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SINGLE-TURN INFLECTOR FOR ELECTRON BEAM-STACKING EXPERIMENTS AT ENERGIES IN THE RANGE 1.5 - 3.5 MeV.

PS/1692

SINGLE-TURN INFLECTORS FOR 2 MeV ELECTRON BEAM STACKING EXPERIMENTS.

1. Introduction.

The proposed storage ring (PS/Int. AR/60-6) has straight sections whose basic dimensions are as shown in Fig. 1.

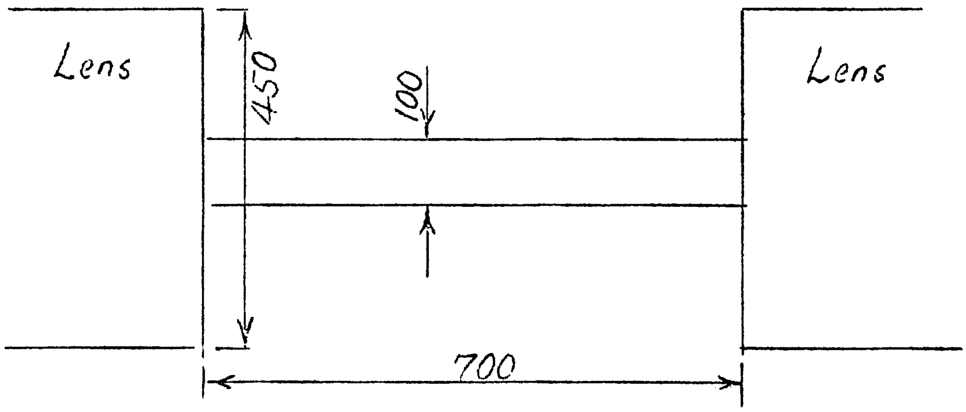

Fig. 1.

The problem studied in this report is the basic design of a single-turn pulsed inflector which could be located in such a straight section. It would be required to bring the centre of the injected beam into coincidence with the equilibrium orbit at injection energy within specified limits of error, and to produce a negligible disturbance in the region of the stacking orbit.

It will be assumed that the injection orbit lies 1 cm inside the outer edge of the vacuum chamber, and that the stacking orbit lies at the centre of the chamber, i.e. 4 cm from the injection orbit.

The time for one revolution is taken to be 80 ns.
Electrons traversing the inflector while the inflector field has the correct value will be placed on orbit and will remain on orbit provided that the inflector field is zero during the second and all subsequent transversals. The implications of this requirement are made clear in Fig. 2.

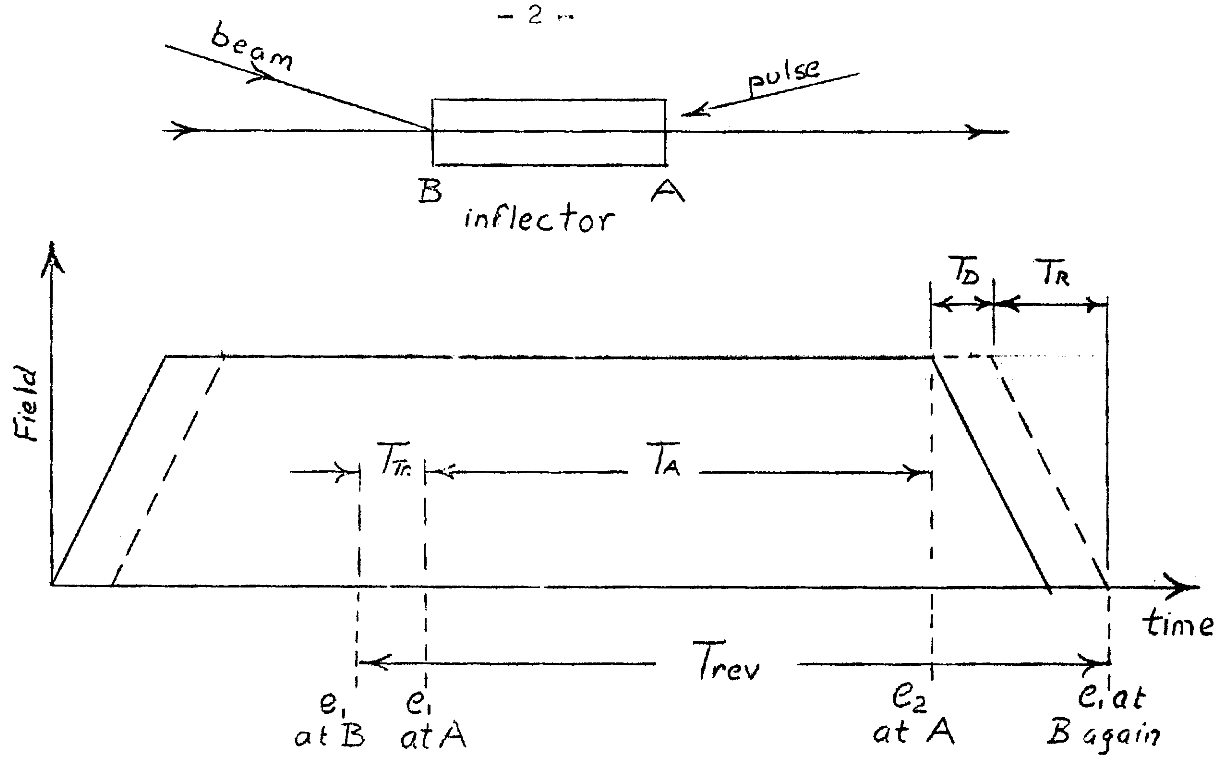

Fig. 2.

In Fig. 2 , e_1 is the earliest and e_2 the latest acceptable electron. The acceptance time T_A is evidently given by

$$T_A = T_{rev} - (T_{Tr} + T_D + T_R) \qquad (1)$$

in which T_{Tr} is the time of transit of an electron through that part of the inflector which is transversed by the equilibrium orbit, T_D is the delay time for propagation of the pulse along the same part of the inflector, and T_R is the rise time of the inflecting pulse.

In the design to be described in this report, $T_{Tr} \simeq T_D = 1.2$ ns. We thus require that T_R should be less than $(8 - 2.4) = 5.6$ ns for the inflection efficiency, defined as

$$\eta_I = \frac{T_A}{T_{rev}} \times 100 \qquad (2)$$

to be not less than 90 o/o.

The factors determining T_R are discussed in a separate paragraph, but it seems reasonable to hope that with two or three-stage pressurized trigge ed spark gaps and suitable design of the charging line, rise-times appreciably shorter than 5 ns should be achievable.

2. Unloaded or Loaded Transmission Lines?

The "delay-line" type of inflector described by O'Neill (Princeton University Internal Report GKO'N-10, VK-3, December 18th, 1957) and by Kuiper and Plass (CERN 59-30) would appear to suit our requirements. The basic arrangement is as shown in Fig. 3.

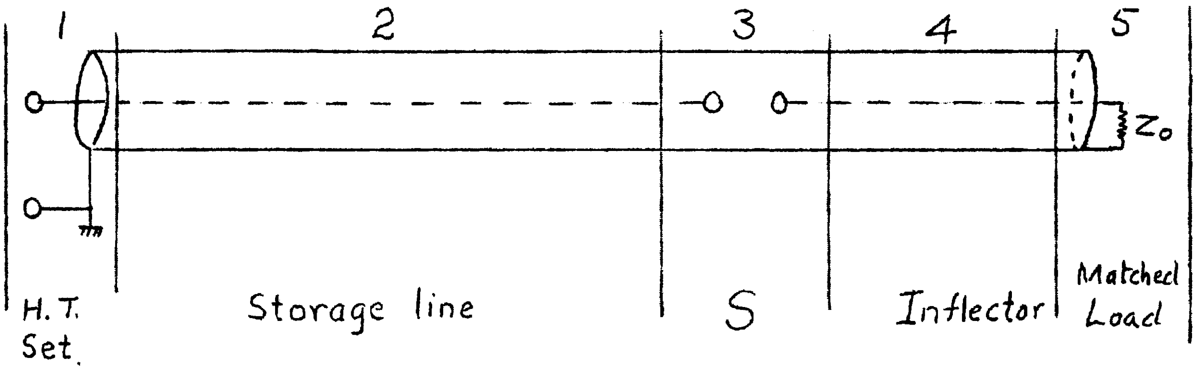

Fig. 3

It consists of essentially five components:

1. A high-voltage rectifier set charges
2. a storage line whose length determines the length of the pulse,
3. A triggered spark gap switch S connects the storage line to
4. the inflector line, down which, or part of which, the electrons are directed, and are deflected by the constant electromagnetic field which exists in it during the pulse.
5. The inflector line is terminated by a matched load.

In the inflectors described by O'Neill and by Kuiper and Plass, ferrites are used to increase the inductance or, in other words, to increase the magnetic induction (and hence the kick per unit length) per unit current. As is shown by the calculations in Appendix 1, this is probably unavoidable if one is concerned with, for instance, 100 MeV electrons or 25 GeV protons.

When, as with an electron storage ring, rise-times appreciably less than 10 ns are required, the use of ferrites begins to be of dubious value.

To produce a given deflecting field the current I_F required in a ferrite-loaded inflector compared with the current I required in an unloaded inflector is given by

$$\frac{I_F}{I} = \frac{h}{2\pi r} + \frac{1}{\mu_s'}\left(1 - \frac{h}{2\pi r}\right) \qquad (3)$$

where h is the gap width in the ferrite loaded inflector, r the mean radius in either inflector, (see Fig. 4), and μ_s' is the real part of the complex permeability of the ferrite. Fig. 16 in the Mullard book on Ferroxcube (1955) gives curves showing the variation of μ_s' with frequency for all grades of Ferroxcube. The best for high frequency application, namely grade B5, shows that $\mu_s' \simeq 20$ in the range of interest up to $\simeq 10^8$ c/s, after which it drops rather rapidly.

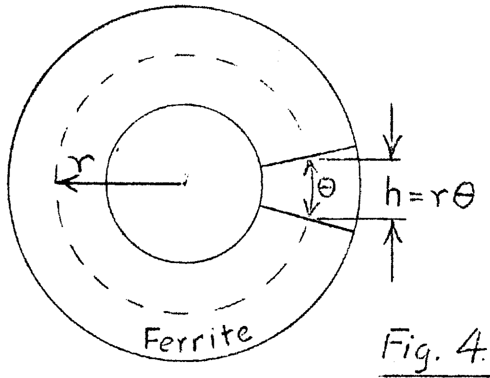

Fig. 4

Thus with $h = 0.5$ cm and $r = 2.25$ cm, for example, we would have

$$\frac{I_F}{I} \simeq 0.0353 + \frac{0.9647}{20} = 0.0835,$$

i.e. a gain of a factor of about 12. At the same time, as the curves show, the loss factor

$$\tan \delta_r = \frac{\mu_s''}{\mu_s}$$

varies very rapidly with frequency in the region 10^6 to 10^8 c/s, from about 1/200 at the lower frequency to about 1/2 at the higher. Consequently the ferrite would produce considerable distortion of the pulse, by progressively increasing attenuation of the higher-frequency components, i.e. prolongation of the effective rise time. *) (see footnote p. 5.)

For these reasons it was considered advisable to design an unloaded inflector (i.e. without ferrite) for use at 2 MeV, and meanwhile to start some experimental studies on the behaviour of a ferrite loaded inflector, which latter might have to be used in the event of a higher injection energy being required.

A consideration which must be taken into account in any distributed-parameter transmission-line inflector, is the existence of the electric field component, which may either add to or subtract from the magnetic deflection, depending upon relative directions of particle motion and wave propagation. See Fig. 5.

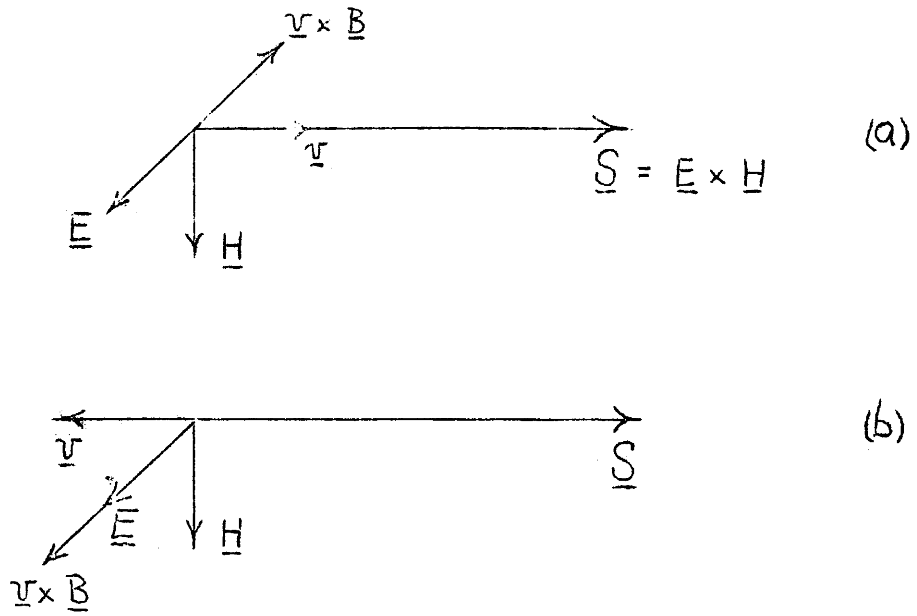

Fig. 5.

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x) (See preceding page).
It is shown in Appendix 2 that the ratio of the energy stored in the ferrite to that stored in a radial gap of angular width θ is $U_F/U_g = \frac{1}{\mu}\left(\frac{2\pi}{\theta} - 1\right)$, where μ is the real part of the complex permeability of the ferrite. Thus, for instance, with $2\pi/\theta = 36$ and $\mu = 20$, $U_F/U_g = 1.75$. This shows that ferrite losses, and particularly the frequency dependence of these losses, must play a significant part in distorting the pulse.

Since $\underline{F} = \underline{q} \, (\underline{E} + \underline{v} \times \underline{B})$ and the Poynting vector $\underline{S} = \underline{E} \times \underline{H}$, it is obvious from the figure that if $\underline{S}$ and $\underline{v}$ are in the same direction, the forces subtract, whereas if $\underline{S}$ and $\underline{v}$ are in opposite directions, the forces add.

Since in a principal mode field configuration in vacuo

$$B = \mu_0 H = E \sqrt{\mu_0 \varepsilon_0} = \frac{E}{c}$$

$$F = q E \left(\underline{n} + \frac{\underline{v} \times \underline{n}}{c} \right)$$

where $\underline{n}$ is a unit vector in the direction of $\underline{E}$, and the two force components tend to equality as $v \rightarrow c$.

Thus for relativistic particles the effective magnetic deflecting field is practically double the actual magnetic field.

In the ferrite-loaded distributed-parameter line shown in Fig. 4, it is shown in Appendix 2 that for $v \simeq c$ the magnetic and electric deflecting forces are in the ratio

$$\frac{F_H}{F_E} \simeq \left[\frac{1}{\mu} + \frac{\theta}{2\pi} \left(1 - \frac{1}{\mu} \right) \right]^{-1/2} \tag{4}$$

Thus, for instance, with $\mu = 20$ and $\theta/2\pi = 1/36$,

$$\frac{F_H}{F_E} \simeq 3.6$$

In this case the effective deflecting force with the correct choice of direction of propagation and particle motion would be $4.6/2.6 = 1.77$ times larger than with the incorrect choice.

If, of course, the inflector is designed with lumped parameters in such a way as to practically separate in space the electric and magnetic fields, then the above consideration would not apply.

3. Perturbation of the Stacked Beam by Leakage Field from the Inflector.

In the region where the injection orbit intersects the inflector, the outer wall of the latter must be provided with a slot to allow the beam (including its betatron oscillations) to circulate without hitting the inflector.

This is illustrated in Fig. 6.

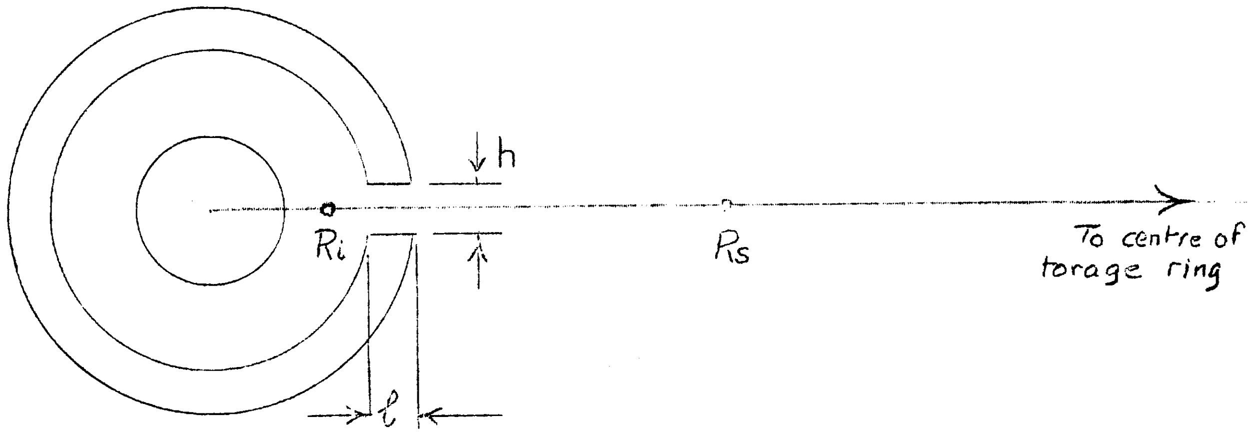

Fig. 6.

If the slot length ℓ is made sufficiently long, possibly by prolonging it with "lips" projecting a little way in towards the stacking orbit, propagation in the direction $R_i \rightarrow R_s$ can be greatly attenuated for all the significant Fourier components of the pulse, which will be of lower frequency than the cut-off frequency.

For a parallel plane wave-guide of separation h the cut-off frequency of the n^{th} order mode is

$$f_c = \frac{nc}{2h}$$

Thus with $h = 0.5$ cm the lowest $(n = 1)$ cut-off frequency will be

$$f_{c_1} = 3 \times 10^{10} \text{ c/s} \qquad (\lambda_c = 1 \text{ cm})$$

If the pulse rise-time is 8 ns, the highest frequency component of interest would be $f \sim \frac{0.45}{8 \times 10^{-9}} = 5.625 \times 10^7$ c/s with the corresponding wavelength $\lambda \sim 533$ cm.

the attenuation is

$$\alpha = \frac{n\pi}{h} \sqrt{1 - \left(\frac{2h}{n\lambda}\right)^2} \quad \text{cm}^{-1} \qquad (5)$$

For waves far below cut-off $n\lambda \gg 2h$ and

$$\alpha \approx n\pi/h \quad \text{cm}^{-1} \qquad (6)$$

Then with $n = 1$ $h = 0.5$

$$\alpha = 2\pi \quad cm^{-1}$$

The field attenuation factor for three values of ℓ is as follows:

ℓ =	1	1.5	1.25 cm
Attenuation factor	5.43×10^2	1.26×10^4	2.63×10^3

In the design described in the next section, the magnetic field just inside the coaxial line is about 30 gauss. The extimated tolerable field at the stacking radius is about 2×10^{-2} gauss.

Thus with $\ell = 1.25$ cm one would have at the exit from the parallel plate guide a field of $30/2.6 \times 10^{-3} = 1.14 \times 10^{-2}$ gauss. There would be a further inverse-square-law reduction of the field strength between this point and the stacking radius, since we would here be dealing with the induction field.

It therefore seems that an extension of ~ 1.25 cm of the slot should reduce the disturbance at the stacking radius to a value comfortably below tolerance.

4. Design of an Unloaded Transmission Line Inflector.

(a) General features.

The general layout of the inflector which is to be built and tested in the immediate future is shown in Fig. 7.

The inflector itself consists of a coaxial line of inner diameter 3 cm and outer diameter (inside the line) 6 cm. It will accordingly have a characteristic impedance $Z_o = 60 \ln \frac{b}{a} = 41.6$ Ohm.

The radius of curvature of the orbit, which will be mid-way between the inner and outer conductors, will be $\rho_d = 91.5$ cm. The angle of deflection $\theta_d = 22.5$ deg.

For inflecting electrons of 1.75 MeV kinetic energy, for which $\beta = 0.975$ and $\gamma = 4.43$ the effective magnetic field must be

$$B_{eff} = 1.705 \times 10^{-3} \beta\gamma/\rho_d = 7.36 \times 10^{-3}/\rho_d \quad W/m^2.$$

and so

$$B_{eff} = \frac{7.36 \times 10^{-3}}{0.915} = 80.4 \times 10^{-4} \quad W/m^2$$

The actual magnetic field thus needs to be

$$B = \left(\frac{1}{1+\beta}\right) B_{eff} = \frac{1}{1.975} B_{eff} = 40.7 \times 10^{-4} \ W/m^2$$

The current required to produce this field inside the coaxial line at radius r_o is

$$I = \frac{2 \pi r_o B}{\mu_o} = \frac{2 \pi \times 2.25 \times 10^{-2} \times 40.7 \times 10^{-4}}{4 \pi \times 10^{-7}} \ A = 458 \ A$$

The corresponding line voltage will be

$$V = I Z_o = 458 \times 41.6 \times 10^{-3} = 19.1 \ kV$$

The inflector must be terminated by its characteristic impedance in order to avoid ringing of the inflector - storage line system, since the storage line must be effectively open circuited at its its input end, where it is connected to the HT set.

This termination must be capable of dissipating a pulse power of

$$W_p = 19.1 \times 458 \times 10^{-3} = 8.75 \ MW$$

The pulse length would be about 0.1 μs and the repetition rate about 100 pps. Th duty cycle would thus be 10^{-5} and the mean power

$$W_{av} = 87.5 \ Watt.$$

Resistors of the metal film on ceramic type are available which should be capable of dissipating such power and of presenting uniform resistance up to about 1 kMC.

The termination to be used is the standard exponential taper type described, for instance, by Zaccheroni (CERN 58-27).

To permit higher injection energies, and correspondingly higher power dissipation in the termination, it is proposed to use a resistor capable of dissipating 300 W with refrigerated oil cooling.

The inflector is connected to the pulse-forming line via a triggered spark gap. A special two-stage gap designed by Schneider is now about to be tested. We may use a pressurised version of this gap, or a more elaborate three-stage gap, depending on the test results on the present prototype.

The pulse-forming line will be 15 metres long, giving a pulse length of 100 ns, and will have the same cross-section dimensions as the rest of the system. Since it will have to be charged to 38 kV D.C. (for inflecting 1.75 MeV electrons), it will be pressurised in order to avoid breakdown along the surfaces of the ceramic discs supporting the inner conductor.

The electrons will enter the inflector after having been deflected some 67° by a bending magnet.

The vertical aperture of the inflector slot will be 0.5 cm, and in the design study prototype we will have detachable lips 1 to 1.5 cm wide, in order to study the field outside the inflector.

It would be interesting to see whether the same inflector can be used for higher injection energies. At 3.5 MeV (kinetic) the charging voltage would have to be 76 kV. It is accordingly proposed to buy an H.T. set capable of going up to about 80 kV.

At this level, the power dissipation in the termination would be 350 W, and it remains to be seen whether the 300 W resistors can handle this. If not, longer resistors of the same diameter are available, which go up to 1 kW.

(b) Tolerances.

We have $B_\rho = \frac{p}{e}$

Thus $\rho \Delta B + B \Delta\rho = \frac{1}{e} \Delta p$.

The tolerance on B is accordingly given by putting $p = 0$ and so $\frac{\Delta B}{B} = -\frac{\Delta\rho}{\rho}$; and the tolerance on ρ by putting $B = 0$ and so $\frac{\Delta p}{p} = \frac{\Delta\rho}{\rho}$.

If the deflection angle is Θ , it may easily be shown that an error $\Delta\rho$ will cause a positional error

$$\Delta x \simeq \Delta\rho\,(1 - \cos\theta)$$

and a divergence $\Delta\alpha \approx \frac{\Delta\rho}{\rho}\sin\theta$ (See Fig. 8).

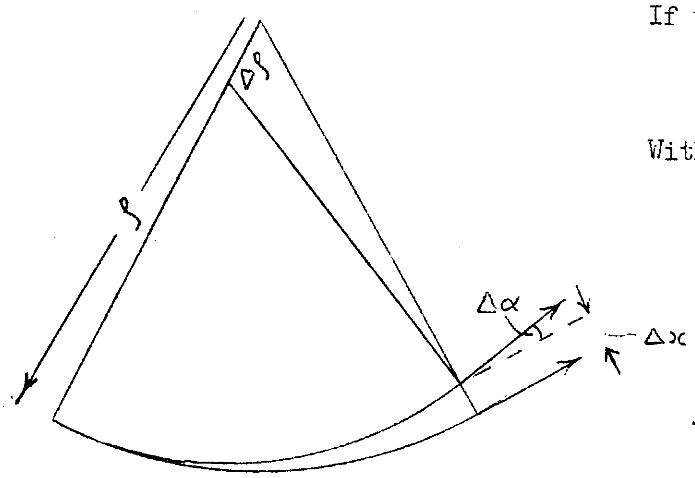

Fig. 8

If we require

$$\Delta x < 1 \text{ mm},$$

With $\theta = 22.5^{\circ}$

$\cos\theta = 0.92$

$\sin\theta = 0.38$

$\rho = 915$ mm ~ 1000 mm

$$\frac{|\Delta B|}{B} = \frac{|\Delta p|}{p} = \frac{|\Delta\rho|}{\rho} < 10^{-3}$$

The upper acceptable limit on the beam emittance will be $\sim 5 \times 10^{-4}$ cm.rad. With a beam of 1 mm radius, this corresponds to a divergence of 5×10^{-3} rad.

If $\frac{\Delta\rho}{\rho} < 10^{-3}$ then $\Delta\alpha < 4 \times 10^{-4}$ rad, an order of magnitude less than the divergence, and so quite acceptable.

Thus to inflect within 1 mm of the correct position and with negligible divergence error we shall require that the inflector field and the particle momentum be constant (and correctly related) to within 1 part in 10^3.

This will require (a) a stabilised charging voltage on the inflector storage line, (b) a stabilised injector, and (c) some means of correlating the inflector charging voltage and the injection energy.

The requirement (a) can be satisfactorily met either with a series regulator valve or with a magnetic amplifier system similar to that used in the Linac Modulator Units (CERN-PS /BWM-1, 1957)

Requirement (b) implies a pulse-to-pulse stability of $< \pm 2$ keV (in 2 MeV) as well as an energy spread not larger than the same amount in each pulse. This requirement can be met with a Van de Graaff enerator (with a liner), but the capability of a linac in this respect is not yet certain.

Regarding (c), it should be possible to devise an arrangement (e.g. an error signal from a magnetic analyser) which can adjust the inflector charging voltage in accordance with slow drift (or for that matter deliberate alteration) of the injector voltage.

(c) Defocusing.

The field inside the inflector is radially defocusing, since both H_φ and E_r decrease as $1/r$. The n-value, defined as

$$n = \frac{R}{B} \frac{dB}{dR}$$

may be shown to be equal to $- R_o/r_o$ where R_o is the radius of curvature of the orbit in the inflector, and r_o is the radial position of this orbit measured from the inflector axis.

In the present case $R_o = 91.5$, $r_o = 2.25$, so $n = - 40.7$.

The growth of radial betatron amplitude during the traversal of the inflector, which subtends an angle $\theta_d = 22.5^o = \frac{2\pi}{16}$, is, accordingly

$$\exp \left[\frac{1}{16} \sqrt{41.7} \right] = \simeq 1.5$$

This does not seem large enough to worry about, but, if necessary, it could be compensated in advance of the inflector by a suitable radially focusing lens.

(d) Factors affecting the rise-time.

The rise time T_R (see Fig. 2) will be determined by the breakdown time of the spark gap and by losses and reflections in the storage-line, switch and inflector system. We expect the breakdown time to be the most important factor. This time can be reduced, perhaps to $\sim$ 1 ns, by shielding the second gap of a two-stage triggered gap, so that there is no pre-ionisation. After the first gap has broken down and the second gap is subjected to an overvoltage, the latter will break down suddenly and rapidly, after a randomly variable delay, when an ion-producing event occurs. The concomitant of fast breakdown is thus relatively large jitter. If this jitter is unacceptably large, we might consider triggering the second gap breakdown by means of an X-ray pulse produced by one of the Van de Graaff beams. In any case the breakdown time may be shortened and/or the jitter reduced by pressurising the spark gap, and we should try this. Tests on spark-gaps of this type (at present not pressurised) are now starting.

APPENDIX 1.

Inflectors for Electrons in the Energy Range 10 MeV - 1 GeV and for Protons in the Energy Range 1 GeV - 25 GeV.

The inflector voltage is given by the formula x):

$$V = \left(\frac{m_o c^2}{e}\right) \frac{r_{av}}{\rho} F_\mu \beta \gamma \ln \frac{b}{a} \qquad (1)$$

where $m_o c^2/e$ is the particle rest energy in eV,

$r_{av} = \frac{a+b}{2}$

ρ = radius of curvature of particle trajectory in the inflector

$\beta = v/c$, $\gamma^2 = (1 - \beta^2)^{-1}$

b = outer radius, a = inner radius of co-axial inflector

$$F_\mu = \frac{\left[\mu\left\{1 + \frac{\theta}{2\pi}(\mu - 1)\right\}\right]^{1/2}}{\mu + \left[1 + \frac{\theta}{2\pi}(\mu - 1)\right]^{1/2}}$$

μ = real part of ferrite permeability

θ = angle subtended by radial gap in ferrite

$= h/r_{av}$

where h = mean height of radial gap.

The maximum field gradient in the coaxial line is

$$E_{max} = V/a \ln \frac{b}{a} \qquad (2)$$

$$= \left(\frac{m_o c^2}{e}\right) \frac{r_{av}}{\rho} F_\mu \beta \gamma / a \qquad (3)$$

Equations (1) and (3) may conveniently be normalised in terms of $(\beta\gamma)$ and ρ

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x) See Appendix 2.

$$\left(\frac{\rho V}{\beta\gamma}\right) = \left(\frac{m_o c^2}{e}\right) r_{av} F_\mu \ln \frac{b}{a} \qquad (4)$$

$$= 5.12 \times 10^5 \; r_{av} F_\mu \ln \frac{b}{a} \quad \text{V.m. for electrons} \qquad (4a)$$

$$= 9.4 \times 10^7 \; r_{av} F_\mu \ln \frac{b}{a} \quad \text{V.m. for protons} \qquad (4b)$$

$$\left(\frac{\rho E_{max}}{\beta\gamma}\right) = \frac{m_o c^2}{e} \; r_{av} F_\mu / a \qquad (5)$$

$$= 5.12 \times 10^5 \; r_{av} F_\mu / a \quad \text{V for electrons} \qquad (5a)$$

$$= 9.4 \times 10^7 \; r_{av} F_\mu / a \quad \text{V for protons} \qquad (5b)$$

These formulae have been used to calculate the inflector voltages required to inflect electrons in the energy range 10 - 1000 MeV and protons in the energy range 1 - 25 GeV.

These calculations have been made with the following assumptions and restrictions:

The vertical aperture of the inflector measured at the mean radius r_{av} (i.e. the mean height h) must not be less than 0.5 cm for the electron machines and 1.0 cm for the proton machines.

The radial aperture (b - a) of the inflector must not be less than 1 cm for the electron machines and 2 cm for the proton machines.

Four grades of "Ferroxcube" are considered, namely B_2, B_3, B_4, and B_5 (Mullard Ferroxcube, 1955; page 41).

These grades have approximately constant real permeabilities, of 300, 100, 50, and 20 respectively, up to frequencies of 7, 20, 40 and 90 Mc/s respectively.

If we take the final orbit frequency of the accelerator concerned and require that the ferrite should be good up to a frequency at least 10 times higher than this (giving rise-times somewhat less than 10 o/o of the revolution period), we obtain the following frequencies for representative accelerators: (see Table 1).

Table 1.

Accelerator	Particle	Energy GeV	Max. Freq. Mc/s	Ferrite Grade	Permeability
CERN PS	p	25	5	B2	300
Nimrod	p	7	20	B3	100
Saturne	p	2.5	42	B4	50
DESY	e	6	9.5	B2	300
Frascati	e	1	109	B5	20
Bonn	e	0.5	181	B5	$<$ 20

Under the above assumptions the voltage and the voltage gradient required for a simple, ferrite-loaded, distributed parameter inflector have been calculated for each of the above cases. For the proton machines the inflector dimensions were taken to be:

$$a = 2 \text{ cm} \quad b = 4 \text{ cm} \quad h = 1 \text{ cm} ;$$

and for the electron machines

$$a = 2 \text{ cm} \quad b = 3 \text{ cm} \quad h = 0.5 \text{ cm}.$$

The results are shown in Table 2:

Accelerator	ρV (MVm)	ρE_{max} (MV)
CERN PS	10	724
Nimrod	2.75	200
Saturne	0.9	65
DESY	9	1110
Frascati	2.2	272
Bonn	1.2	148
100-MeV Electron S.R.	0.22	27

The magnitude of ρ , the radius of curvature of the particle trajectory in the inflector, will be determined by the straight-section geometry and by whether the inflector must bring the beam onto orbit without making use of betatron oscillations, or whether, for instance, a bending magnet located a quarter betatron wavelength upstream can be used in conjunction with the inflector to place the beam on orbit. Such a scheme (in reverse) is proposed by Kuiper and Plass (CERN 59-30) for the CERN PS fast extractor, and with this arrangement they obtain effectively $\rho \approx 323$ m. This method depends, of course, on Q_R being constant. For our storage ring experiments we will probably want to be able to vary the Q's, and so should, if possible, avoid an inflection system which makes use of betatron oscillations.

If, for all except the last machine in Table 2, we assume that, as in the PS, ρ can be made about 3.2 times larger than the accelerator orbit radius, we arrive at the following very rough estimates of voltage and gradient:

Table 3.

Accelerator	V (kV)	E_{max} (kV/cm)
CERN PS	31	22.5
Nimrod	46	33.3
Saturne	33	23.9
DESY	89	110
Frascati	190	243
Bonn	141	174
(a) 100 MeV S.R. (ρ = 1 m)	220	275
(b) 100 MeV S.R. (ρ = 12 m)	18	23

For the 100 MeV storage ring it is assumed (a) that the straight section geometry would be not much different from what is now proposed, which would limit ρ to about 1 m unless some way could be found to bring the inflector through the side of one of the lenses.

Version (b) of the 100-MeV storage ring assumes that use can be made of a quarter betatron oscillation, as in the other examples calculated. In this case $\rho \sim 12$ m. Evidently we would not be able to retain the feature of variable Q if we want to use the storage ring at energies approaching 100 MeV.

It is interesting to observe that the CERN PS presents the least difficult ejection/inflection problem of all the machines considered, which is due, of course, to its comfortably large size.

A more detailed design study should perhaps be made of a distributed parameter type of ejector/inflector for the PS and a possible 25 GeV storage-ring.

What voltage gradients can in fact be reached in evacuated, ferrite-loaded lines subjected to short pulses? This is a question which we will probably have to study experimentally, unless this has already been done, e.g. at Philips (Eindhoven).

The case of an unloaded line corresponds, of course, to $\mu = 1$ and $\Theta = 2\pi$. Then $F_\mu = 0.5$, and, for electrons,

$$\left(\frac{\rho V}{\beta\gamma}\right) = 2.56 \times 10^5 \; r_{av} \ln \frac{b}{a}$$

and with $r_{av} = 2.5$ cm, $b/a = 1.5$ as before, we obtain

$$\frac{\rho V}{\beta\gamma} = 2.6 \times 10^3 \text{ Vm}$$

and with $(\beta\gamma) = 195$, $\rho V = 0.51$ MVm.

This would mean that with the proposed storage ring dimensions, for which $\rho \sim 1$ m, we would have $V \sim 500$ kV, which would be impossible.

If on the other hand, we can keep Q_R constant and use betatron oscillations for inflection, ρ could be increased to say 10 metres, and V would be ~ 50 kV, which would be acceptable.

Thus it may be concluded that at 100 MeV in the proposed storage ring it might be possible to make a ferrite-loaded inflector without depending upon Q_R, an unloaded inflector would certainly require a Q-dependent system.

APPENDIX 2.

Electromagnetic Fields in Ferrite-loaded Distributed-parameter Coaxial Lines.

Consider a distributed-parameter ferrite-loaded line of the form shown in Fig. 4 of the text.

Call the region filled with ferrite (2) and the empty region (1).

Then assuming that the field configurations are those of the principal mode, we have an azimuthal magnetic field H_1 in (1) and H_2 in (2) for which

$$H_1 = C_1/r \qquad , \qquad H_2 = C_2/r \tag{1}$$

Across the boundary surface the induction must be continuous, i.e.

$$\mu_1 H_1 = \mu_2 H_2$$

or

$$H_1/H_2 = C_1/C_2 = \mu_2/\mu_1 \tag{2}$$

If the centre conductor carries a current I , then

$$\oint \underline{H} \cdot d\underline{\ell} = I$$

or

$$H_1 r \theta + H_2 r (2\pi - \theta) = I$$

$$H_1 r \theta + H_1 \frac{\mu_1}{\mu_2} r (2\pi - \theta) = I \qquad \text{from (2)}$$

or

$$H_1 = \frac{I}{2\pi r} \frac{1}{\left[\frac{\mu_1}{\mu_2} + \frac{\theta}{2\pi}\left(1 - \frac{\mu_1}{\mu_2}\right)\right]} \tag{3}$$

The stored magnetic energy per unit length of line is

$$U = 1/2 \int_V \underline{B} \cdot \underline{H} \, dv$$

$$= 1/2\, \mu_o \left[\mu_2 (2\pi - \theta) \int_a^b r H_2^2 \, dr + \mu_1 \theta \int_a^b H_1^2 \, r \, dr \right]$$

$$= 1/2\, \mu_o H_1^2 r^2 \ln \frac{b}{a} \left[(2\pi - \theta) \frac{\mu_1^2}{\mu_2^2} + \mu_1 \theta \right] \qquad (4)$$

If the inductance per unit length is L , then $1/2\, L\, I^2 = U$, and by substituting for I from (3) and u from (4) we obtain

$$L = \frac{\mu_o}{2\pi} \ln \frac{b}{a} \left[\frac{\mu_1}{\frac{\mu_1}{\mu_2} + \frac{\theta}{2\pi} \left(1 - \frac{\mu_1}{\mu_2}\right)} \right] \qquad (5)$$

Assuming the permittivity of the ferrite to be unity, and again assuming quasi-static fields, the capacitance per unit length will, as usual, be

$$C = 2\pi\, \varepsilon_o / \ln \frac{b}{a} \qquad (6)$$

From (5) and (6) the characteristic impedance is

$$Z_o = \sqrt{\frac{L}{C}} = \frac{1}{2\pi} \sqrt{\frac{\mu_o}{\varepsilon_o}} \ln \frac{b}{a} \left[\frac{\mu_1}{\frac{\mu_1}{\mu_2} + \frac{\theta}{2\pi} \left(1 - \frac{\mu_1}{\mu_2}\right)} \right]^{1/2} \qquad (7)$$

We want to know the electric field in the radial gap. This is given by

$$E = \frac{V}{r \ln \frac{b}{a}} = \frac{I Z_o}{r \ln \frac{b}{a}}$$

Thus

$$E = \frac{I}{2\pi r} \sqrt{\frac{\mu_o}{\varepsilon_o}} \left[\frac{\mu_1}{\frac{\mu_1}{\mu_2} + \frac{\theta}{2\pi} \left(1 - \frac{\mu_1}{\mu_2}\right)} \right]^{1/2} \qquad (8)$$

From (8) and (3)

$$\frac{E}{H_1} = \sqrt{\frac{\mu_o}{\varepsilon_o}} \left\{ \mu_1^{1/2} \left[\frac{\mu_1}{\mu_2} + \frac{\theta}{2\pi} \left(1 - \frac{\mu_1}{\mu_2}\right) \right]^{1/2} \right\} \qquad (9)$$

In an unloaded line $E/H_1 = \sqrt{\mu_o \mu_1/\varepsilon_o}$, and as $v \to c$ the magnetic and electric forces approach equality, i.e. $F_H \to F_E$.

In the present case, therefore,

$$\frac{F_H}{F_E} \approx \frac{1}{\sqrt{\left[\frac{\mu_1}{\mu_2} + \frac{\theta}{2\pi}\left(1 - \frac{\mu_1}{\mu_2}\right)\right]}} \qquad (10)$$

$(v \approx c)$

When, as in the practical case, $\mu_1 = 1$, $\mu_2 = \mu$

$$\frac{F_H}{F_E} \approx \frac{1}{\sqrt{\frac{1}{\mu} + \frac{\theta}{2\pi}\left(1 - \frac{1}{\mu}\right)}} \qquad (11)$$

Thus, for instance, with $\mu = 20$ and $\theta/2\pi = 1/36$, $F_H/F_E \approx 3.6$. In this case one can reduce the voltage required for a given deflection by a factor $3.6/4.6 = 0.78$ by a correct choice of the relative directions of particle and wave propagation.

It is evident from equation (4) that the ratio

$$\frac{\text{Energy stored in ferrite}}{\text{Energy stored in gap}} = \frac{(2\pi - \theta)\, \mu_1^2/\mu_2}{\mu_1\, \theta}$$

$$= \frac{\mu_1}{\mu_2}\left(\frac{2\pi}{\theta} - 1\right)$$

$$= \frac{1}{\mu}\left(\frac{2\pi}{\theta} - 1\right) \qquad (12)$$

$$\begin{pmatrix} \mu_1 = 1 \\ \mu_2 = \mu \end{pmatrix}$$

Equation (5) may be written

$$L = \frac{\mu_o\, \mu_{eff}}{2\pi} \ln \frac{b}{a} \qquad (13)$$

where

$$\mu_{eff} = \frac{\mu_1}{\frac{\mu_1}{\mu_2} + \frac{\theta}{2\pi}\left(1 - \frac{\mu_1}{\mu_2}\right)} \tag{14}$$

And, correspondingly

$$Z_o = \frac{1}{2\pi}\sqrt{\frac{\mu_o}{\varepsilon_o}}\ \ln\frac{b}{a}\left[\mu_{eff}\right]^{1/2} \tag{15}$$

Similarly, from equation (3)

$$I = \frac{2\pi\, r_{av}\, B}{\mu_o\, \mu_{eff}} \tag{16}$$

where $B = \mu_o\mu_1 H_1$

$r = r_{av}$

Hence, the line voltage will be

$$V = I\, Z_o = \frac{c\, r_{av}\, B \ln b/a}{\sqrt{\mu_{eff}}} \tag{17}$$

where

$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

Now the magnetic field required for given radius of curvature ρ of the inflector trajectory is given by

$$B\,\rho = c\,\frac{m_o}{e}\,\gamma\,\beta\, f_\mu \tag{18}$$

where the factor f_μ is put in to allow for the contribution of the electric field to the deflection;

$$f_\mu = \frac{F_H/F_E}{1 + F_H/F_E} \tag{19}$$

and, for $v \approx c$

$$f_\mu \approx \frac{1}{1+\sqrt{\frac{1}{\mu} + \frac{\theta}{2\pi}\left(1 - \frac{1}{\mu}\right)}} \tag{20}$$

Substituting (18) in (17), we obtain

$$V = \frac{m_o c^2}{e} \frac{r_{av}}{\rho} F_\mu \ln \frac{b}{a} \beta \gamma \tag{21}$$

where $F_\mu = \dfrac{f_\mu}{\sqrt{\mu_{eff}}}$

$$= \frac{\sqrt{\frac{1}{\mu} + \frac{\theta}{2\pi}\left(1 - \frac{1}{\mu}\right)}}{1 + \sqrt{\frac{1}{\mu} + \frac{\theta}{2\pi}\left(1 - \frac{1}{\mu}\right)}} \tag{22}$$

$$= \frac{\sqrt{1 + \frac{\theta}{2\pi}\left(1 - \frac{1}{\mu}\right)}}{\sqrt{\mu} + \sqrt{1 + \frac{\theta}{2\pi}(\mu - 1)}} \tag{23}$$

M.J. Pentz.

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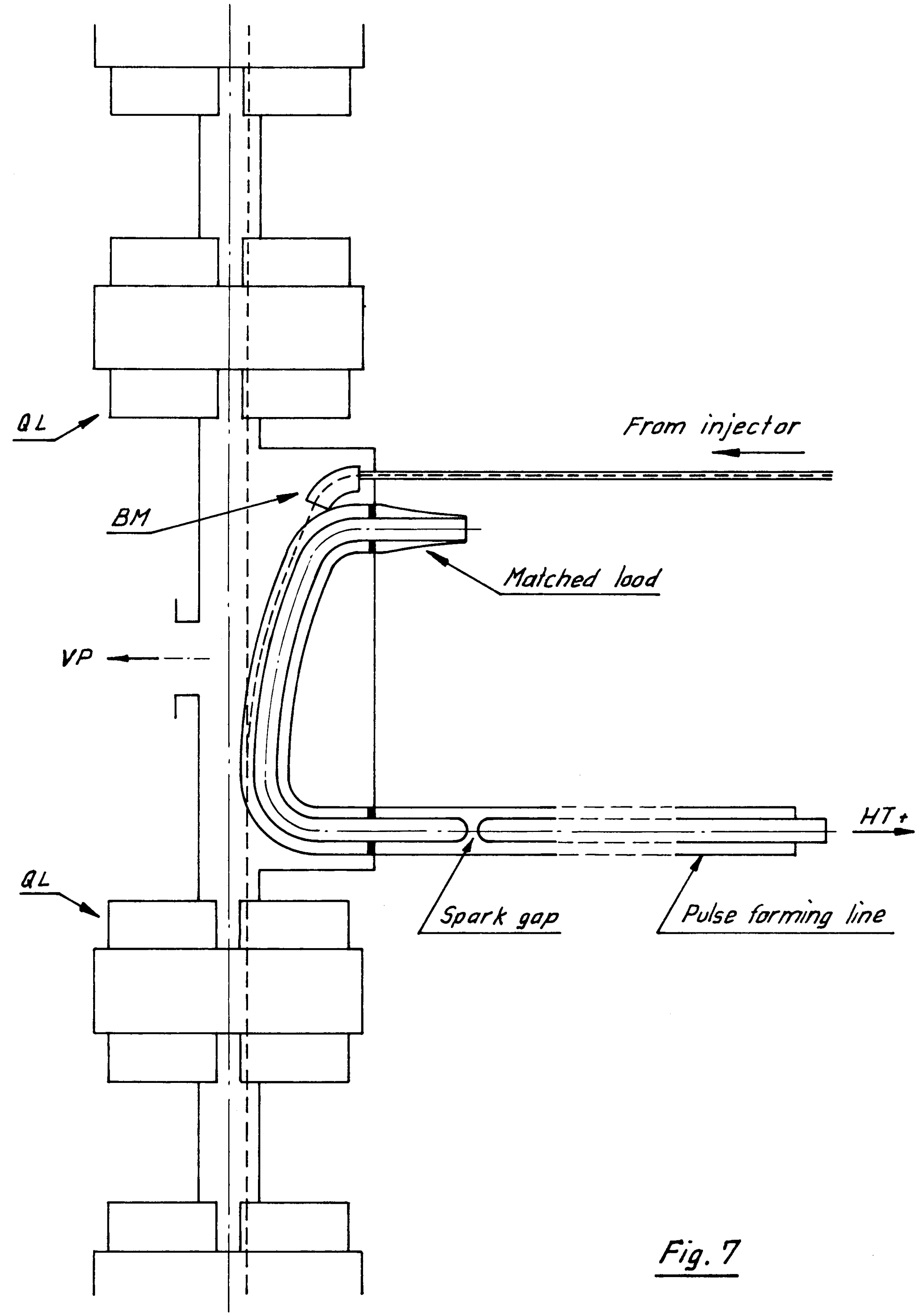

QL
From injector
BM
Matched load
VP
HT+
QL
Spark gap
Pulse forming line

Fig. 7