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SYNCHROTRON FREQUENCIES
CALCULATED FOR PRESENT PS-OPERATION

by

H. H. Umstätter

Summary

The available tables of synchrotron frequencies [1] had become obsolete when the number of accelerating cavities had been reduced from 16 to 15 and their sum voltage stepped up from initially 108 kV to 150 kV per turn (since March 1966). The tables had been calculated under the assumption of a nominal rate of rise of the magnetic field $\dot{B}$ of 12 kGauss/sec. Actually, $\dot{B}$ varies between 14 and 11 kG/sec during the acceleration cycle and accordingly the phase of the synchronous particle must also vary in the PS. By consideration of this fact, more precise tables of the expected synchrotron frequencies have been calculated and plotted for 150 kV accelerating voltage and less. From these curves one can find the theoretical synchrotron frequencies also for the case that the accelerating voltage changes during the cycle.

Comments

In fig. 1 and 2 the synchrotron frequencies are plotted versus the B trigger-pulses which are used as a reference in most measurements (one pulse occurring for every 10 gauss increased field) and which are related in a simple way to the particle momentum

$$p = 2.1009 \frac{\mathrm{GeV/c}}{\mathrm{kGauss}} \cdot B$$

The kinetic energy is indicated in a scale on top of the graphs. The curves have been drawn as accurately as possible, so that they can replace tables because curves are more useful for interpolation between various accelerating voltages.

More details of the transition region are plotted in fig. 3. For a momentum compaction factor $\alpha = 0.027$, transition occurs shortly after B = 2680 Gauss. If α and B_{trans} change with the radial position of the beam, one should shift the abscissa in fig. 3 horizontally. ~~If B and the voltage on the cavities will be doubled in the future, the synchrotron frequencies will be the same.~~

Method of calculation

The differential equation of phase oscillations during adiabatic acceleration is

$$\frac{d^2\varphi}{dt^2} = +\left(\frac{c}{R}\right)^2 \frac{heV}{2\pi\gamma E_o} \left(\alpha - \frac{1}{\gamma^2}\right) \left[\cos\varphi - \cos\varphi_s\right] \tag{1}$$

where φ is the phase angle of the particle counted from the crest of the sinusoidal RF voltage, φ_s is the synchronous phase, h = 20 the harmonic number, V the sum voltage of all accelerating cavities, α the momentum compaction factor, E_o the proton rest energy, $\gamma = E/E_o$ and (c/R) the angular frequency of a particle orbiting with the velocity of light on an orbit of R = 100 m.

Since this is a nonlinear differential equation, the phase oscillations are unharmonic, particles with large amplitudes oscillating more slowly *). However, it is well known that in the case of short bunches and small amplitudes, one can linearize the right hand side of the differential equation (1)

$$\ddot{\varphi} = -\left(\frac{c}{R}\right)^2 \frac{heV}{2\pi\gamma E_o} \left(\alpha - \frac{1}{\gamma^2}\right) \sin\varphi_s (\varphi-\varphi_s) = -\omega_s^2 (\varphi-\varphi_s) \qquad (2)$$

which means that one considers only harmonic, coherent synchrotron oscillations around the stable phase φ_s with frequency

$$\underline{f_s = \frac{\omega_s}{2\pi} = \frac{c}{2\pi R}\sqrt{\frac{heV\sin\varphi_s}{2\pi E_o\gamma}\left(\alpha - \frac{1}{\gamma^2}\right)}} \qquad (3)$$

There are two quantities γ and $\sin\varphi_s$ which vary as the magnet field rises

$$\gamma = \frac{E}{E_o} = \sqrt{\frac{p^2c^2+E_o^2}{E_o^2}} = \sqrt{\frac{e^2r^2B^2c^2}{E_o^2} + 1}$$

$$\underline{\gamma = \sqrt{\left(\frac{rcB}{E_o/e}\right)^2 + 1}} \qquad (4)$$

The phase φ_s is related to the rise of the magnetic field because $\dot{p}$, the rate of increase of particle momentum, is equal to the energy gain per turn $eV\cos\varphi_s$ divided by the orbit length.

$$\dot{p} = er\dot{B} = \frac{eV\cos\varphi_s}{2\pi R} \qquad (5)$$

where r = 70.079 m is the radius of curvature of the beam in the magnets.

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*) The factor by which they are slower can be obtained from tables of Gumowski [2] or Koziol [3].

Thus we have

$$\underline{\sin\varphi_s} = \sqrt{1 - \cos^2\varphi_s} = \underline{\sqrt{1 - \left(\frac{2\pi Rr\dot{B}}{V}\right)^2}} \qquad (6)$$

Now we must know $\dot{B}$ as a function of B(t). B(t) has been measured recently and its time derivative is plotted in fig. 4. We can see that $\dot{B}$ is not constant, but we get a good approximation by the function

$$\underline{\dot{B} = 14\ \text{kG s}^{-1} - 0.32\ \text{s}^{-1} \cdot B} \qquad (7)$$

Equations (7), (6) and (4) have been substituted in (3) and programmed for the computer to calculate tables of the synchrotron frequency f_s. The results are based on the following data:

R	=	100.000 m	h	=	20
r	=	70.079 m	α	=	0.027
c	=	$299.7925 \cdot 10^6$ m/s			
E_o	=	938.256 MeV	V	=	150 kV, 130 kV .... 70 kV

Acknowledgement

I thank Mr. Ley for the measurement of B(t) on May 20, 1968. It confirms the data given in Mr. Isch's report MPS/Int.RF 66-2 fig. 3.4 and MPS user's handbook section F2 page 3.

References

1) J. A. Geibel : Table of Dynamic Parameters of the CERN Protron Synchrotron RF Note 27 - PS/Int.RF 58-8 (revised)

2) I. Gumowski : CPS RF-Bucket, Width, Height and Area. MPS/Int.RF 67-1

3) H. Koziol : Periods of Phase Oscillations as a Function of Amplitude. CERN 67-29

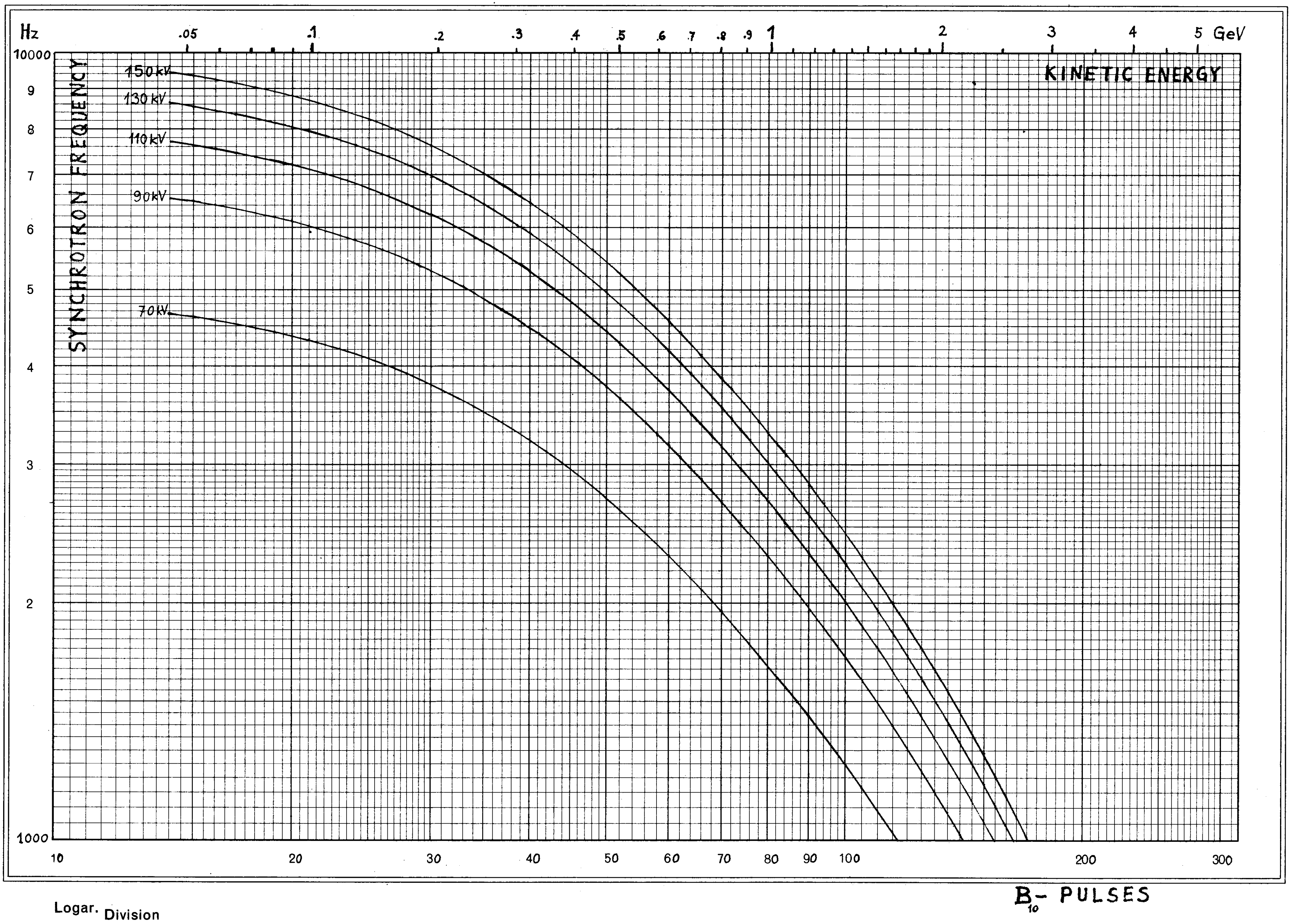

FIG.1

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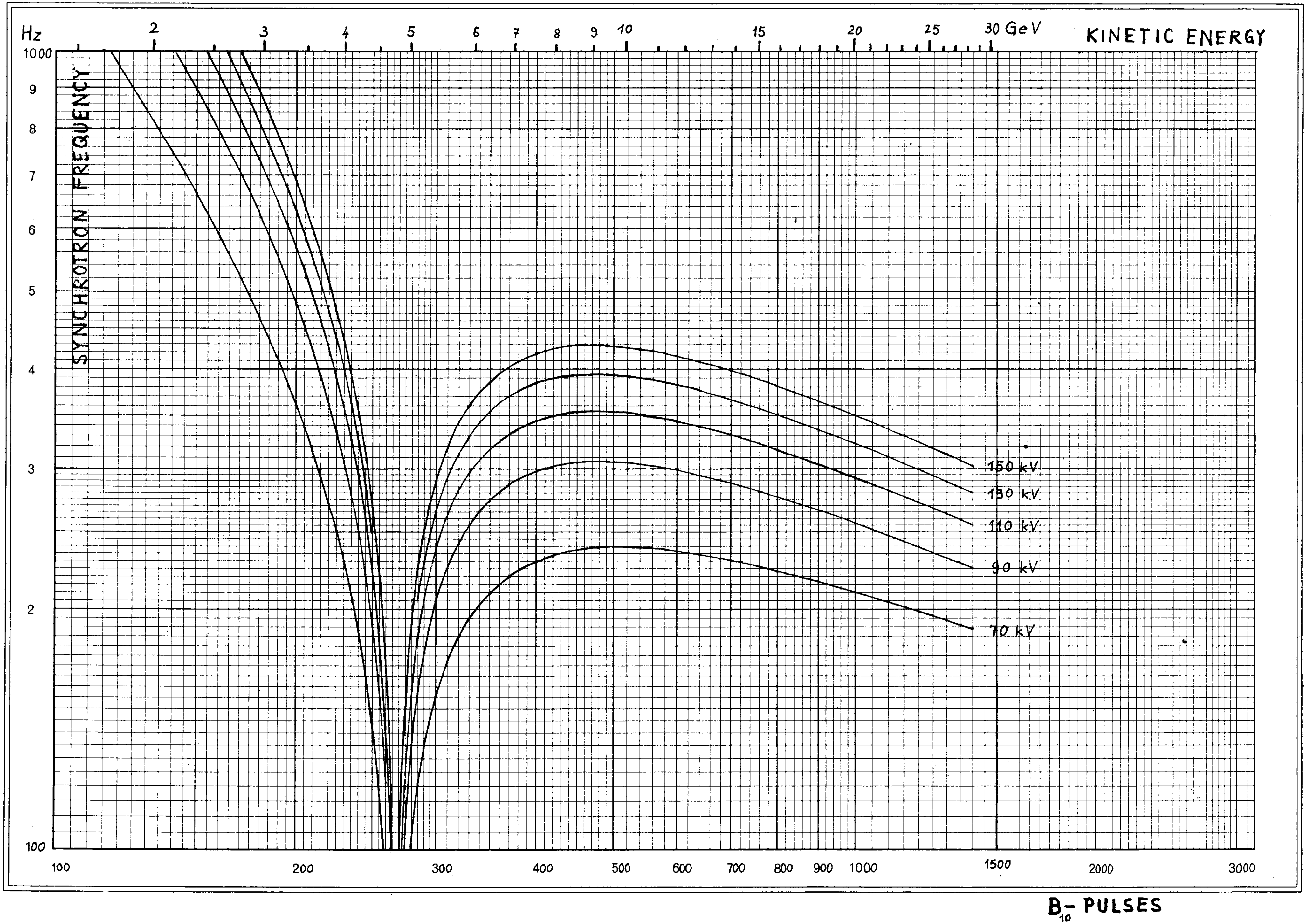

Hz
1000
9
8
7
6
5
4
3
2
100
SYNCHROTRON FREQUENCY
2
3
4
5
6
7
8
9
10
15
20
25
30 GeV
KINETIC ENERGY
150 kV
130 kV
110 kV
90 kV
70 kV
100
200
300
400
500
600
700
800
900
1000
1500
2000
3000
B_{10}- PULSES

FIG. 2

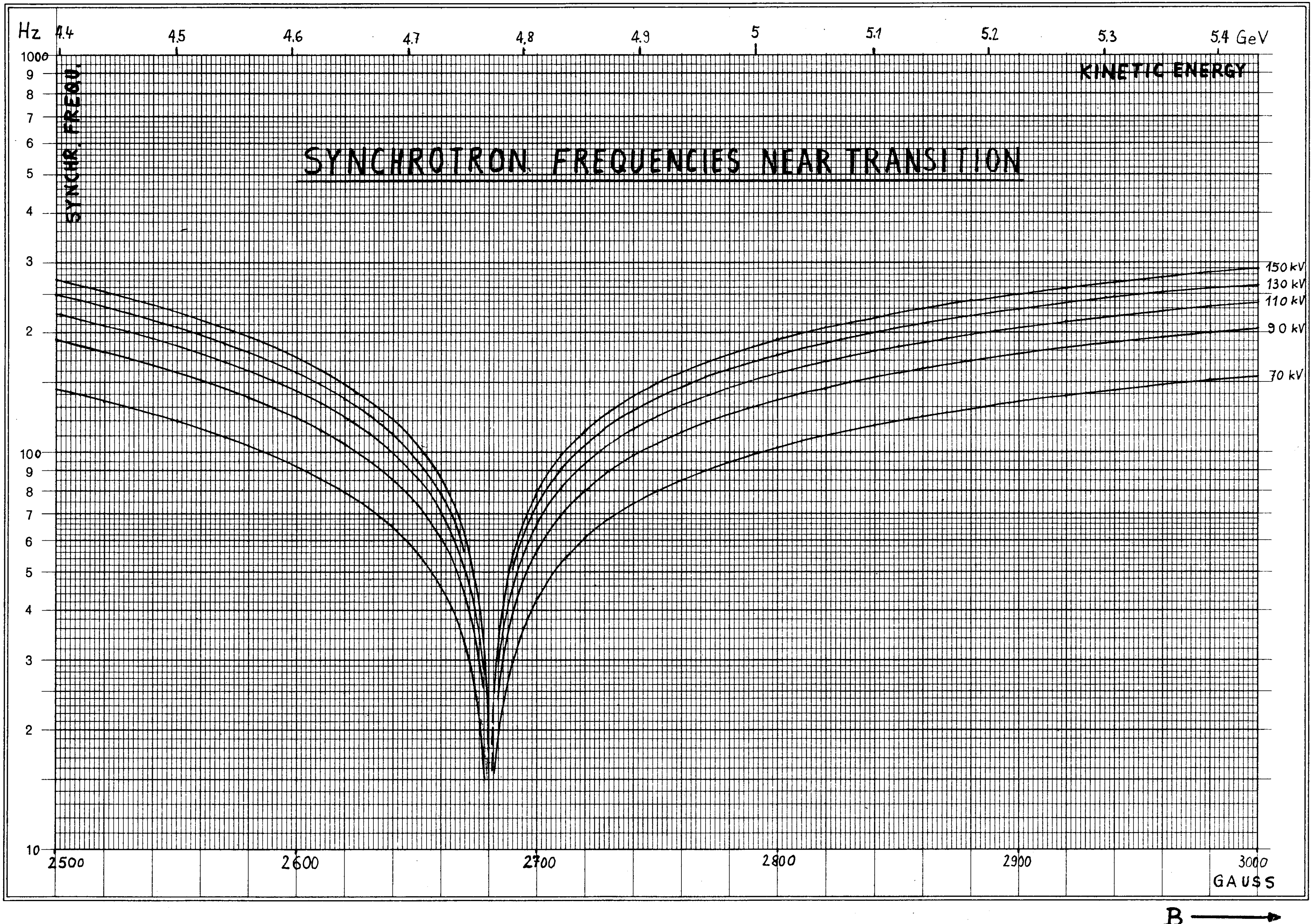
SYNCHROTRON FREQUENCIES NEAR TRANSITION
KINETIC ENERGY
SYNCHR. FREQU.
Hz
GeV
4.4
4.5
4.6
4.7
4.8
4.9
5
5.1
5.2
5.3
5.4
1000
100
10
150 kV
130 kV
110 kV
90 kV
70 kV
2500
2600
2700
2800
2900
3000
GAUSS
B

FIG. 3

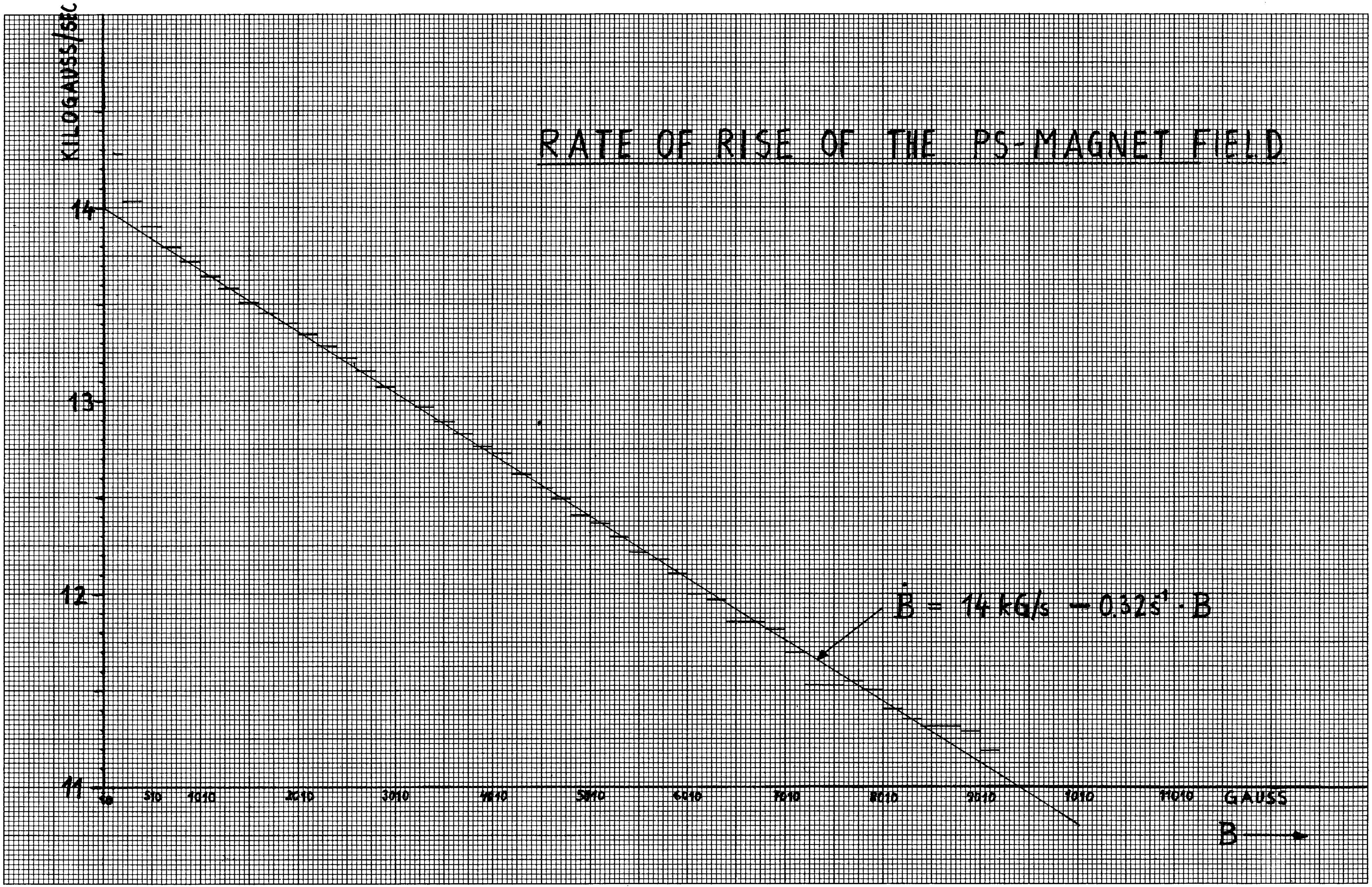

FIG.4

21.5.1968