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DATA ON LONGITUDINAL PHASE SPACE MATCHING FOR BEAM TRANSFER FROM THE BOOSTER INTO THE PS AT 800 MeV

by

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Summary

Booster and PS data at 800 MeV are given and formulae for matching φ and $\dot{\varphi}$ (or $\pm \Delta E$) to the bucket size in the booster and in the PS are derived. The following cases are discussed: Booster bunches of 145° and 220° length are accepted. Then either the PS RF-voltage V and the magnet-$\dot{B}$ have to be drastically reduced in order to obtain a sufficiently large bucket and matched phase space trajectories or the bunch is rotated by 90° during a quarter phase oscillation with a suitable pair of V and $\dot{B}$ to match the bunch to the nominal PS-bucket.

PS/8046

1. General data at 800 MeV

$$E = 800\ \text{MeV} + E_0 = 1738.256\ \text{MeV}$$

$$\gamma = E / E_0 = 1.852646$$

$$\beta = \sqrt{1 - \gamma^{-2}} = 0.8418134$$

$$\beta\gamma = cp/E_0 = 1.559582$$

$$p = 1.463287\ \text{GeV/c}$$

$$\omega_{RF} = \beta \cdot h \cdot c/R = 50.474\ \text{MHz} = 2\pi \cdot 8.0332\ \text{MHz}$$

for both synchrotrons

2. Booster data before transfer (Figs. 1, 2)

According to the Booster parameter list [1] the bunches have the following dimensions in the longitudinal phase space:

	length $(\varphi_2 - \varphi_1)$	height $\pm\Delta E$
after trapping	230° = 4.02 radians	0.34 MeV
at 800 MeV	145° = 2.53	1.45 MeV
dto., lengthened	220° = 3.84	0.97 MeV

Fig. 1 illustrates the phase space area occupied by the trapped particles within the RF-bucket of the booster and shows how the bunch length shrinks due to adiabatic damping during acceleration (V = 12 kV, energy gain per turn 1 kV, updated version of an earlier computer plot [2]). The phase is counted from the crest of the RF-voltage but this is unimportant in this context.

After arrival on the magnet flat top at 800 MeV the synchronous phase φ_s changes by 4.8° and one obtains the large non-accelerating RF-bucket shown in Fig. 2, containing the bunch to be transferred into the PS. Note that although the bucket is larger after this phase jump the boundary of the phase space area of the bunch is not noticeably affected. The vertical scale of the plot indicates the RF-frequency error $\Delta\omega_{RF} = \dot{\phi}$ of the non-synchronous particles in units of the synchrotron frequency $\Omega_s = 2\pi f_s$ and can easily be converted into units of energy or momentum spread:

$$\Delta E = v \cdot \Delta p \quad \text{and} \quad \Delta p = \frac{p}{\eta \cdot \omega_{RF}} \Delta\omega_{RF} = \frac{p}{\eta\omega_{RF}} \dot{\phi} \qquad (1),(2)$$

by definition of η. This notation will simplify the matching procedure because the phase space trajectories for small oscillation amplitudes are circles during all adiabatic changes of energy and RF-voltage. Only the multiplier of the vertical scale varies (for compatibility with phase space conservation). On the booster flat top the bunches can be lengthened to 220° = 3.84 r by reducing adiabatically the RF-voltage and hence the synchrotron frequency and the energy spread from 1.45 MeV to 0.97 MeV. Fig. 2 shows that even the long bunches are enclosed by a fairly good elliptical phase space trajectory. However, the ratio b/a of the vertical to horizontal axis is less than 1. It can be shown that this axis ratio of an elliptical phase space trajectory represents the coefficient by which the large non-linear oscillations are slower than the small linear oscillations (circles, frequency Ω_s).

3. Magnetic field and other PS-data at transfer

The guiding field for transfer at 800 MeV is reached at B = 696.5 gauss, i.e. between the B-pulses 69 and 70. If we refer to a measurement [3] of the present variable $\dot{B}$ = 1.38 Tesla/sec at this B-pulse and assume that $\dot{B}$ will be doubled (90 % → 180 % $\dot{B}$) the injection field will be reached 20 ms later than our present injection field of 147 gauss and we can expect at transfer

$$\dot{B} = 2.76 \text{ T/s} = 2.76 \text{ V/m}^2.$$

The momentum compaction factor $\alpha = 0.027$ of the PS yields at transfer

$$\eta = \alpha - \gamma^{-2} = - 0.26435$$

and relations (1) and (2) enable us to calculate the maximum frequency error $\dot{\varphi} = \Delta\omega_{RF}$ of the transferred bunches:

$$\dot{\varphi} = \frac{\eta \cdot \omega_{RF}}{p} \cdot \Delta p = \frac{\eta \cdot \omega_{RF}}{\beta^2 \, E} \cdot \Delta E = 10.83 \, \frac{\text{kHz}}{\text{MeV}} \cdot \Delta E \qquad (2a)$$

$$\dot{\varphi}_{max} = \begin{cases} \pm \, 15.7 \text{ kHz short bunch} \\ \pm \, 10.5 \text{ kHz lengthened bunch} \end{cases}$$

4. Size of nominal RF-bucket during PS-acceleration (Fig. 3)

Fig. 3 shows a computer plot of the nominal RF-bucket of the PS with $\dot{B} = 2.76$ T/s and RF-sum voltage V = 240 kV.

One can see that the lengthened booster bunches exceed the bucket size and the short bunches can be included by the separatrix but are mismatched. This means that after a quarter period of a synchrotron oscillation ($\Omega_s = 2\pi \cdot 4.8$ kHz) the particles would rotate inside the bucket by $\approx 90^{\circ}$ along the plotted trajectories and we shall have a minimum bunch length and maximum energy spread of roughly $\pm$ 4 MeV instead of 1.45 MeV because the boundary of the phase space area occupied by particles does not match the trajectories. We would observe large synchrotron oscillations and during the following oscillation periods the outer particles with non-linear oscillations which rotate more slowly in phase space will lag behind and distort the ellipse into a spiral which finally will occupy a much larger phase space area (dilution). Obviously some matching is necessary.

In the first draft of the proposal of a 2-ring booster (TART) the problem of longitudinal phase space matching had been investigated by K. Leeb [4]. Since the TART-project has been replaced by the present 4-ring booster project with different design parameters, the problem is now different. However, he mentioned a second possibility of matching the booster bunch to the PS by a rotation of 90°. If the larger energy spread and space charge density implied by this proposal is accepted, it may be an interesting possibility because it is much easier to match a short bunch with large momentum spread. On the other hand, one can ask whether it would not be better not to lengthen the booster bunches before transfer because the lower momentum spread would be only a fictitious gain if it should turn out that by mismatch due to hardware limitations in the PS the beam would soon get a 3-4 times larger momentum spread anyway.

5. Providing sufficient bucket length and height (Fig. 4)

The bucket length and the normalized height depend only on the ratio

$$\Gamma = \frac{V_t}{V} = \frac{\text{voltage gain per turn}}{\text{RF-sum voltage}} \qquad (3)$$

Since we cannot increase $V \leqslant 240$ kV we must accelerate more slowly reducing $\dot{B}$ in order to accomodate also the adiabatically lengthened booster bunches in the PS. For other PS-buckets than the one in Fig. 3 the length and the height of the phase stable area enclosed by the separatrix is plotted in Fig. 4 versus Γ. The bucket length is given in degrees and radians. The voltage gain per turn is related to $\dot{B}$ by the PS-geometry:

$$\underline{V_t} = 2\pi R \cdot \rho \cdot \dot{B} = \underline{44031.85 \ m^2 \cdot \dot{B}} \qquad (4)$$

For the nominal $V = 240$ kV there is a second abscissa in Fig. 4 indicating $\dot{B}$. If V is reduced, $\dot{B}$ must be reduced by the same proportion. A second vertical scale for V = 240 kV indicates the bucket height also in MeV. One can see that with our $\dot{B} = 27.6$ kGauss/sec and $\Gamma = 0.51$ we can trap only the short booster bunch of 145°.

In order to trap the long 220° bunch we have to reduce Γ to < 0.365 and $\dot{B} < 19.7$ kGauss/sec. A smaller $\dot{B}$ increases not only the length but also the bucket height. Thus we have to reduce the multiplier of the vertical energy scale reducing the RF-voltage V and further reduce $\dot{B}$ while maintaining Γ constant. Then the bucket height shall decrease only as the square root of V.

6. Analytical matching formula for small linear oscillations

Since the separatrix in the PS is quite different from the quasi-elliptical boundary of the phase-plane area of the booster bunch and since there is even a singular point on the separatrix, the conditions of bucket length and height are necessary but not sufficient for matching.

Fig.s 2 and 3 show that the majority of particles move on almost circular trajectories and the outer more "elliptical" trajectories are longer than high because the larger non-linear oscillations are slower than the small linear oscillations. So we can linearize the differential equation of synchrotron oscillations(adiabatically slowly changing γ):

$$\frac{d^2\varphi}{dt^2} = \left(\frac{c}{R}\right)^2 \eta \frac{heV}{2\pi E} (\cos\varphi - \underbrace{\cos\varphi_s}_{\Gamma}) \tag{5}$$

where the phase φ and synchronous phase φ_s are counted from the crest of the RF-waveform.

$$V(\cos\varphi - \underbrace{\cos\varphi_s}_{\Gamma}) \simeq -V(\varphi-\varphi_s) \sin\varphi_s = -V(\varphi-\varphi_s)\sqrt{1-\Gamma^2}$$

(before transition)

and taking into account equ. (3)

$$V(\cos\varphi - \cos\varphi_s) \simeq -(\varphi-\varphi_s)\sqrt{V^2 - V_t^2} \tag{6}$$

So we obtain the linearized synchrotron equation

$$\frac{d^2\varphi}{dt^2} = -\Omega_s^{\;2} (\varphi-\varphi_s) \tag{5a}$$

with

$$\Omega_s = \frac{c}{R}\sqrt{-\eta\frac{he\sqrt{V^2-V_t^2}}{2\pi E}} \tag{7}$$

and elliptical or circular phase space trajectories

$$\varphi - \varphi_s = \varphi_o \sin\Omega_s (t-t_o) \tag{8}$$

$$\dot{\varphi} = \Omega_s \varphi_o \cos \Omega_s (t - t_o) \tag{9}$$

for which the ratio of the vertical to horizontal axis is given by

$$\Omega_s = \frac{\dot{\varphi}_{max}}{(\varphi-\varphi_s)_{max}} = \frac{\Delta\omega_{RF}}{(\varphi-\varphi_s)_{max}} \tag{10}$$

Ω_s measures the longitudinal focusing forces. Note that the synchronous phase is eliminated from the expression for the synchrotron frequency (7) and we can solve explicitely for the voltage V which we need for matching:

$$V = \sqrt{\left[\left(\frac{\Omega_s}{c/R}\right)^2 \frac{2\pi E}{-\eta h e}\right]^2 + V_t^{\,2}} \tag{11}$$

for $V_t = \dot{B} = 0$ we have

$$V_o = \left(\frac{\Omega_s}{c/R}\right)^2 \cdot \frac{2\pi E}{-\eta h e} \tag{12}$$

7. Two matching conditions

A) The axis ratio $2\dot{\varphi}_{max}/(\varphi_2 - \varphi_1)$ of the almost elliptical phase area occupied by the booster bunches determines the value Ω_s in the PS to match the trajectories equ. (8) and (9).

B) For the large non-linear oscillations the trajectories are distorted and we have to pay attention that the largest amplitude φ_o does not exceed the largest closed trajectory, i.e. the separatrix. Since the bucket is longer than high we have to check the height.

The result can finally be checked by an exact computer calculation as in Figs. 1, 2, 3.

8. Numerical results

Substituting PS-data in equ. (11) we obtain

$$V = \sqrt{\left[\left(\frac{\Omega_s}{2.99793\ \mathrm{MHz}}\right)^2 \cdot 2.063 \cdot 10^6\ \mathrm{kV}\right]^2 + V_t^2}$$

$$= \sqrt{\left[\left(\frac{\Omega_s}{2.086\ \mathrm{kHz}}\right)^2 \mathrm{kV}\right]^2 + V_t^2} \qquad (11a)$$

If we do not accelerate ($\dot{B} = 0$, $V_t = 0$) we need only the voltage

$$V_o = \left(\frac{\Omega_s}{2.086\ \mathrm{kHz}}\right)^2 \cdot \mathrm{kV} \qquad (12a)$$

The choice of V_t or $\dot{B}$ will be limited by the second matching condition which determines the ratio $V_t/V = \Gamma$. Therefore we express V as a function of Γ

$$V = \sqrt{V_o^2 + V_t^2} = \sqrt{V_o^2 + \Gamma^2 V^2}$$

$$V = \frac{V_o}{\sqrt{1 - \Gamma^2}} \qquad (13)$$

With the phase space area of the booster bunch we obtain for $\Omega_s = \dot{\varphi}_{max}/\varphi_o$

short bunch	long bunch
$\varphi_o = 145^o/2 = 1.265$ radian	$\varphi_o = 220^o/2 = 1.92$
$\Omega_s = 15.7$ kHz/1.265 = 12.4 kHz	$\Omega_s = 10.5$ kHz/1.92 = 5.47 kHz
$V_o = 35.2$ kV	$V_o = 6.86$ kV
$V = \sqrt{(35.2\ \mathrm{kV})^2 + V_t^2}$	$V = \sqrt{(6.86\ \mathrm{kV})^2 + V_t^2}$
$V = 35.2\ \mathrm{kV}\ \sqrt{1 + (\frac{\dot{B}}{0.80\ \mathrm{T/s}})^2}$	$V == 6.86\ \mathrm{kV}\ \sqrt{1 + (\frac{\dot{B}}{0.156\ \mathrm{T/s}})^2}$
$V = \frac{35.2\ \mathrm{kV}}{\sqrt{1-\Gamma^2}}$	$V = \frac{6.86\ \mathrm{kV}}{\sqrt{1-\Gamma^2}}$

If we choose the bucket height equal to the oscillation amplitude φ_o in Fig. 4 we find for Γ

$\Gamma = 0.447$ $\qquad\qquad$ $\Gamma = 0.05$

and from equ. (13), (3), and (4)

$V = 39.4$ kV $\qquad\qquad$ $V = 6.87$ kV

$\dot{B} = 0.400$ T/s $\qquad\qquad$ $\dot{B} = 0.0078$ T/s

This means that for the shorter bunch the voltage on all RF-cavities is reduced to 16.4 % which is feasible, whereas for the long bunch it means a reduction to 2.86 %. A further disadvantage is the very small $\dot{B}$ imposed by the long bunch, which must be kept small until the bunch length has shrunk by adiabatic damping (unless we increase the voltage for adiabatic bunch shortening).

9. Matching by 90° rotation in phase space

In order to obtain short bunches in the PS it had been proposed first to lengthen the bunches adiabatically in the booster and then rotate them by 90° in the PS (K. Leeb)[4]. This rotation is done best with a bucket of maximum length ($\dot{B} = 0$) in order to avoid spiralling.

For a bunch length $2\varphi_o$ and maximum frequency spread $\pm \dot{\varphi}_{max}$ we obtain after the 90° rotation (in linear approximation) another ellipse with

$$\dot{\varphi}'_{max} = \varphi_o \, \Omega_s \tag{14}$$

$$\varphi_o' = \dot{\varphi}_{max} / \Omega_s \tag{15}$$

$$\Omega_s = \sqrt{\left(\frac{\dot{\varphi}'_{max}}{\varphi_o'}\right) \left(\frac{\dot{\varphi}_{max}}{\varphi_o}\right)} \tag{16}$$

If we would perform the bunch rotation with full RF voltage V = 240 kV, $\dot{\varphi}_{max}$ would become too high (Fig. 5), but if we start with a smaller voltage, we can choose the vertical scale factor Ω_s such that after the 90° rotation the area of the bunch fits to a trajectory of the nominal PS-bucket of 240 kV and $\dot{B} = 2.76$ T/s in Fig. 3 for which we have

$$V_t = 44031.85 \text{ m}^2 \cdot \dot{B} = 121.5 \text{ kV/turn}$$

$$\Omega_s = \frac{c}{R} \sqrt{- \eta \frac{he \sqrt{V^2 - V_t^2}}{2 \pi E}} = 30.0 \text{ kHz } (= 2\pi \cdot 4.78 \text{ kHz})$$

The geometric mean value between this desired Ω_s and the axis ratio Ω_s = 12.4 kHz of the short bunch received from the booster is

$$\Omega_s = \sqrt{30.0\ \text{kHz} \cdot 12.4\ \text{kHz}} = 19.3\ \text{kHz}.$$

According to equs. (12), (12a) we start with a reduced voltage of

$$V_o = \left(\frac{\Omega_s}{2.086\ \text{kHz}}\right)^2 \cdot \text{kV} = 85.6\ \text{kV}$$

The $\pi/2$-rotation occurs during

$$\tau = \frac{\pi/2}{19.3\ \text{kHz}} = 81.4\ \mu\text{sec}.$$

This matching procedure resembles the case in optics or electromagnetic wave theory where a wave coming from a medium of refraction index n_1 or characteristic impedance Z_1 has to be matched before entering another medium characterized by n_2 resp. Z_2 and matching is accomplished by insertion of a $\lambda/4$-layer or $\lambda/4$-transmission line for 90^o phase rotation and refraction index $n = \sqrt{n_1 n_2}$ or characteristic impedance $\sqrt{Z_1 Z_2}$. In our case $\Omega_s = \sqrt{\Omega_{s1}\ \Omega_{s2}}$ plays this rôle.

Neglecting non-linear distortions, the short booster bunch (φ_o = 1.265, $\dot{\varphi}_{max}$ = 15.7 kHz) after 90^o rotation becomes

$$\varphi_o = \frac{\dot{\varphi}_{max}}{19.3\ \text{kHz}} = 0.814\ \text{radians} = 46.6^o. (2\varphi_o = 93^o\ \text{bunch length})$$

$$\dot{\varphi}_{max} = \varphi_o \cdot 19.3 \text{ kHz} = \pm 24.4 \text{ kHz}$$

$$\Delta E = \pm 2.25 \text{ MeV}$$

This bunch would fit well inside the bucket in Fig. 3. An inconvenience is, however, $\dot{B} = 0$ and the low voltage. Fig. 3 shows that the 90^o-rotation of the short booster bunch could also be accomplished within this accelerating bucket. In this case of $\dot{B} > 0$ the voltage V is higher :

$$V = \sqrt{(85.6 \text{ kV})^2 + V_t{}^2} = \sqrt{(85.6 \text{ kV})^2 + (44032 \text{ m}^2 \cdot \dot{B})^2}$$

$V = V(\dot{B})$ is plotted in Fig. 6 and yields <u>149 kV for $\dot{B} = 2.76$ T/s.</u>

The most feasible proposal is probably to avoid any jump in $\dot{B}$ (which is very harmful because magnet transients and ripple affect the control of the synchronous phase φ_s) and start with a reduced voltage.

The time $\tau = 81.4$ µs is about the fastest linear rise time observed on the AVC control of the present PS cavities and unless the AVC of the new RF-cavities is much faster than this we would have to replace the voltage step by a continuously increasing voltage of the same average value. We can expect that even this (non adiabatic) reduction will work, since it is known that in $\lambda/4$-line transformers for matching one can replace the discontinuity in Z by a smooth change of impedance provided the discontinuity capacity is equivalent. Although the theory is more complicated, the physical situation is equivalent or even more favourable for a smooth change of characteristic impedance or Ω_s in our case.

10. Bunching factors and momentum spread

The bunching factor B is the ratio of average particle density to peak density in one period. Since the density distribution in phase space at transfer is unknown, we assume a uniform phase space density inside the elliptical phase space area of the bunch which yields

$$B = \frac{\pi}{4} \cdot \frac{\text{bunch length}}{360^o}$$

bunch length $2\varphi_o$	220^o	145^o	93^o (in PS only)
$\pm$ ΔE	0.97 MeV	1.45 MeV	2.25 MeV
$\pm$ $\Delta p/p$	1.71 ‰	2.56 ‰	3.97 ‰
bunching factor (unif. phase space dens.)	0.480	0.318	0.203
bunching factor (Bigliani)	0.4	0.25	

The smaller bunching factors [6,7] are based on a density distribution resulting from Bigliani's studies of adiabatic trapping of a linac beam with gaussian distribution of momentum spread and subsequent scaling (adiabatic changes) to 800 MeV.

11. Conclusions

After discussion in the longitudinal working party (1.12.1969) the case in section 8, of a reduction of the RF-sum-voltage in the PS to 35.2 - 39.4 kV for a booster bunch of 145^o length has been retained for more detailed studies.

Acknowledgements

I wish to thank Prof. I. Gumowski for fertile discussions on the dimensions of the booster bunch specified in the SI-parameter list and for the use of his tables.

References

1) C. Bovet : CPS Synchrotron Injector Parameter List. SI/Note/DL/69-1

2) H.H. Umstätter : Accelerating bucket of the Booster. MPS/RF/B-Note 67-19

3) H.H. Umstätter : Synchrotron Frequencies .... MPS/Int. SR/68-6, fig. 4

4) O. Barbalat et al. : Draft (internal). Comparison between two possible CPS-injectors DIR/TM/14 (3.3.1966), chapter 3.3, p.21.

5) I. Gumowski : CPS RF-bucket width, height and area MPS/Int. RF 67-1.

6) U. Bigliani : Système HF du booster. Capture dans l'espace de phase longitudinal. SI/Int. EL 68-2.

7) O. Barbalat : PS Study and Improvement Programme. MPS/Int. DL 69-4, p.6 bis.

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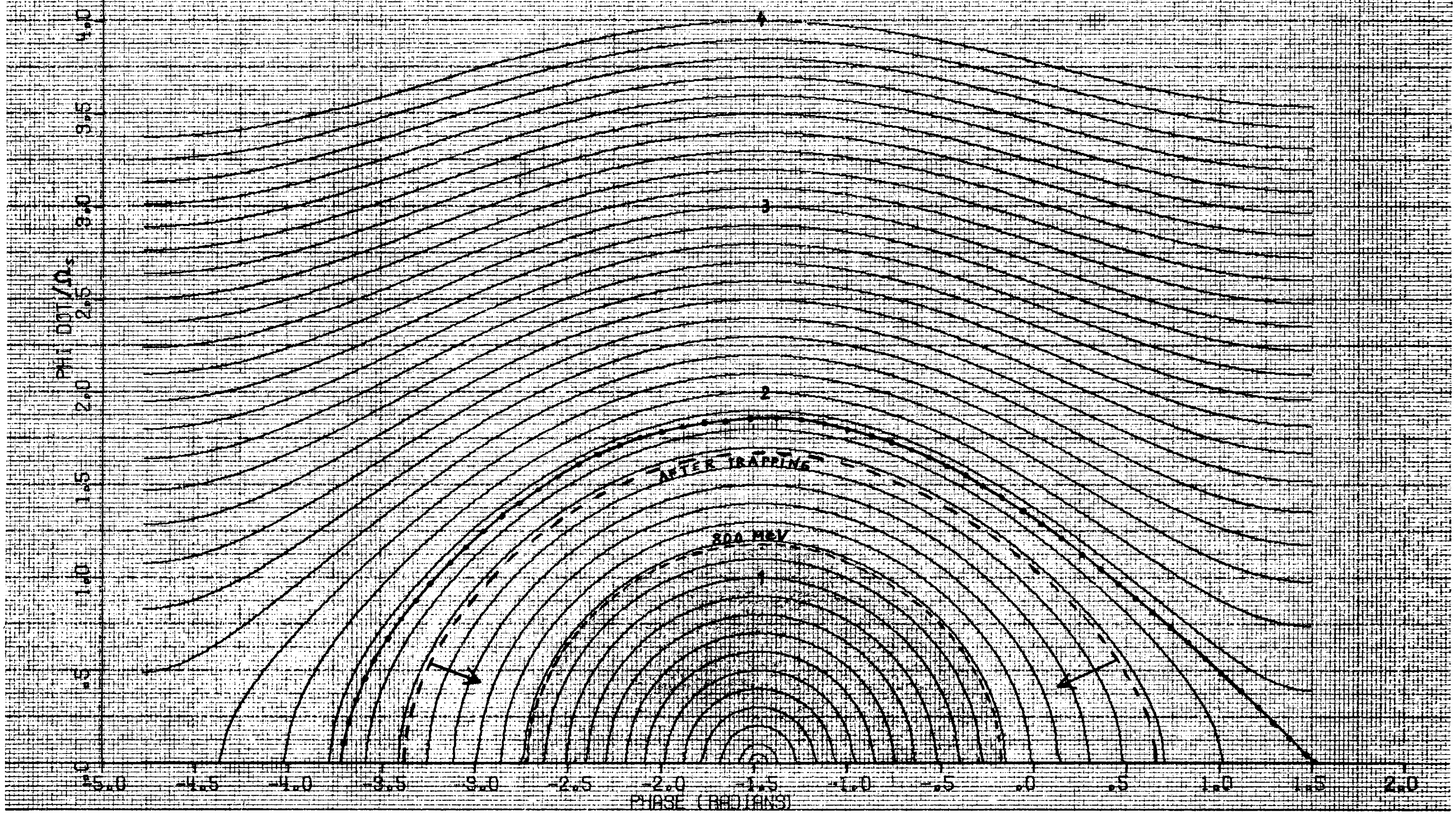
FIG. 1

BOOSTER BUCKET BEFORE FLAT TOP

$V = 12\,kV$, $V_t = 1\,kV/TURN$, ADIABATIC DAMPING



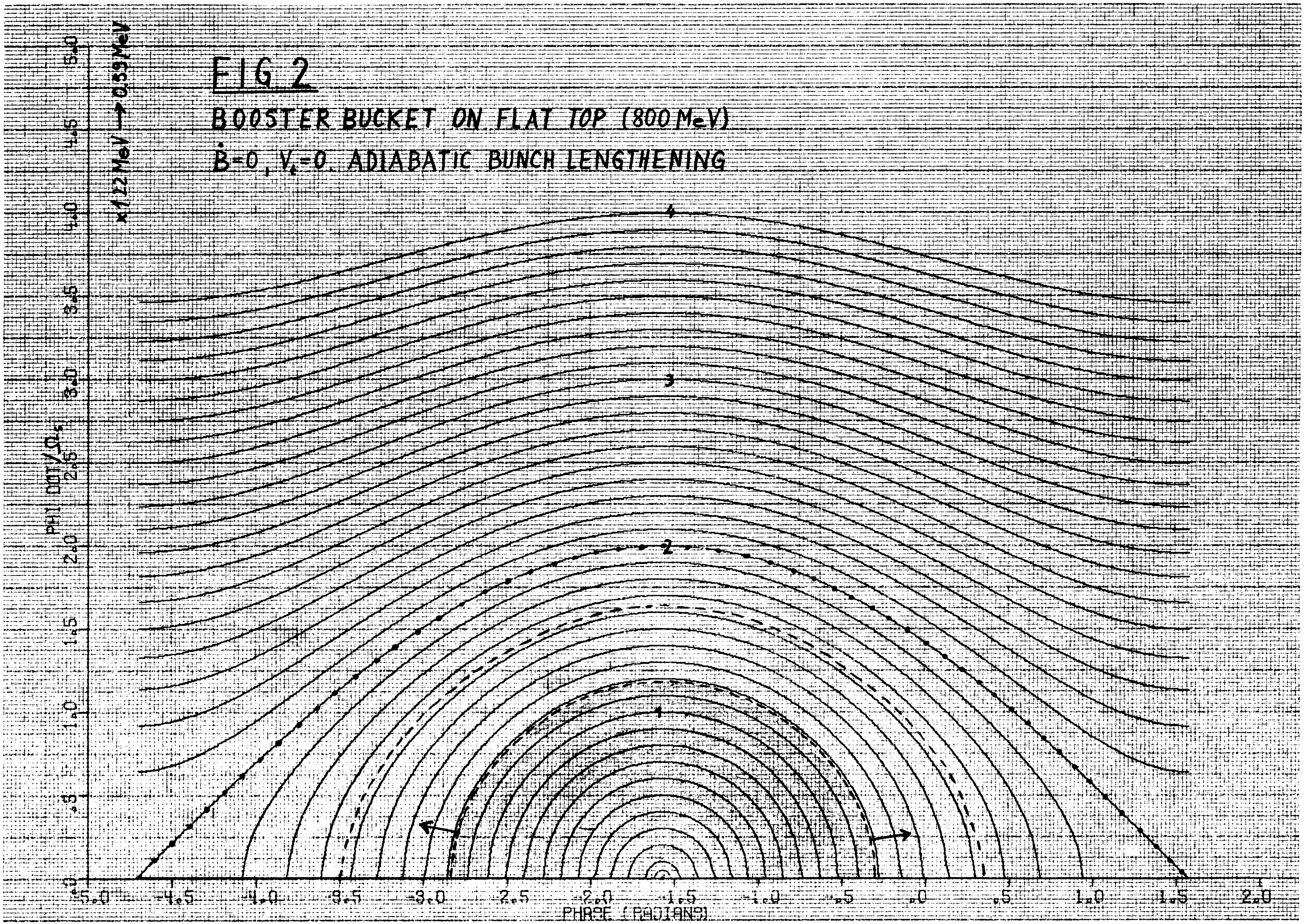


FIG 2

BOOSTER BUCKET ON FLAT TOP (800 MeV)

$\dot{B}=0$, $V_t=0$. ADIABATIC BUNCH LENGTHENING

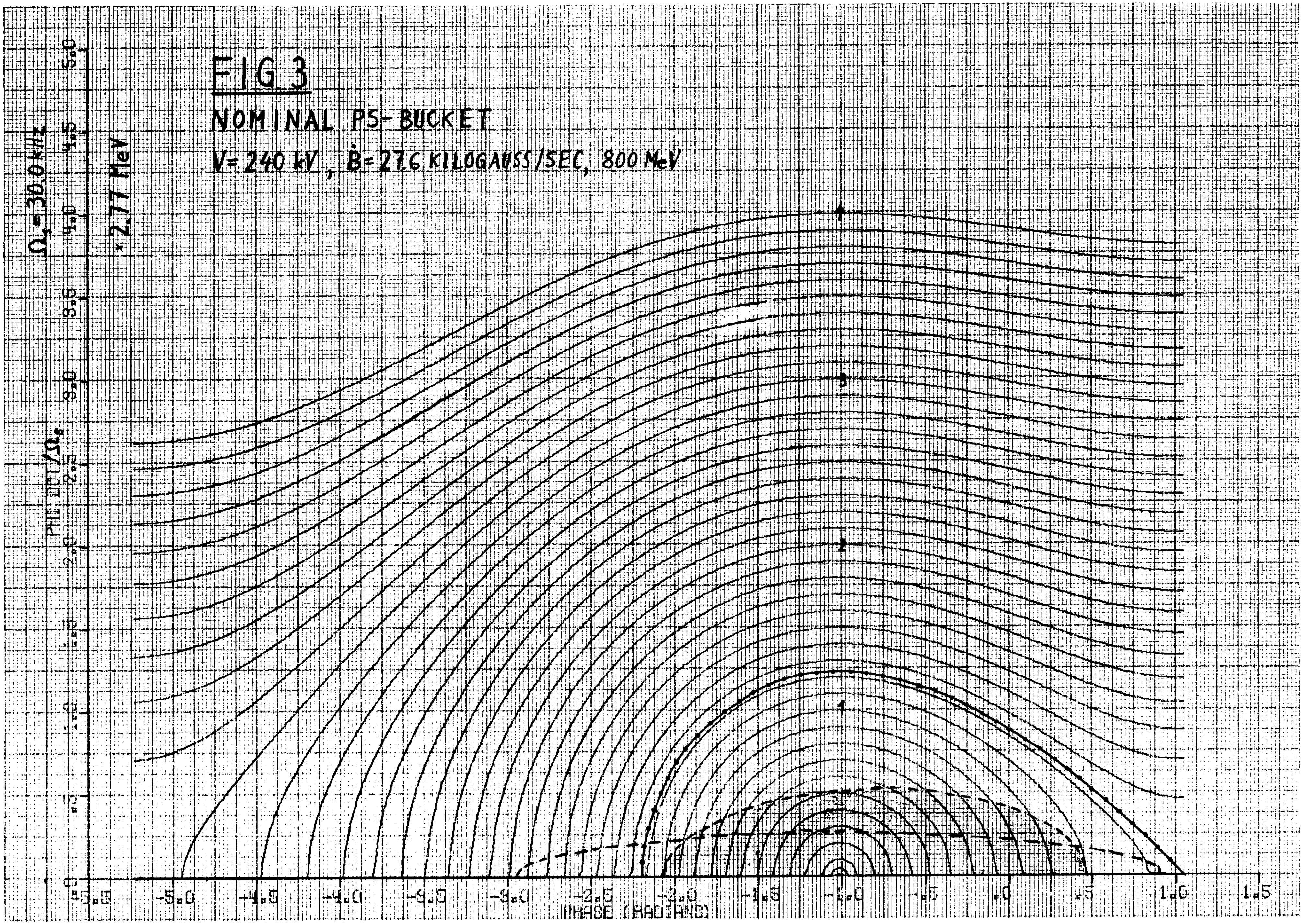


FIG 3
NOMINAL PS-BUCKET
V= 240 kV , Ḃ=27.6 KILOGAUSS/SEC, 800 MeV
Ω_s = 30.0 kHz
* 2.77 MeV
PHI DOT/Ω_s
PHASE (RADIANS)

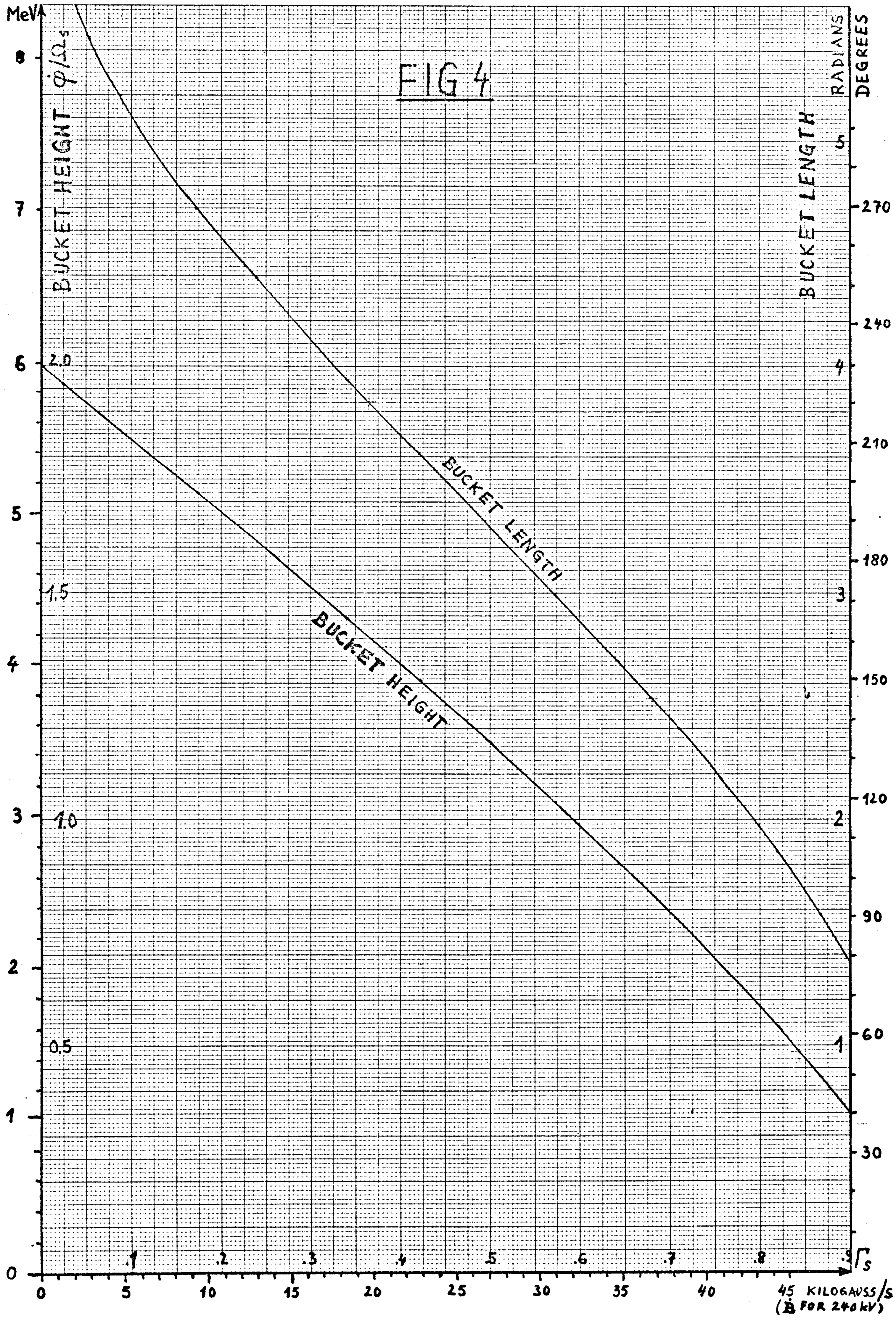


FIG 4
MeV
BUCKET HEIGHT $\dot{\varphi}/\Omega_s$
BUCKET LENGTH
RADIANS
DEGREES
BUCKET LENGTH
BUCKET HEIGHT
8
7
6
5
4
3
2
1
0
2.0
1.5
1.0
0.5
5
4
3
2
1
270
240
210
180
150
120
90
60
30
.1
.2
.3
.4
.5
.6
.7
.8
.9
Γ_s
0
5
10
15
20
25
30
35
40
45
KILOGAUSS/S
($\dot{B}$ FOR 240kV)

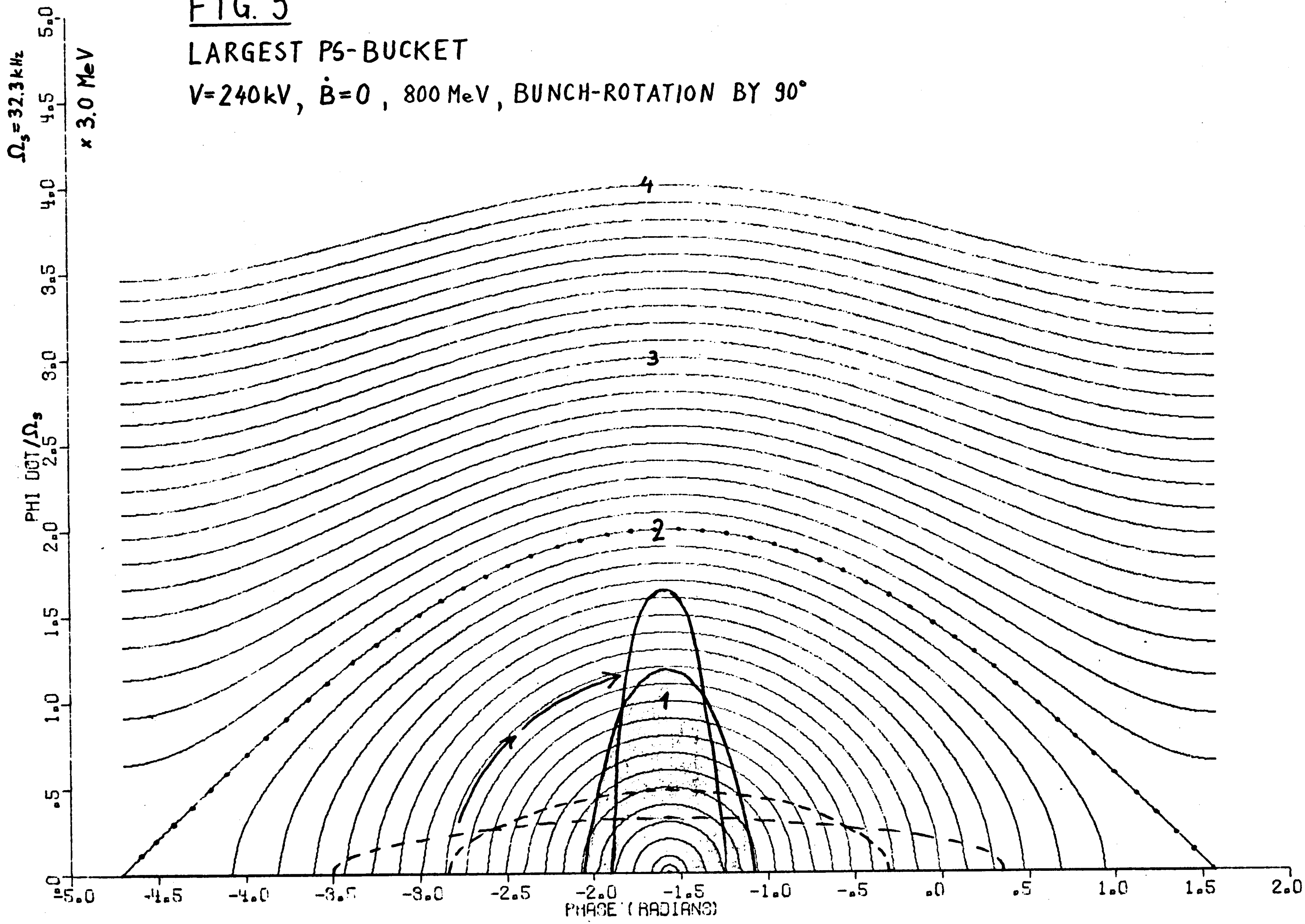


FIG. 5

LARGEST PS-BUCKET

V=240kV, $\dot{B}=0$, 800 MeV, BUNCH-ROTATION BY 90°

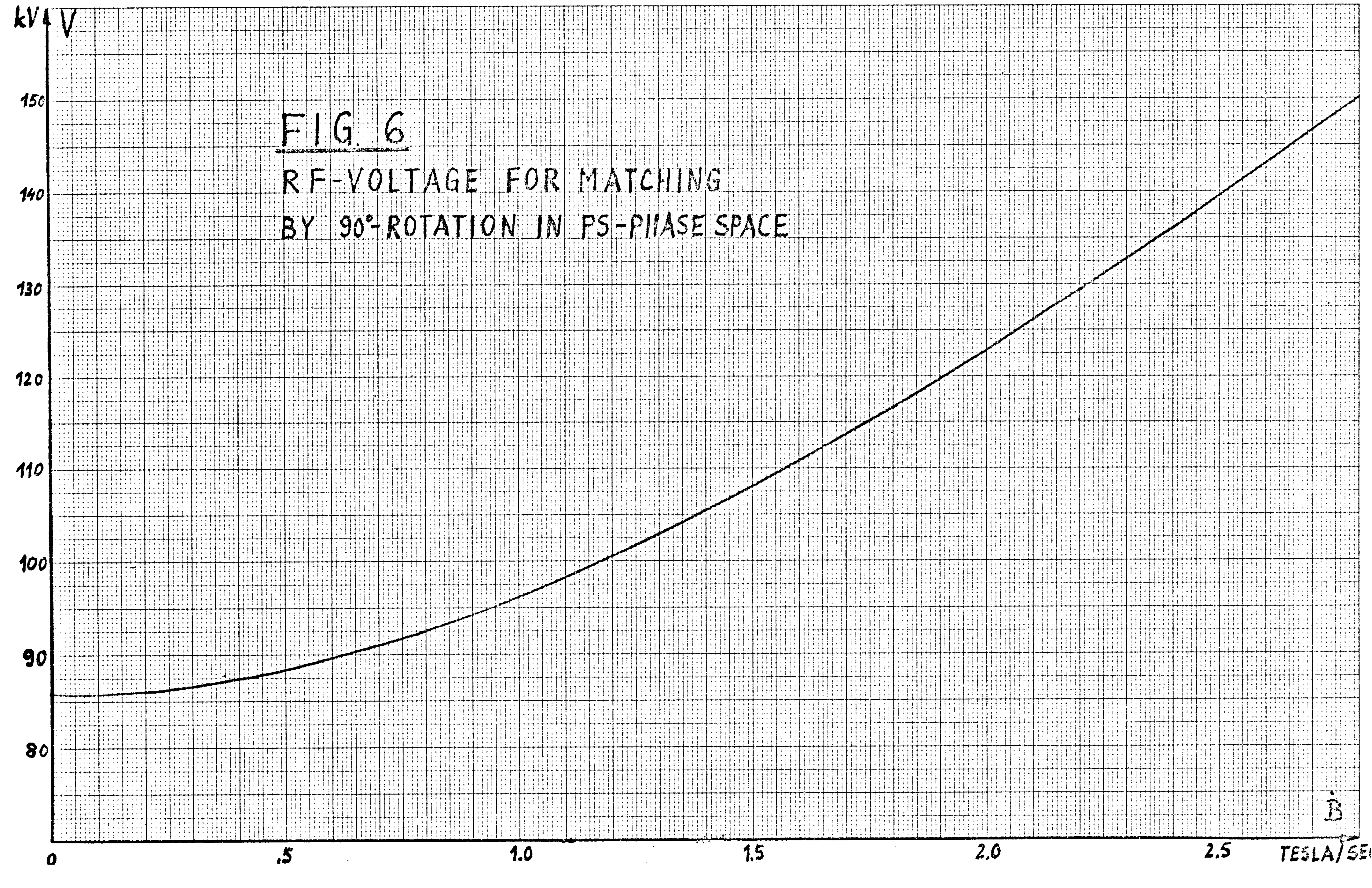


FIG. 6
RF-VOLTAGE FOR MATCHING
BY 90°-ROTATION IN PS-PHASE SPACE
kV
V
150
140
130
120
110
100
90
80
0
.5
1.0
1.5
2.0
2.5
TESLA/SE
Ḃ