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MPS/SR/Note 73-41
29.11.1973

EDDY CURRENT EFFECTS IN ELLIPTICAL VACUUM CHAMBERS OF DIPOLE AND QUADRUPOLE (γ TRANSITION)

A.K. Mitra

1. Summary

This note describes the magnetic field and field gradient distortions due to eddy currents in elliptical vacuum chambers of the dipole and the quadrupole (γ transition) lenses. Mainly two methods, namely "Magnet Programme" and "Complex Transformation", have been used to calculate the eddy current effect.

2. Eddy Currents in the Vacuum Chamber of the Dipole

We assume an ideal dipole field $\bar{B}$ (0, By, 0) and the rise in field with time to be constant i, e $\frac{dB}{dt}$ = constant. From Maxwell's equation in cartesian coordinate :

$$\text{curl}\,\bar{E} = -\frac{\partial \bar{B}}{\partial t} \quad \text{where} \quad \bar{E}(0,0,Ez) \qquad (1)$$

$$E_z = \dot{B}_y x \quad (2) \quad \text{where} \quad \dot{B}_y = \frac{\partial B_y}{\partial t}$$

Current density $j_z = \frac{1}{\rho} E_z = \frac{\dot{B}_y}{\rho} x$ (3) , ρ = resistivity of the chamber material, and the eddy current $I_z = \frac{\dot{B}_y}{\rho} \cdot x \cdot \Delta x \cdot \Delta y$ (4).

The chamber is assumed to be infinitely long. One quadrant of the chamber is divided into 25 elements (n1 - n25) as shown in Fig. 1. Using the numerical values :

$$\frac{\partial B_y}{\partial t} = 7 \frac{V}{m^2} \text{ (i.e. 7 kGauss in 100 ms)}, \ \rho = 10^{-6} \Omega \text{ m.}$$

Current density $j_z = 7x \cdot 10^{-3} \frac{A}{mm^2}$ (5),(x in mm). The field has been calculated due to these eddy currents by three methods.

a) Using "Magnet" [1]

25 elements of the vacuum chamber (shown in Fig. 1) with their current densities are introduced in the "Magnet" as coil parameters and the field, due to eddy currents, is obtained (with infinite permeability of the iron). The results are whown in Fig. 2.

b) Using Image Method [2]

A current I in open air has a complex potential function at location Z :

$$U(Z) = A(Z) + iV(Z) = -\frac{\mu_o I}{2\pi} \ln\left(\frac{z - z_o}{C}\right) \quad (6)$$

Z_o = location of the current
A = vector potential parallel to the current
V = scalar potential of the magnetic induction B
C = arbitrary constant.

Taking two parallel currents symmetric with respect to the median plane and their mirror images (Fig. 3).

$$U(Z) = -\frac{\mu_o I}{2\pi} \left\{ \ell n \frac{\pi}{2R} (Z - Z_o) + \ell n \frac{\pi}{2R} (Z - \bar{Z}_o) + \right.$$

$$\sum_{K=1}^{\infty} \left\{ \ell n \frac{Z - (Z_o + 2iKR)}{C} + \ell n \frac{Z - (\bar{Z}_o + 2iKR)}{C} + \ell n \frac{Z - (Z_o - 2iKR)}{C} \right.$$

$$\left. + \ell n \frac{Z - (\bar{Z}_o - 2iKR)}{C} \right\} \qquad (7)$$

$$= -\frac{\mu_o I}{2\pi} \ell n \left\{ \frac{\pi (Z - Z_o)}{2R} \sum_{K=1}^{\infty} \left(\left[\frac{(Z - Z_o)\pi}{2R} \right]^2 \frac{1}{\pi^2 K^2} + 1 \right) + \right.$$

$$\left. \frac{\pi (Z - \bar{Z}_o)}{2R} \sum_{K=1}^{\infty} \left(\left[\frac{(Z - \bar{Z}_o)\pi}{2R} \right]^2 \cdot \frac{1}{\pi^2 K^2} + 1 \right) \right\} \qquad (8)$$

using

$$\frac{\text{Sinh}}{x} = \prod_{x=1}^{\infty} \left(\frac{x^2}{\pi^2 K^2} + 1 \right) \qquad (9)$$

$$U(Z) = -\frac{\mu_o I}{2} \ell n \left\{ \text{Sinh} \frac{\pi}{2R} (Z - Z_o) \cdot \text{Sinh} \frac{\pi}{2R} (Z - \bar{Z}_o) \right\} \qquad (10)$$

Since $\bar{B}^* = Bx - i\, By = i \frac{dU}{dZ}$ (11)

$$\bar{B}^* = Bx - i\, By = -i \frac{\mu_o I}{4R} \left\{ \text{Coth} \frac{\pi}{2R} (Z - Z_o) + \text{Coth} \frac{\pi}{2R} (Z - \bar{Z}_o) \right\} \qquad (12)$$

Field along x-axis, for any point x,

$$\Delta By = \frac{\mu_o I}{2R} \cdot \frac{\text{Sinh} \frac{\pi}{R} (x - x_o)}{\text{Cosh} \frac{\pi}{R} (x - x_o) - \cos \frac{\pi}{R} y_o} \qquad (13)$$

Taking into account the current elements symmetrically placed about y-axis above the median plane and attributing negative value to these current elements (since total current in the vacuum chamber must vanish), we obtain the field due to the eddy currents for the whole chamber

$$By = \frac{\mu_o}{2R} \sum_{n1}^{n25} I(x_o) \left[\frac{\text{Sinh}\ \frac{\pi}{R}(x - x_o)}{\text{Cosh}\ \frac{\pi}{R}(x-x_o) - \text{Cos}\ \frac{\pi}{R}\ y_o} - \frac{\text{Sinh}\ \frac{\pi}{R}(x + x_o)}{\text{Cosh}\ \frac{\pi}{R}(x+x_o) - \text{Cos}\ \frac{\pi}{R}\ y_o} \right] \quad (14)$$

From equ. (4) the currents for all the 25 elements have been evaluated and the fields for different values of x have been obtained by a simple computer programme. The field values are shown in Fig. 2. The field thus obtained is exactly the same as calculated by "Magnet" programme.

c) <u>Using Image Method with Infinite Series</u> [3]

Referring to Fig. 4. The magnetic field $\Delta By(x)$ at the point x considering the right hand plane of y axis is given :

$$\Delta By_1(x) = \frac{\mu_o}{\pi} \sum_{n1}^{n25} \left\{ \frac{I(x_o)(x-x_o)}{(x-x_o)^2 + y_o^2} + \sum_{p=1}^{\alpha} \frac{(x-x_o)\ I(x_o)}{(2pH-y_o)^2 + (x-x_o)^2} + \sum_{p=1}^{\alpha} \frac{(x-x_o\ I(x_o)}{(2pH+y_o)^2 + (x-x_o)^2} \right. \quad (15)$$

Similarly, considering the left hand plane of y-axis,

$$\Delta By_2(x) = -\frac{\mu_o}{\pi} \sum_{n1}^{n25} \frac{I(x_o)(x+x_o)}{(x + x_o)^2 + y_o^2} + \sum_{p=1}^{\alpha} \frac{(x+x_o)\ I(x_o)}{(2pH-y_o)^2 + (x+x_o)^2} + \sum_{p=1}^{\alpha} \frac{(x+x_o)\ I(x_o)}{(2pH+y_o)^2 + (x+x_o)^2} \quad (16)$$

Field at any point along x axis, form equation 15 and 16,

$$By(x) = \Delta By_1\ (x) + \Delta By_2\ (x) \quad (17)$$

The infinite summation has been done in a computer and sufficiently accurate results can be obtained by adding approx. 10000 images. The result is shown in Fig. 2.

3. Eddy Currents in the Vacuum Chamber of the Quadrupole

We assume an ideal quadrupole field $\bar{B}$

$$\frac{\partial By}{\partial x} = \frac{\partial Bx}{\partial y} = g_o \tag{18}$$

From Maxwell's equation : $\text{curl}\ E = -\frac{\partial \bar{B}}{\partial t}$, $\bar{E} = (0,0,E_z)$ we obtain

$$E_z = \frac{\dot{g}_o}{2}(x^2 - y^2) + C \tag{19}$$

$\dot{g} = \frac{d}{dt} g_o$ and C is a constant. Current density $j_z = \frac{1}{\rho} E_z$.

Since the total current in the vacuum chamber must

$$C = -\frac{\dot{g}}{2} \frac{\iint (x^2 - y^2)\, dxdy}{\iint dxdy} \tag{20}$$

For the chamber shown in Fig. 5

$$C = -\frac{\dot{g}}{2} \cdot \frac{\left(\text{atan}\left(\frac{\beta}{\alpha}\right)\right)\left(\alpha^2 R1 + \beta^2 R2\right) - \frac{\pi}{2}\beta^2 R2 + \left(\frac{\alpha\beta}{R2-R1}\ R2^2 + R1^2 - R2R1\right)}{\left(\text{atan}\left(\frac{\beta}{\alpha}\right)\right)(R1-R2) + \frac{\pi}{2} \cdot R2} \tag{21}$$

$$C = -\frac{\dot{g}}{2} \cdot 1580.56$$

Current density
$$j_z = \frac{\dot{g}_o}{2\rho}\left[(x^2 - y^2) - 1580.56\right] \tag{22}$$

Using the numerical value :

$g_o = 1.5\ \mathrm{Tm}^{-1}$, $\dot{g}_o = 5.65 \cdot 10^3\ \mathrm{Tm}^{-1}\mathrm{s}^{-1}$, $\omega_o = 3705\ \mathrm{s}^{-1}$, $\rho = 10^{-6}\ \Omega\mathrm{m}$

$$j_z = 2.825 \cdot 10^3 \left[(x^2 - y^2) - 1580.56\right]\ \mathrm{A/m}^2 \text{ , x,y, in mm}$$

$$\text{and } I_z = 2.825 \cdot 10^{-3} \left[(x^2 - y^2) - 1580.56\right] \cdot \Delta x\, \Delta y \text{ Amps} \qquad (23)$$

The fields due to these eddy currents have been calculated by the following methods :

a) Using "Magnet"

The same process as outlined for dipole is used here. The gradient of field is shown in Fig. 6.

b) Using Image Method

The field due to the eddy currents along x-axis

$$B_y = \sum_{n1}^{n25} \frac{\mu_o I(x_o)x}{R^2} \frac{\sinh \frac{\pi}{R^2}(x^2 - x_o^2 + y_o^2)}{\cosh \frac{\pi}{R^2}(x^2 - x_o^2 + y_o^2) - \cos \frac{\pi}{R^2}(2x_o y_o)}$$

and the field along y-axis.

$$B_x = -\sum_{n1}^{n25} \frac{\mu_o I(x_o)y}{R^2} \frac{\sinh \frac{\pi}{R^2}(y^2 - y_o^2 + x_o^2)}{\mathrm{Cosh} \frac{\pi}{R^2}(y^2 - y_o^2 + x_o^2) - \mathrm{Cos} \frac{\pi}{R^2}(2x_o y_o)}$$

The field values have been calculated by a simple computer programme and the gradients are shown in Fig. 6.

The maximum gradient of field due to eddy current = 35.12 gauss/cm
The eddy current causes a maximum of 23.43% error in gradient.

Acknowledgement

The author wishes to acknowledge the assistance and encouragement given to him by F. Rohner.

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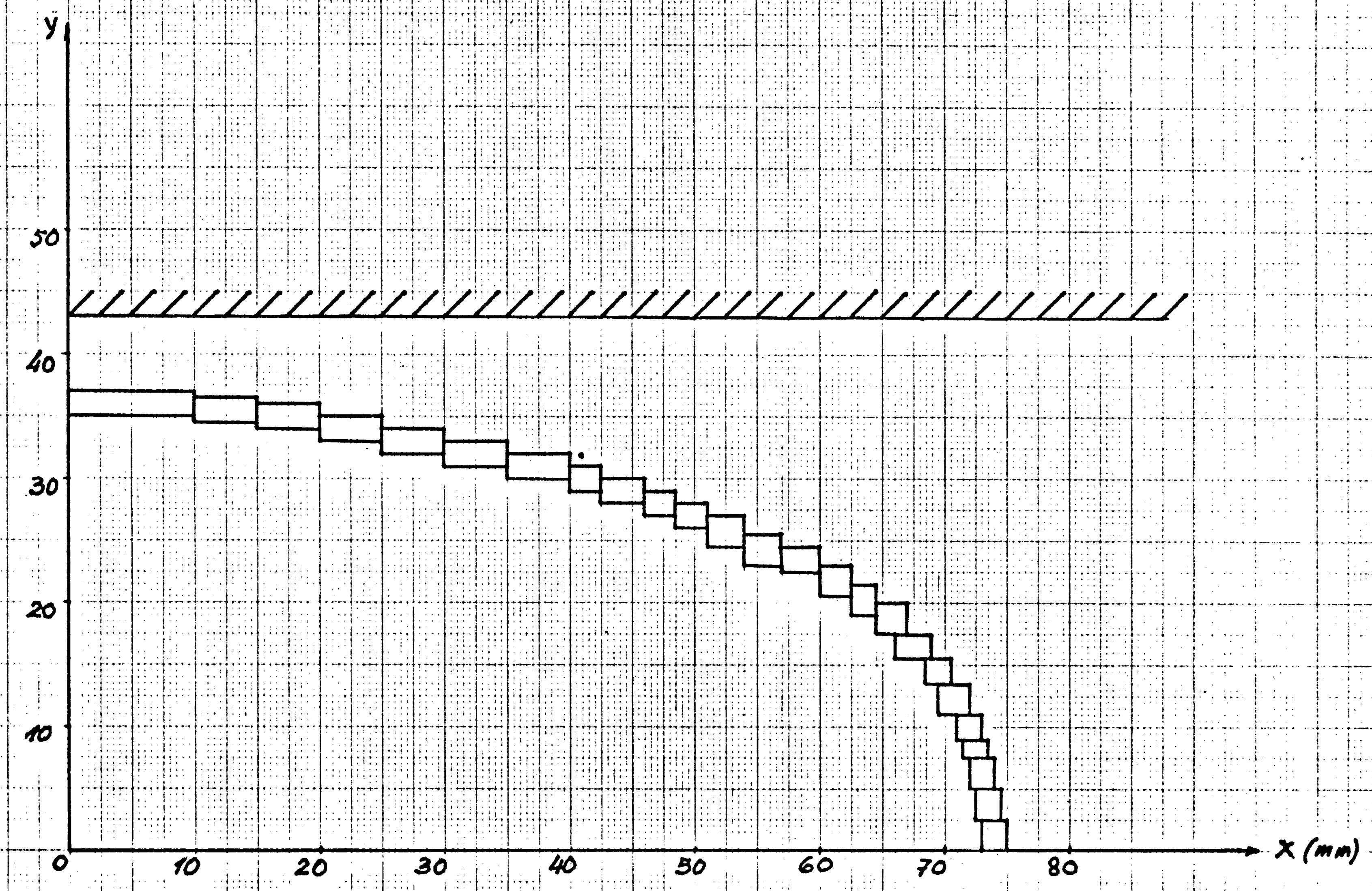

Fig.1 : Twenty-five Sections Of Vacuum Chamber.

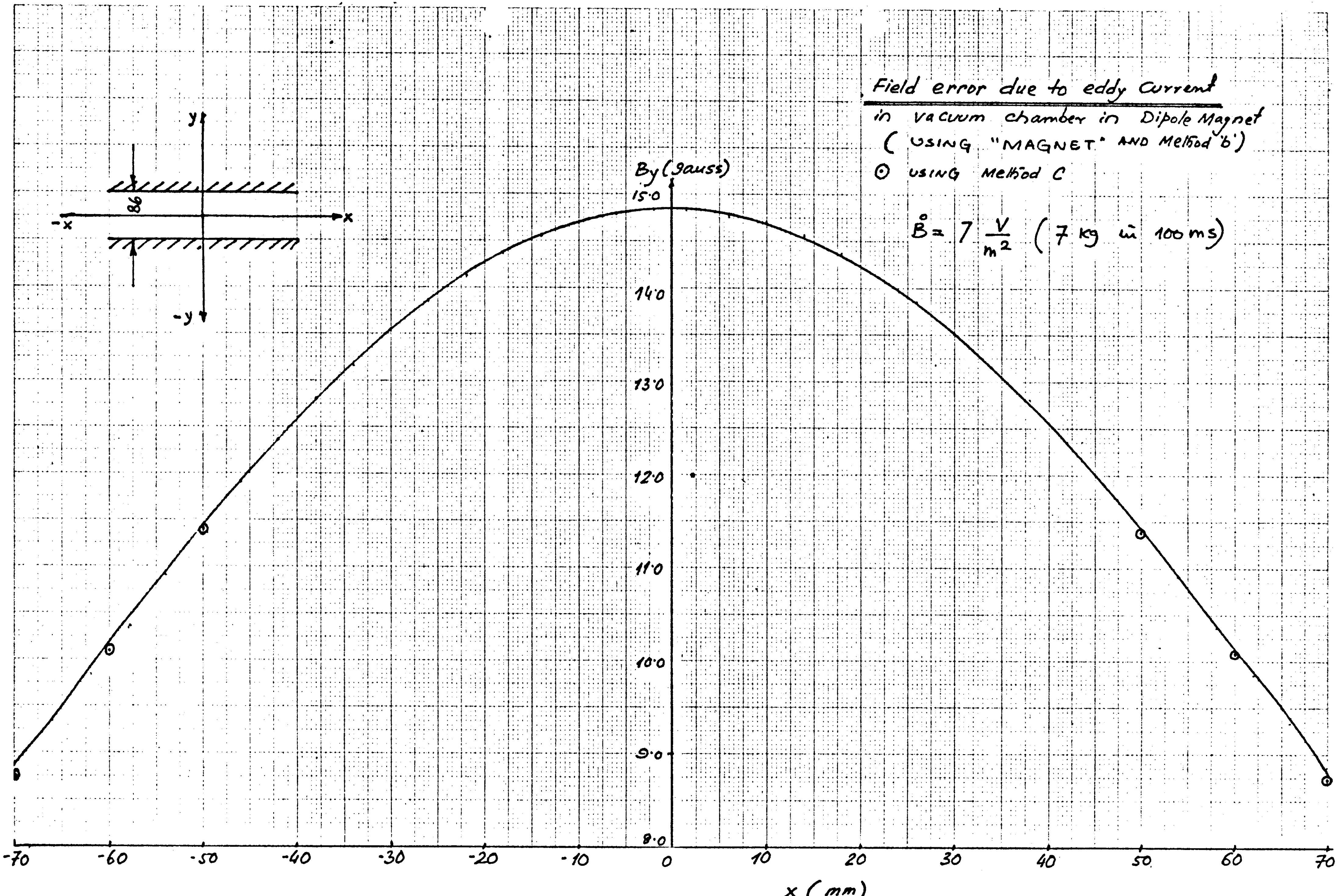

Fig.2 :Field Error Due To Eddy Current.

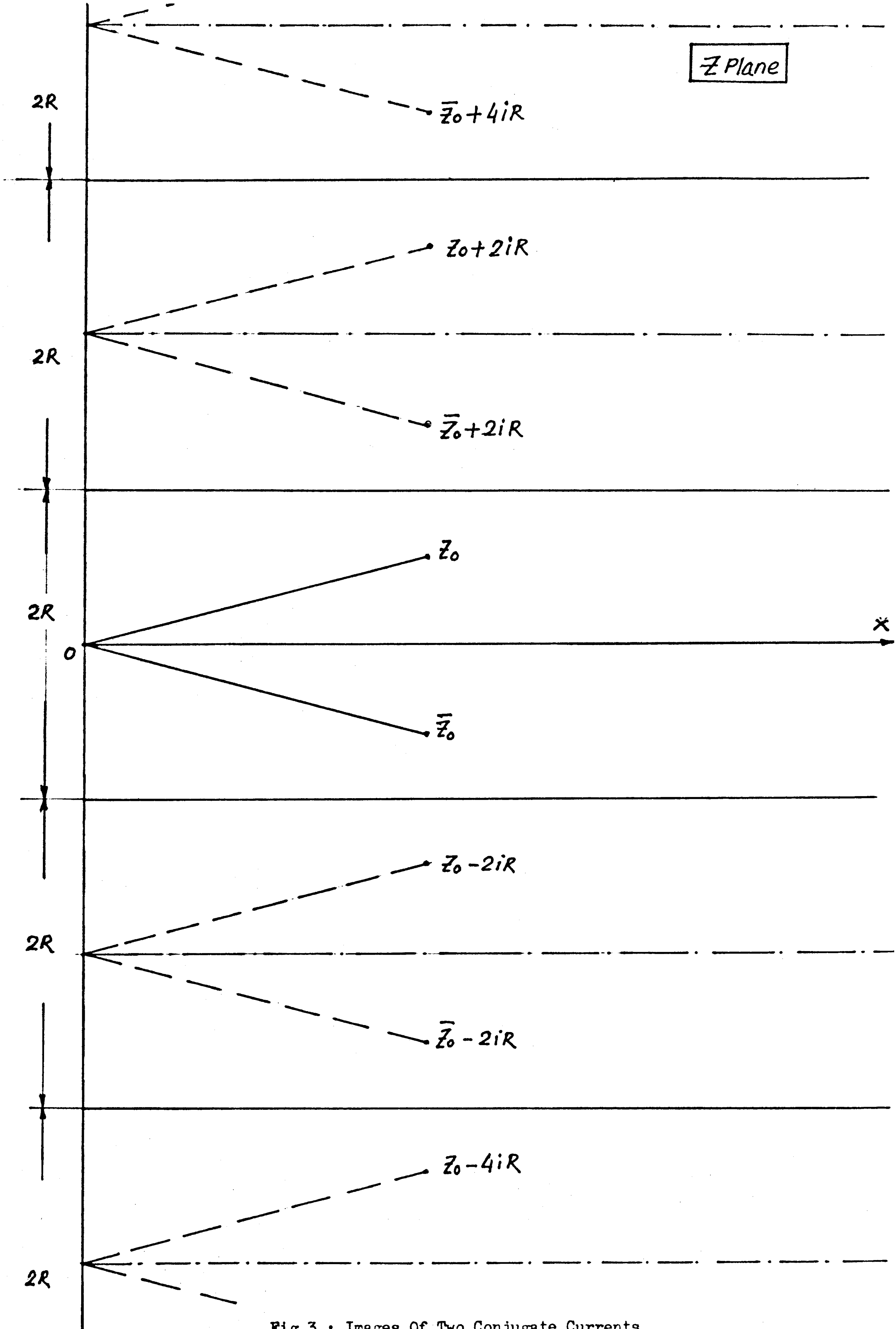

Fig.3 : Images Of Two Conjugate Currents.

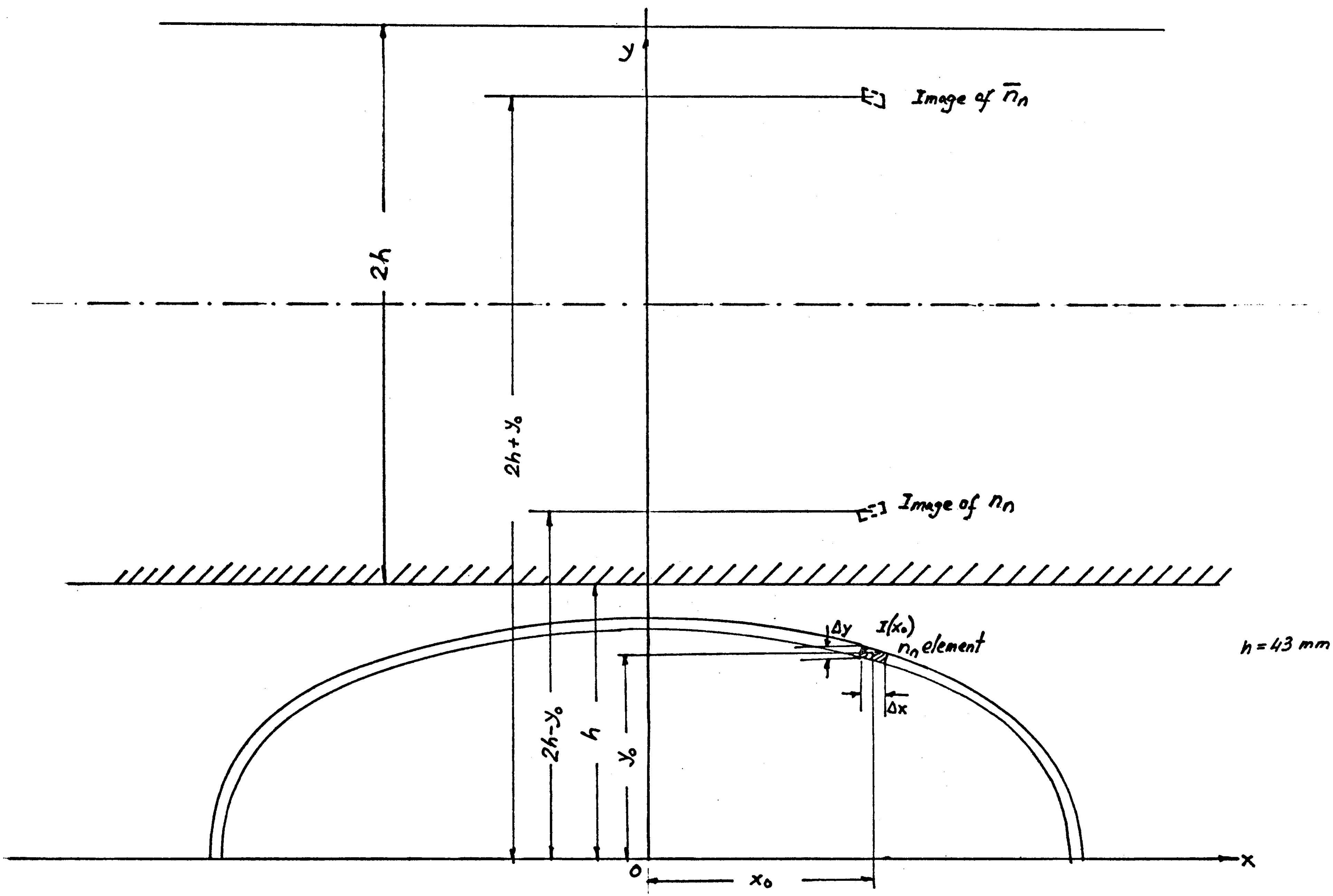

Fig.4 : Images of Two Currents Symmetrically Located About x-Axis.

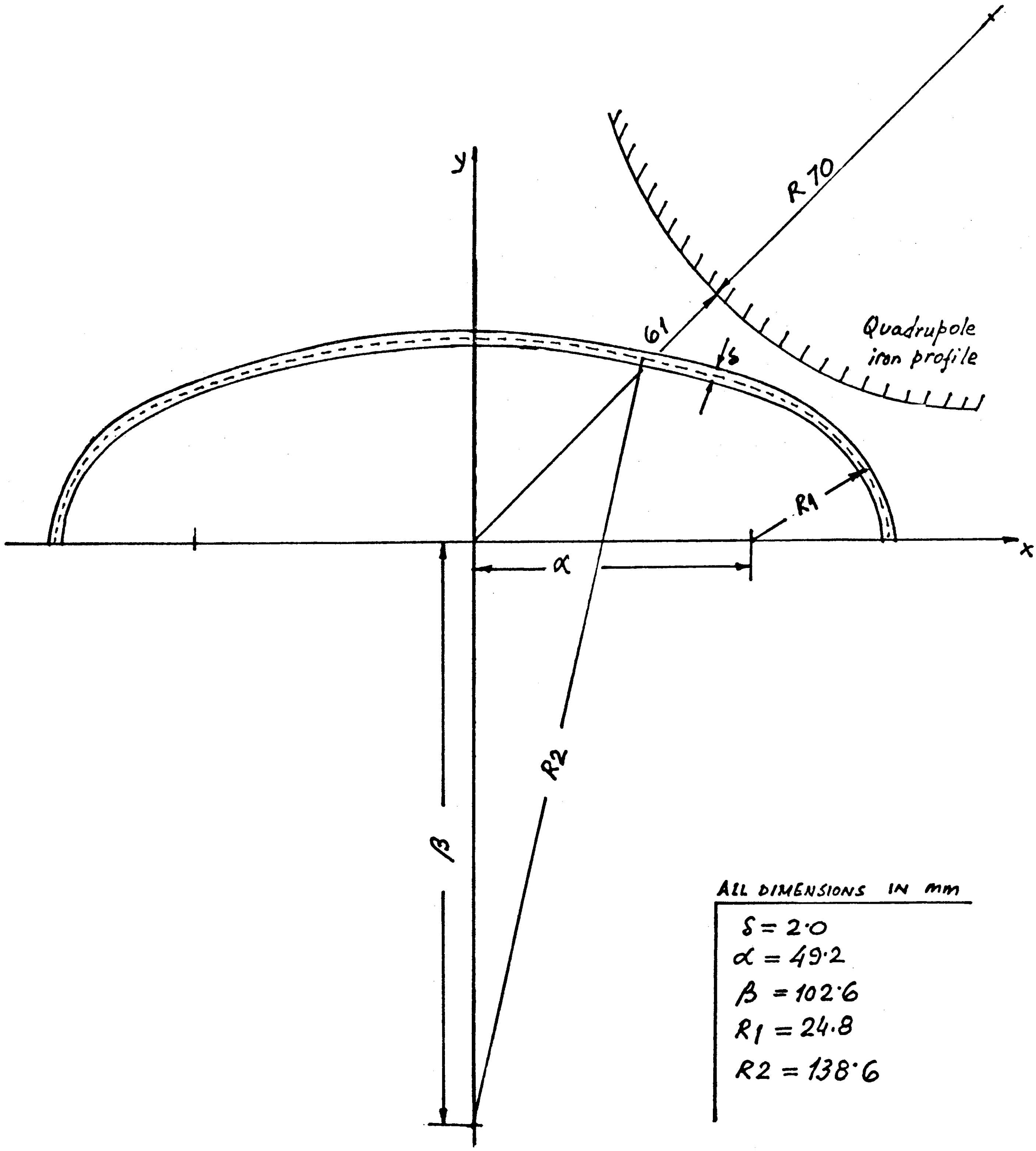

Fig.5 : Vacuum Chamber Dimensions.

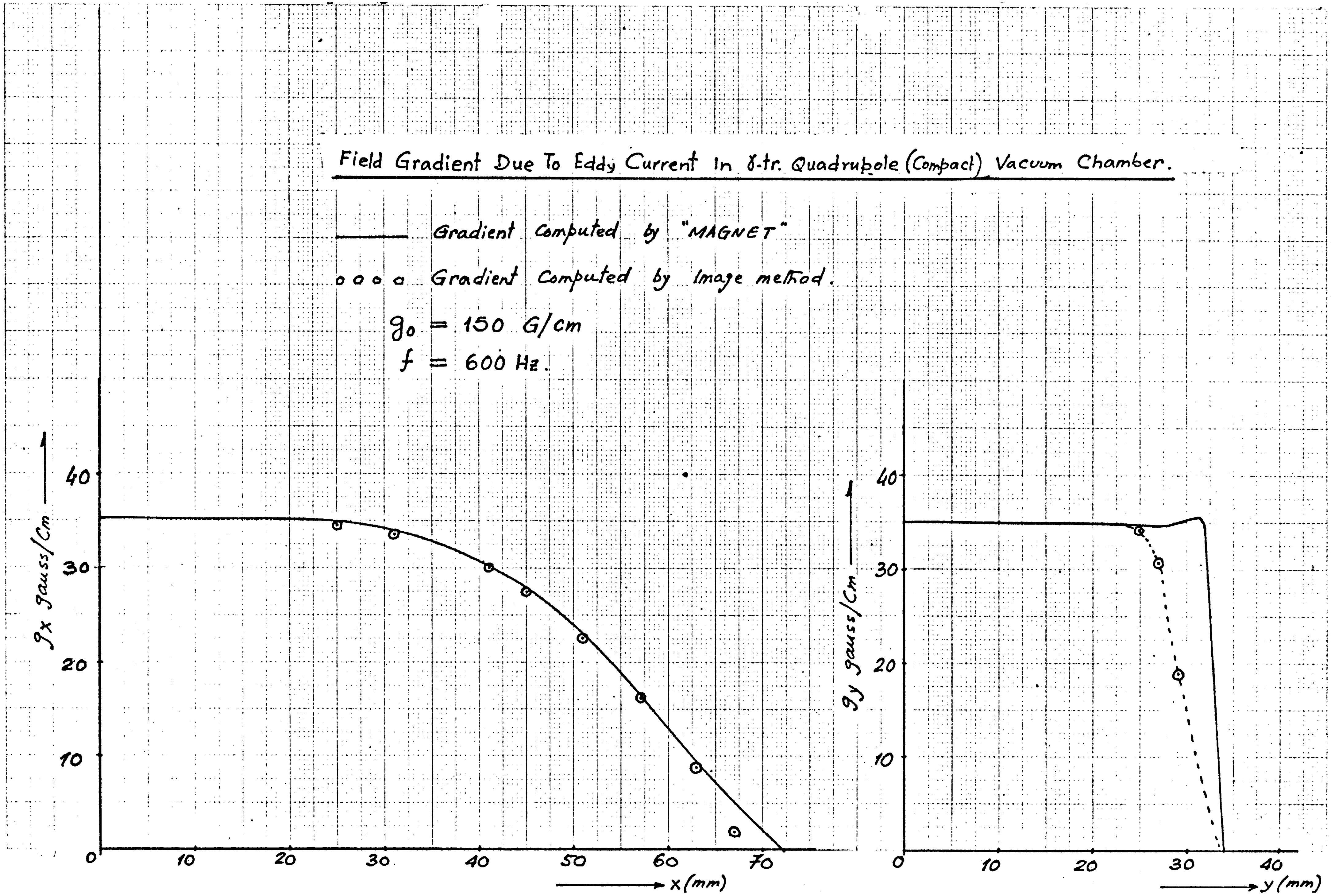

Fig.6 : Gradients Due To Eddy Current.