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PS/Int. AR/60-9
31st May, 1960.

SOME PROBLEMS OF INJECTION INTO A 2 MeV STORAGE RING.

Currently, two types of storage ring are being discussed. One is an A.G. storage ring with separated bending magnets and focusing lenses (See K. Johnsen's report PS/Int. AR/60-6, 6 May 1960).

The other is a symmetrical FFAG storage ring for injection at about 2 MeV and stacking at about 10 MeV (Reports describing the parameters of rings of this type are in preparation).

The present notes discuss some of the problems of injecting a "reasonable" beam current into such storage rings. By "injection" is meant the whole process which starts when a pulse of electrons starts coming out of the Van de Graaff generator and finishes when some of these electrons have been trapped in R.F. buckets and have started moving in them towards the stack. The word "reasonable" needs to be defined. It means a current which is large enough to permit measurements to be made on stacking processes, but not so large as to cause instabilities which would seriously affect the interpretation of these measurements. We will have to try to make some sort of estimate of the limits within which the injected beam current would, in this sense, be reasonable.

We start by considering an effect which will apparently set an upper limit to the current. This is the longitudinal ("negative-mass") space-charge instability discussed in MURA 441 and in CERN Proceedings 1959, page 239.

1. Longitudinal Space-charge Effects.

The criterion for negative mass instability is

$$(\Delta E)^2 \ngtr \frac{300\, g\, N\, E_0\, e\, (k+1)\,(\gamma^2-1)}{\gamma\, R\, \pi^2\, \left| k+1-\gamma^2 \right|}$$

or, for stability, the number of electrons in a uniform beam

$$N < \frac{\gamma\, R\, \pi^2 \left| k+1-\gamma^2 \right| (\Delta E)^2}{300\, g\, E_0\, e\, (k+1)(\gamma^2-1)} \tag{1}$$

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where $\gamma = E/E_o$

$k + 1 = \gamma_{tr}^2$

ΔE = energy spread in eV

$$g = 1 + 2 \ln \frac{2G}{\pi a}$$

G = vacuum chamber aperture

a = cross-sectional radius of beam

E_o = electron rest energy in eV

e = " charge in esu

R = orbit mean radius in cm.

We consider the following numerical examples:

__Ring 1__ $k + 1 \sim 7$, $\gamma = 4$, $R = 400$ cm; $a = 0{,}1$ cm *), $G = 5$ cm } so $g \simeq 8$

__Ring 2__ $k + 1 \sim 9$, $\gamma = 4$, $R = 270$ cm; $a = 0{,}1$ *), $G = 5$ cm } $g \sim 8$

__Ring 3__ As in Ring 2 but with $\gamma = 20$ and $R = 320$ cm

__Ring 4__ As in Ring 2 but with $\gamma = 6$ and $R = 280$ cm

We then have the following results with $(\Delta E) = 10^3$ eV:

Ring No.	Maximum N	Time for 1 rev. at injection (ns)	Maximum injected current (mA)	Build-up time (μsec)
1	2.3×10^9	80	4.6	50
2.	9.4×10^8	57	1.6	22
3	9.4×10^9	57	16	2160
4	2.4×10^9	57	4	65

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*) We assume here that betatron amplitudes are ~ 1 mm and are much larger than the intrinsic radial spread due to the energy spread of the beam. With the latter equal to 1 keV, the intrinsic radial spread

$$\Delta R \quad \frac{1}{k+1} \frac{\Delta E}{E} R$$

$$1/7 \cdot 1/2000 \times 400 = 1/35 \text{ cm}$$

so that the contribution of this to a would be $\sim 1/70$ cm.

The parameters of Ring 1 correspond approximately to the proposed AG storage ring. Those of Ring 2 apply to the beam at the injection radius in one of the proposed versions of the FFAG storage ring. Ring 3 is the same storage ring at the stacking radius for $E_s \sim 10$ MeV and Ring 4 is the same with $E_s \sim 3.1$ MeV. This last case corresponds to a difference of 10 cm between injection and stacking radius and may be compared with Ring 1 which has about the same radial aperture.

In fact the build-up time of longitudinal instabilities is so short that if the condition (1) is not fulfilled the limit indicated for Ring 2 would also apply to Ring 3 and Ring 4 because there would not be enough time to increase γ enough for stability.

The build-up time for $\Delta E = 0$ is given by

$$T = \frac{1}{n} \left[\frac{\gamma^2 R E (\gamma^2 - 1)(k + 1)}{300\, g\, N\, e\, f^2 \left| k + 1 - \gamma^2 \right|} \right]^{1/2} \text{ sec} \qquad (2)$$

where

E = particle energy in eV

f = particle revolution frequency in cycles per sec.

n = wave-number of azimuthal non-uniformity.

The appropriate values are indicated in the table.

Evidently the build-up time of importance for the FFAG storage rings is the $\sim$ 20 μsec indicated for Ring 2.

We should thus conclude that for the AG ring the injected current should be less than about 5 mA and in the FFAG ring it should be less than about 2 mA, provided that the energy spread is not less than 1 keV.

2. Energy Spread of Injected Beam.

The Van de Graaff generator has a terminal capacity of 100 pF. There is thus a voltage drop of 10 kV per micro-Coulomb of charge removed from the top terminal.

The voltage drop during the 80 ns injection pulse of Ring 1 will thus be $\sim$ 4 volt and for the 57 ns pulse of Rings 2 - 4 , it will be $\sim$ 1 volt, if the currents drawn from the top terminal are, respectively 5 mA and 2 mA.

These values are comparable to the energy spread due to thermal and space-charge effects in the injector. The energy spread in a single injected pulse will thus be very small.

Since synchrotron oscillations are rather linear over about 90 o/o of the bucket width, a lot of synchrotron oscillations will be needed before a bucket of 1 keV amplitude can spread out such a narrow beam. In any case, after a quarter of a synchrotron oscillation the situation would be as in Fig. 1a with $h = 1$ and 1b with $h = 10$.

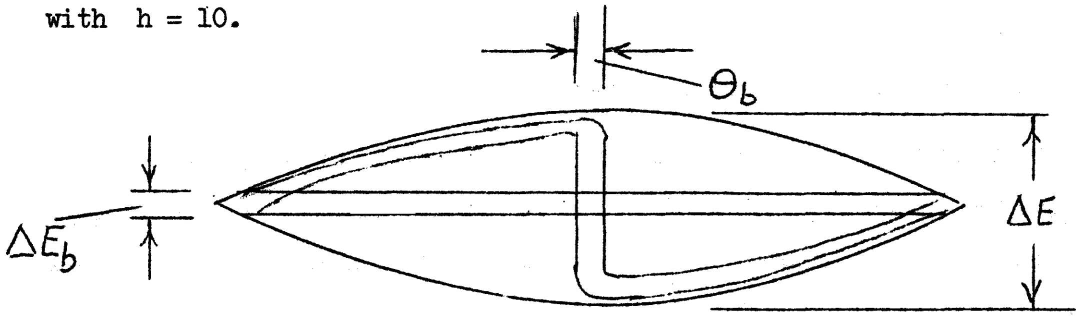


Fig 1a

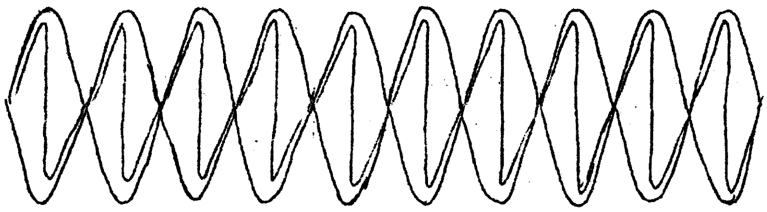


Fig 1b

In the former case, most of the beam will be concentrated within an azimuth of order $2\pi(\Delta E_b/\Delta E)$, i.e. $\sim 3.6^{\circ}$ if we take $\Delta E \sim 10$ eV.

If we call the azimuhtal bunching factor b', we have

$$1/b' = 1/b = \frac{\theta_b}{2\pi} = \frac{\Delta E_b}{\Delta E} \quad \text{in the case } h = 1$$

$$1/b' = \frac{h\,\theta_b}{2\pi} = h\frac{\Delta E_b}{\Delta E} = h\,b \quad \text{in general.}$$

Two effects of this bunching must be considered. First it will increase the linear density of charge by the factor b (for all h). The figures in Table 1 need accordingly to be reduced by this factor if longitudinal instability is to be avoided in the situation of Fig. 1. (This assumes that nothing has been done to produce the required energy spread in the beam before it is injected - something that we will conclude to be necessary).

If we take $\Delta E_b \sim 10$ eV $\Delta E/\Delta E_b \sim 100$, and the injected currents should now be respectively $< 50\ \mu A$ and $20\ \mu A$ in the two rings 1 and 2 - 4.

In addition we must consider the effect on the transverse space charge limit, which is normally given by

$$N \quad \frac{2\pi a^2 \beta^2 \gamma^3 Q}{\Delta Q\ R\ r_e} \tag{3}$$

where ΔQ is the permissible shift in Q

$r_e = 2.8 \times 10^{-13}$ cm

a = beam cross-section radius in cm

R = orbit mean radius in cm

N = total number of electrons in orbit.

With a bunched beam the net defocusing force is increased by the factor b, so

$$N < \frac{2\pi a^2 \beta^2 \gamma^3 Q}{b\ \Delta Q\ R\ r_e} \tag{4}$$

As numerical examples we may take the parameters of Ring 1 and Ring 2.

For Ring 1:

$a \sim 0.1$ cm	$R = 400$ cm	$b = 100$
$\beta \sim 1$	$Q = 2.75$	
$\gamma \sim 4$	$\Delta Q = 0.2$	

$$N < \frac{2\pi \times 64 \times 2.75}{10^2 \times 100 \times 0.2 \times 400 \times 3 \times 10^{-13}}$$

$$< 4.6 \times 10^9 \quad (\text{Corresponding } I_{inj} < 9.2 \text{ mA})$$

For Ring 2:

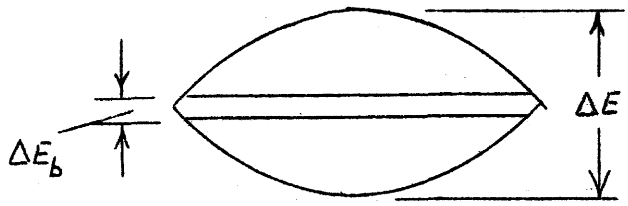
$a \sim 0.1$ cm	$R = 270$ cm	$b = 100$
$\beta \sim 1$	$Q \sim 6$	
$\gamma \sim 4$	$\Delta Q = 0.2$	

$$N < \frac{2\ \pi \times 64 \times 6}{100 \times 100 \times 0.2 \times 270 \times 3 \times 10^{-13}}$$

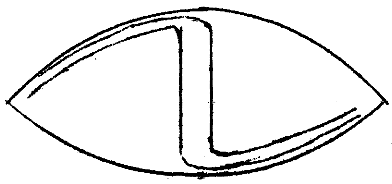
$$< 1.49 \times 10^{10} \quad \text{(corresponding } I_{inj} < 30 \text{ mA)}$$

In taking $b = 100$ we are imagining an intrinsic energy spread $\Delta E_b \sim 10$ eV and a bucket spread $\Delta E \sim 1$ keV.

In fact the longitudinal limits would now be given by ΔE_b instead of ΔE in the initial situation:



and would consequently be b^2 times lower than the values given in Table 1, and b times lower in the situation:



since now we have a linear charge density b times greater even though the energy spread is now ΔE.

Evidently the longitudinal limit will be much more serious than the transverse if nothing has been done to give the injected beam the required energy spread ΔE. If this has been done, the longitudinal limits are then as given in Table 1 and the transverse limits are 100 times larger than the figures given above. Once again the longitudinal limits are the serious ones.

If, however, the longitudinal limit is exceeded and the beam starts to bunch up under the influence of the space charge forces, then we would expect to lose particles through exceeding the transverse limits within times of the order of 5 x 50 = 250 μsec in Ring 1 and 7 x 22 = 154 μsec in Ring 2.

It must be concluded

(a) the injected currents cannot exceed the values indicated in Table 1, viz. about 4.5 mA for the A.G. ring and about 1.5 mA for the FFAG ring.

(b) the energy spread for which the above limits apply, viz. 1 keV must be present in the beam before it is injected.

We cannot use the van de Graaff terminal voltage droop for this purpose. Even if we drew, say, 1.25 A from the terminal during an 80 ns injection pulse, thereby producing an energy droop of 1 keV over the pulse, the effect would be as indicated in Fig. 2.

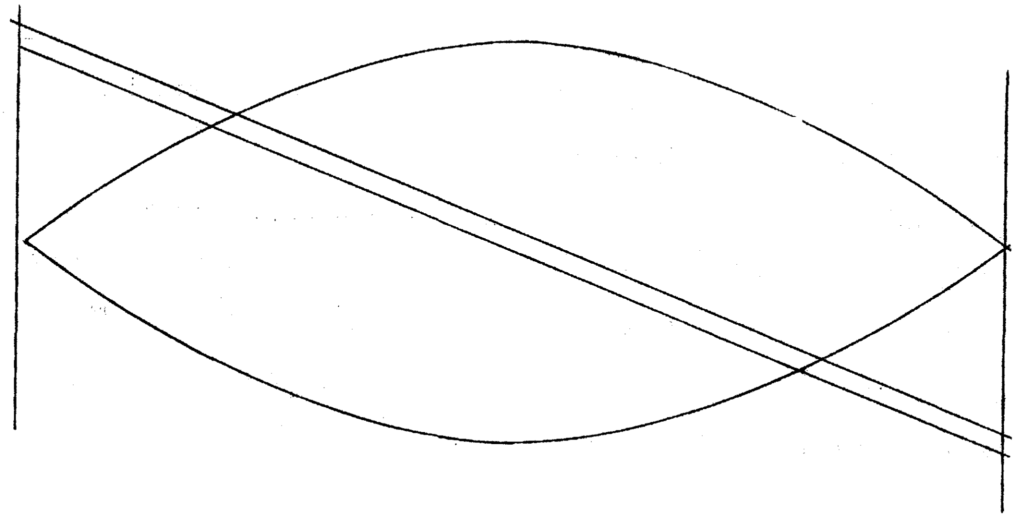


Fig. 2 (a)

which would become in a little less than a quarter synchrotron period

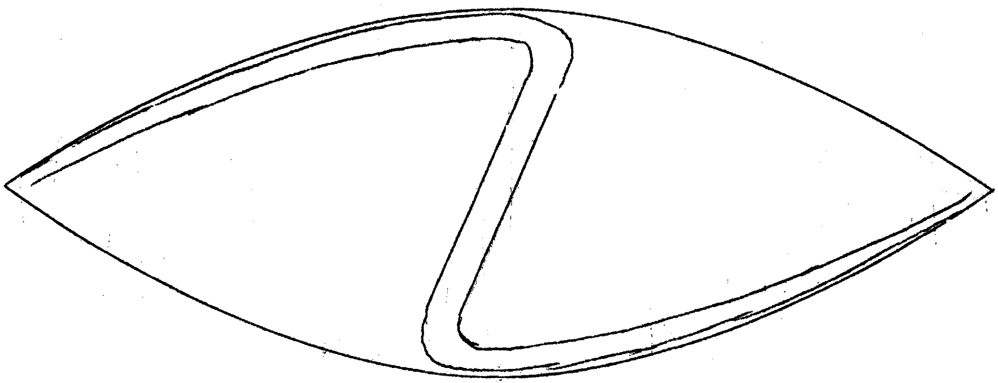


Fig. 2 (b)

and the longitudinal limit would at best be given by the situation of Fig. 2b and would be the value given by Table 1 reduced by a bunching factor of the same order (though a bit less) than before.

For high harmonic operation we would have the state of affairs shown in Fig.3.

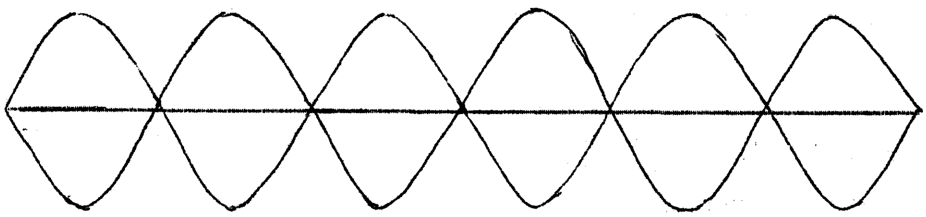
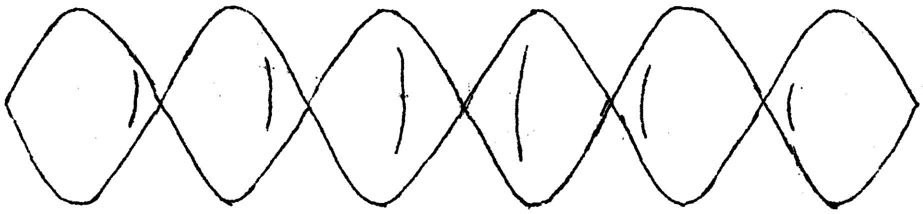


Fig 3a (h = 6)

becoming in about a quarter synchrotron oscillation something like this:



(h = 6)

Fig 3b

with similar consequences. These considerations apply to single-turn injection. Multi-turn injection will be discussed later.

It will therefore be necessary to pass the injected beam through an energy spreading cavity which would have an amplitude of about 500 V and a frequency very much higher than the R.F. bucket frequency. A 10-cm cavity would perhaps be convenient.

With $h = 10$ the bucket frequency would be 125 Mc/s (for Ring 1) and we would have 24 waves per bucket, as illustrated in Fig. 4.

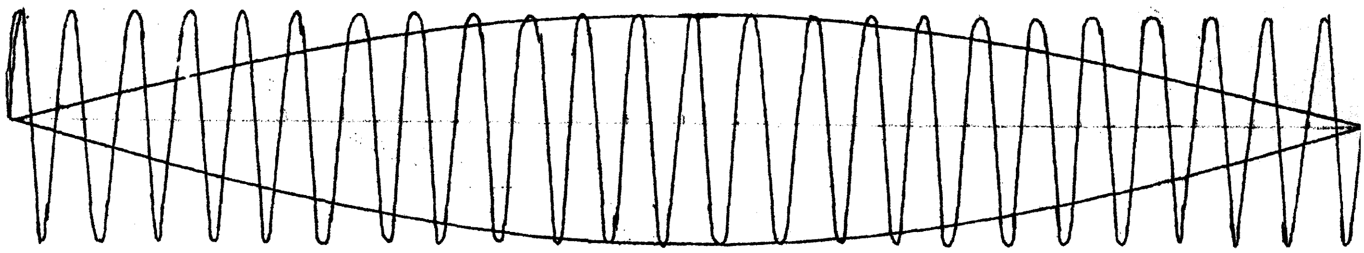


Fig. 4

Obviously the situation is improved by using a lower value of h or a higher frequency cavity for the spreader.

If the time for one synchrotron oscillation divided by the number of spreader waves per bucket is much less than the build-up time for longitudinal space-charge instability (with $\Delta E = 0$) we should expect to have a stable situation.

The phase oscillation period is given by

$$T_{ph} = 2\pi \left[\frac{E(\gamma^2 - 1)(k+1)}{hV \left| k + 1 - \gamma^2 \right| 2\pi f^2} \right]^{1/2} \qquad (5)$$

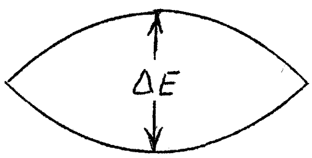
$$(\Gamma \equiv \sin \varphi_s = 1)$$

Combining (5) with (2) we obtain

$$f_{sp}^2 \gg \frac{150\, n^2\, g\, N\, e\, h\, c^2}{\pi\, \gamma^2\, R^3\, V} \qquad (V \text{ in volts}) \qquad (6)$$

where f_{sp} is the frequency of the spreader cavity.

Now for a stationary bucket of amplitude $\Delta E/2$



$$V = \frac{(\Delta E)^2\, \pi h \left| k + 1 - \gamma^2 \right|}{8 E_o\, \gamma(\gamma^2 - 1)(k+1)} \qquad (7)$$

With $\Delta E = 10^3$, $h = 10$, $k + 1 \doteq 7$, $\gamma = 4$
$E_o = 0.51 \times 10^6$

$$V = 0.17 \text{ volt}$$

And with $n \simeq 10$, $g \simeq 8$, $N = 10^9$, $e = 4.8 \times 10^{-10}$
$c = 3 \times 10^{10}$, $\gamma = 4$, $R = 400$, $h = 10$

$$f_{sp} \gg \sim 1 \text{ k Mc}$$

Thus it seems that whereas an S-band cavity might be all right ($f \sim 3$ kMc) it would probably be better to go to X-vand ($f \sim 10$ kMc) or even Q-band ($f \sim 30$ kMc). Since we already have S-band equipment in the lab, perhaps this should be tried first. Q-band would probably have to be excluded because of the very small beam cross-section that would be required.

It has been assumed up to now that the R.F. bucket frequency is always correctly related to the mean energy of the electrons during the injected pulse. Since the terminal voltage will be subject to a "belt-ripple" (frequency spectrum at present not known) which may be as much as $\pm$ 2 kV, we have to consider what can be done about this. This will be done in the next part of this report.

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