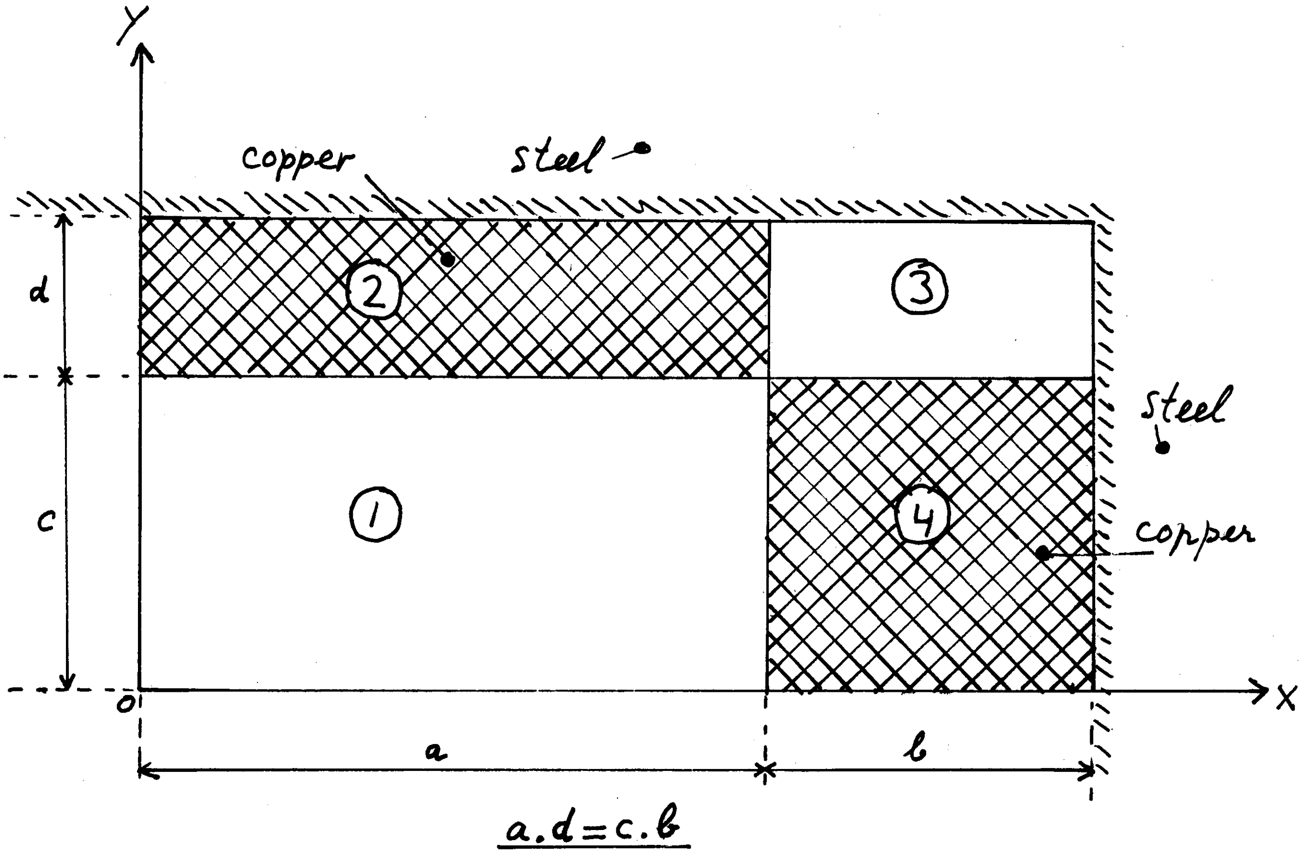

Fig. 1

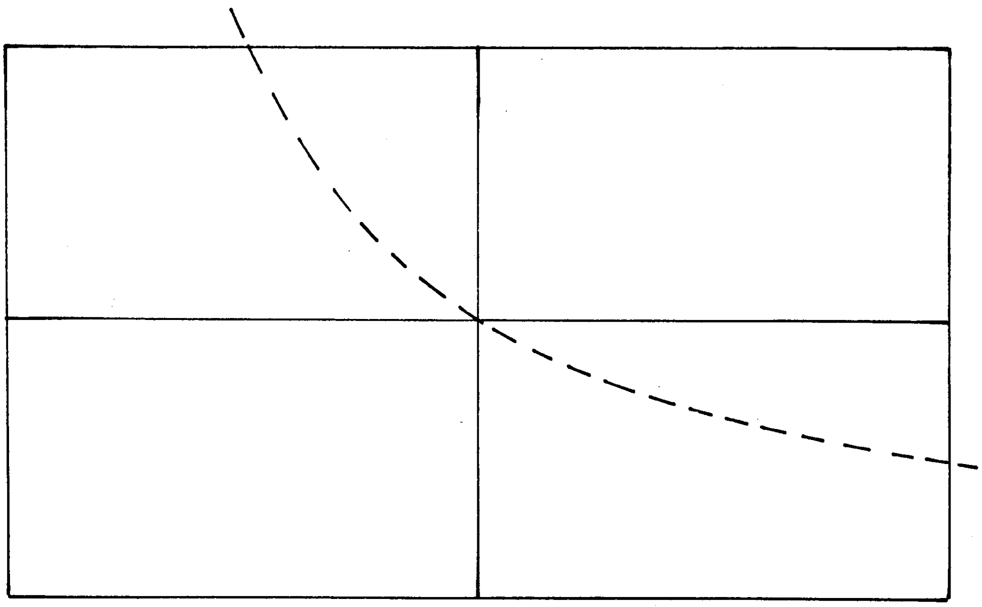

Fig. 2