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ON THE MEASUREMENT OF THE SLOW EJECTED BEAM STRUCTURE

by

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TABLE OF CONTENTS:

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1. Introduction

The slow ejected beam from the synchrotron is not an ideal constant flux of protons, but shows considerable intensity fluctuations. These fluctuations present a handicap to the experimental physicists. In order to determine the effect of the structure on an experiment, and to assist in the adjustment of the ejection, some quantitative measure of the structure is required.

This report discusses the use of the autocorrelation function and the effective spill time as measures of the burst structure. These give a measure of the average structure over the burst, this being of most relevance to the physics experiments as they involve a considerable stochastic element. The instantaneous time structure of the burst is, of course, of great interest but is unsuitable as a quantitative measure of the structure.

Three methods of measuring the high frequency structure are examined and compared. It is shown how these high frequency measurements can be combined with measurements of the low frequency structure to give the overall characteristics of the burst.

2. The effective spill time

A major problem in most experiments is that of chance events: events which cannot be distinguished from real events but which result from more than one primary particle. This problem is accentuated by the presence of structure on the beam.

It will be shown later that the number of chance events C resulting from two primary particles increases with the square of the beam intensity. If we define the proton flux of the ejected burst as the signal $m(t)$ starting at $t = 0$ and finishing at $t = T_s$ then we can write

$$C \propto \int_0^{T_s} m^2(t)\, dt \tag{1}$$

whereas the number of real events is

$$R \propto \int_0^{T_s} m(t)\, dt \tag{2}$$

The fraction R/C is a sort of signal to noise ratio and can always be increased by decreasing the intensity and hence increasing the duration of the experiment. For best efficiency, however, the mean intensity $\overline{m(t)}$ and the ratio R/C should both be as high as possible, which leads to the figure of merit

$$D_{PS} = \overline{m(t)} \cdot \frac{R}{C} \tag{3}$$

and if we write

$$\overline{m(t)} = \frac{I}{T_{PS}} \int_0^{T_s} m(t)\, dt \tag{4}$$

where T_{PS} is the PS repetition time, we have

$$D_{PS} = \frac{\left[\int_0^{T_s} m(t)\, dt\right]^2}{\int_0^{T_s} m^2(t)\, dt} \cdot \frac{1}{T_{PS}} \tag{5}$$

The first factor in this expression for D has the dimensions of time and is called the effective spill time (1), (2), (3) T_e where

$$T_e = \frac{\left[\int_0^{T_s} m(t)\, dt\right]^2}{\int_0^{T_s} m^2(t)\, dt} \tag{6}$$

T_e is the time over which the same number of protons, evenly spread out in time, would give the same number of chance coincidences. Thus

$$D_{PS} = \frac{T_e}{T_{ps}} < 1 \tag{7}$$

is the machine duty factor which should be as high as possible. It can never be unity due to the time required to accelerate the protons and the duty factor of septa, etc. We can, however, define an ejection duty factor D_e, where

$$D_e = \frac{T_e}{T_s} \tag{8}$$

D_e depends only on the burst structure and could conceivably reach unity.

3. The auto-correlation function

The burst auto-correlation function (or its frequency domain equivalent, the power spectrum) gives information about the average frequency structure of the burst. This is of interest for two reasons. Firstly not all experiments depend on the structure in the same way; in particular it will be shown later that T_e is often a pessimistic measure of the structure. Secondly knowledge of the frequency structure can help in diagnosing and removing the cause of the structure.

The auto-correlation function can also be used to calculate the effective spill time T_e. Here we define the auto-correlation function as

$$A(\tau) = \int_0^{T_s} m(t)\, m(t-\tau)\, dt \tag{9}$$

and it can be shown that

$$T_e = \frac{\left[\int_0^{T_s} m(t)\, dt\right]^2}{\int_0^{T_s} m(t)\, dt} = \frac{\int_{-T_s}^{T_s} A(\tau)\, d\tau}{A(0)} \tag{10}$$

since $m(t) = 0$ for $t < 0$, $t > T_s$.

Both $A(\tau)$ and T_e are difficult to measure accurately due to the wide range of frequencies and times involved, namely frequencies up to several tens of MHz and burst times of around 200 msec. In the PS, however, we can make the simplifying assumption that the structure consists of a high frequency ripple modulated by the low frequency structure so that

$$m(t) = r(t) \cdot l(t) \qquad (11)$$

where $r(t)$ is the RF structure of period $T = 2.1$ µsec and $l(t)$ is the low frequency shape which has been shown (4),(5) to have no significant frequency components above about 3 kHz. Then

$$A(\tau) = \int_0^{T_s} r(t) \,.\, l(t) \,.\, r(t-\tau) \,.\, l(t-\tau)\, dt$$

$$= \sum_{n=1}^{T_s/T} \int_{(n-1)T}^{nT} r(t) \,.\, r(t-\tau) \,.\, l(t) \,.\, l(t-\tau)\, dt$$

and if $l(t)$ is considered constant over the short time T

$$A(\tau) = \frac{1}{T}\int_0^{T} r(t) \,.\, r(t-\tau)\, dt \;\cdot \int_0^{T_s} l(t) \,.\, l(t-\tau)\, dt$$

$$= A_r(\tau) \,.\, A_l(\tau) \qquad (12)$$

$A_r(\tau)$ is the auto-correlation function of the high frequency structure and gives the information required for debunching (or rebunching) studies.

The effective spill length T_e can be obtained as

$$T_e = \frac{\int_{-T_s}^{T_s} A(\tau)\, d\tau}{A(0)} = \frac{\int_{-T_s}^{T_s} A_r(\tau) \cdot A_l(\tau)\, d\tau}{A_r(0) \cdot A_l(0)}$$

$$= \frac{\sum_{n=-T_s/T}^{T_s/T} \int_{(n-1)T}^{nT} A_r(\tau) \cdot A_l(\tau)\, d\tau}{A_r(0) \cdot A_l(0)}$$

Assuming as before that $A_l(\tau)$ can be considered constant over a time interval T we have

$$T_e = \frac{\frac{1}{T}\int_0^T A_r(\tau)\, d\tau \cdot \int_{-T_s}^{T_s} A_l(\tau)\, d\tau}{A_r(0) \cdot A_l(0)}$$

$$= \frac{\frac{1}{T}\int_0^T A_r(\tau)\, d\tau}{A_r(0)} \cdot T_{elf} = D_r \cdot T_{elf} \tag{13}$$

where T_{elf} is T_e measured neglecting the high frequency components, and D_r is a high frequency duty factor

$$D_r = \frac{\int_0^T A_r(\tau)\, d\tau}{T\ A_r(0)} = \frac{\int_0^T A_r(\tau) \cdot A_l(\tau)\, d\tau}{T\ A_r(0) \cdot A_l(0)}$$

$$= \frac{\int_0^T A(\tau)\, d\tau}{T\, A(0)} \tag{14}$$

as we can assume $A_1(\tau) = A_1(0)$ = constant over the interval $0 \leq t \leq T$. Thus D_r can be obtained by measuring $A(\tau)$ over the first 2.1 µsec.

Now $A_1(\tau)$ and T_{elf} are easily measured using regular computer sampling (4), (5) since only low frequencies are involved. So combining the low and high frequency measurements the complete burst characteristics can be obtained.

As an example consider the effect of a sinusoidal ripple on the burst, of relative amplitude H so that

$$m(t) = 1 + H \cos \omega t$$

The auto-correlation function is

$$A_r(\tau) = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} (1 + H \cos \omega t)\,(1 + H \cos \omega(t+\tau)\, dt$$

$$= 1 + \frac{H^2}{2} \cos \omega\tau \tag{15}$$

Thus the auto-correlation function also shows a sinusoidal ripple but of magnitude $\frac{H^2}{2}$. The duty factor for $0 \leq H \leq 1$ is

$$D_r = \frac{\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\left[1 + \frac{H^2}{2}\cos\omega\tau\right]d\tau}{1 + \frac{H^2}{2}\cos 0}$$

$$= \frac{1}{1 + \frac{H^2}{2}} \tag{16}$$

which gives 66 % for 100 % modulation.

4. Chance Coincidences

As mentioned in section 2 a major experimental problem is that of chance events. In this section expressions for the number of chance events are derived. It is shown that the effective spill time is often pessimistic, i.e. indicates too many chance events. The correct number of chance events for any particular experiment is derived in terms of the auto-correlation function.

The proton beam is used against a target to produce "events". The number of events produced is usually so much smaller than the number of incident protons (~ 10^4 per nano-second) that we can consider the production process as stochastic, with the probability a(t) of an event being proportional to flux of incident protons. Thus

$$a(t) = k\, m(t) \quad \text{and} \quad \int_0^{T_s} a(t)\, dt = N \tag{17}$$

where N is the number of events per burst.

Assume the counting rate is

$$a = \text{constant} \tag{18}$$

Now for the very short time interval dt where

$$a \cdot dt \ll 1 \qquad (19)$$

the probability of observing one particle $P_1(dt)$ will be

$$P_1(dt) = a\ dt \qquad (20)$$

and of observing no particles

$$P_0(dt) = 1 - a\ dt \qquad (21)$$

From this we obtain the probability of observing x particles in the time interval $t + dt$ as

$$P_x(t+dt) = P_1(dt) \cdot P_{x-1}(t) + P_0(dt) \cdot P_x(t) \qquad (22)$$

or with (20) and (21)

$$\frac{P_x(t+dt) - P_x(t)}{dt} = a(P_{x-1}(t) - P_x(t)) \qquad (23)$$

and with $\dfrac{P_x(t + dt) - P_x(t)}{dt} \equiv \dfrac{d\ P_x(t)}{dt}$ (24)

we have $\dfrac{dP_x(t)}{dt} = a(P_{x-1}(t) - P_x(t))$ (25)

This equation can be solved for various values of x to obtain

$$P_x(t) = e^{-at} \frac{(at)^x}{x!}$$

with $$P_0(t) = e^{-at} \tag{26}$$

which is the Poisson distribution.

Consider the probability of a coincidence in a gate time L, related to the occurrence of an event in time dt. This can be written

$$df = a\ dt \cdot (1 - e^{-aL}) \tag{27}$$

i.e. the joint probability of an event in time dt and at least one event in the subsequent gate time L. If

$$aL \ll 1 \tag{28}$$

then equation (27) becomes

$$df = a^2 L\ dt \tag{29}$$

and over the time T_s the total number of coincidences is

$$f = a^2 L\ T_s \tag{30}$$

the well known equation for chance coincidences.

Now consider the case where the probability of observing a particle in the time dt is the time variable a(t) as defined by (17). The derivation of the distribution in this case is more complicated [6], so only the result is quoted here. This gives the probability of x events in the time t starting from t_0 as

$$P_x(t_0,t) = \frac{1}{x!} \left[\int_{t_0}^{t} a(\tau)\, d\tau\right]^x \exp\left[-\int_{t_0}^{t} a(\tau)\, d\tau\right] \tag{31}$$

with

$$P_0(t_0,t) = \exp\left[-\int_{t_0}^{t} a(\tau)\, d\tau\right] \tag{32}$$

Equation (27) for the probability of a coincidence becomes

$$df = a(t) \,.\, dt \left[1 - \exp\left(-\int_{t}^{t+L} a(t)\, dt\right)\right] \tag{33}$$

and for

$$\int_{t}^{t+L} a(t)\, dt \ll 1 \tag{34}$$

this gives

$$df = a(t)\, dt \int_{t}^{t+L} a(t)\, dt \tag{35}$$

and the total number of coincidences is

$$f = \int_{-T_s}^{T_s} a(t) \int_{t}^{t+L} a(t)\, dt \,.\, dt$$

$$= \int_{-T_s}^{T_s} a(t) \int_{0}^{L} a(t+\tau)\, d\tau \,.\, dt$$

$$= \int_{0}^{L} \int_{-T_s}^{T_s} a(t)\, a(t+\tau)\, dt \,.\, d\tau \tag{36}$$

and using equation (17) we have

$$f = k^2 \int_0^L A(\tau) \quad d\tau \tag{37}$$

since the auto-correlation function is an even function of τ. Also from equation (17) we have

$$\int_0^{T_s} k\, m(t)\; dt = N \tag{38}$$

or

$$k = \frac{N}{\int_0^{T_s} m(t)\; dt} \tag{39}$$

so that

$$f = N^2 \frac{\int_0^L A(\tau)\; d\tau}{\left[\int_0^{T_s} m(t)\; dt \right]^2} \tag{40}$$

Equation (40) gives the number of chance coincidences in terms of the auto-correlation function and the total number of counts.

The special case where the gate time L is short compared with any variation in $A(\tau)$ gives for equation (37)

$$f = k^2 L \,.\, A(0)$$

$$= k^2 L \int_0^{T_s} m^2(t)\; dt \tag{41}$$

the result used in section 2 that the number of chance coincidences is proportional to the square of the intensity. From equation (40) for this special case we have

$$f = N^2 L \frac{A(0)}{\left[\int_0^{T_s} m(t)\, dt\right]^2}$$

$$= \frac{N^2 L}{T_e} \tag{42}$$

or

$$T_e = \frac{N^2 L}{f} \tag{43}$$

It can be proved that

$$A(\tau) \leqslant A(0) \tag{44}$$

so that

$$\int_0^L A(\tau)\, d\tau \leqslant L \,.\, A(0) \tag{45}$$

so that if any structure of frequency comparable to or higher than $\frac{1}{L}$ is present then the number of coincidences given by equation (40) will be less than that given by equation (42). This is equivalent to saying that T_e is a worst case measure of the structure and will be pessimistic if the coincidence time of the experiment is large compared with some of the frequencies in the structure. This is especially relevant to spark chamber experiments which have a coincidence time of about 1 µsec, thus being rather insensitive to bunch structure but sensitive to structure at the revolution frequency and below.

It is possible to define an effective spill time for each experiment, T_{exp}, from equation (42) so that

$$f_{exp} = \frac{N^2 L}{T_{exp}} \tag{46}$$

where f_{exp} is the number of coincidences in the particular experiment. From (40) we have

$$T_{exp} = T_e \frac{L \cdot A(0)}{\int_0^L A(\tau)\, d\tau} \tag{47}$$

i.e. T_{exp} is always greater than T_e.

5. Time interval measurement

In this method the time τ between two events τ and τ+L are added together and displayed on a scope. L is the time resolution between ordinates on the scope and the number of ordinates is τ_{max}/L.

The probability of observing a time between τ and τ+L starting at time t in the interval dt is the joint probability of an event in time dt, no event between t and t+τ, at least one event between t+τ and t+τ+L. This gives

$$df = a(t).dt.\exp\left[-\int_t^{t+\tau} a(t)\, dt\right] . \left(1 - \exp\left[-\int_{t+\tau}^{t+\tau+L} a(t)\, dt\right]\right) \tag{48}$$

Assuming the resolution time L short compared with variation in a(t) and L a(t+τ) << 1 (48) becomes

$$df = L.a(t)\ a(t+\tau)\ \exp\left[-\int_t^{t+\tau} a(t)\, dt\right] dt \tag{49}$$

If we now put $$\int_t^{t+\tau} a(t)\, dt = \hat{a}\,\tau \tag{50}$$

and assume $\hat{a}$ is a constant, the total becomes

$$f = L\, e^{-\hat{a}\tau} \int_0^{T_s} a(t)\; a(t+\tau)\; dt \tag{51}$$

$$= L\, e^{-\hat{a}\tau} \,.\, k^2 \,.\, A(\tau) \tag{52}$$

Thus the display on the scope is proportional to the auto-correlation function multiplied by the "wiggle factor" $e^{-\hat{a}\tau}$ which under suitable conditions can be assumed to be a decaying exponential of time constant 1/(Average counting rate).

6. Measurement using delayed coincidences

In this method the pulse train from the detector is put into a coincidence unit of resolution time L along with the pulse train delayed by the time τ. The number of events N and the number of coincidences f are counted. The probability of observing a coincidence derived from an event at time t in the interval dt is the joint probability of an event in dt and at least one event between $t + \tau - \frac{L}{2}$ and $t + \tau + \frac{L}{2}$. This gives

$$df = a(t)\; dt \left(1 - \exp\left[-\int_{t+\tau-\frac{L}{2}}^{t+\tau+\frac{L}{2}} a(t)\; dt\right]\right) \tag{53}$$

which gives, using the same assumptions as before about the shortness of L

$$df = L\; a(t) \,.\, a(t+\tau)\; dt \tag{54}$$

and the total number of coincidences f is

$$f = k^2 \, L \;\; A(\tau) \tag{55}$$

$$= N^2 \, L \frac{A(\tau)}{\int_{-T_s}^{T_s} A(\tau) \, d\tau} \tag{56}$$

$$= \frac{N^2 \, L}{T_e} \cdot \frac{A(\tau)}{A(0)} \tag{57}$$

Using several delay channels, or scanning the delay, enables a relative auto-correlation function to be plotted out. The assumptions made in section 3 about the periodicity of $A_r(\tau)$ and the slowness of change of $A_l(\tau)$ allow us to write here

$$A(2.1 \text{ µsec}) = A(0) \tag{58}$$

so that equation (57) gives

$$T_e = \frac{N^2 \, L}{f_{2.1}} \tag{59}$$

where $f_{2.1}$ is the number of coincidences where the delay is 2.1 µsec, i.e. the period of $A_r(\tau)$. Equation (59) is often used to calculate the effective spill time.

7. Use of two detectors

In practice it is impossible to resolve events closer than about 20 nsec so the above measurement can give no information about $A(\tau)$ for $\tau < 20$ nsec. It must be noted, however, that since we have assumed $A(\tau) \approx A(\tau+2.1\ \mu\text{sec})$, the required information can be obtained by measuring to $\tau = 2.1\ \mu\text{sec} \pm 20$ nsec. At least this is true for the delayed coincidence method. In the time-interval method the "wiggle factor" $e^{-\hat{a}\tau}$ intervenes and it may be desirable to measure right down to $\tau = 0$. This can be accomplished using two detectors and two targets with counting rates

$$a_1(t) = k_1\ m(t)$$

$$a_2(t) = k_2\ m(t + t') \qquad (60)$$

where t' is the time difference between the two detectors. If we assume $a_1(t) = a_2(t) = a(t)$ and $t' = 0$ then the mathematics developed above applies directly. Otherwise some small modifications have to be made.

Now whereas two detectors are feasible, two targets may not be. If the two detectors look at the same target we have the problem of coincidences in roughly a $\pm$ 10 nsec region being due to secondary particles from the same event. Suppose we assume that each proton hitting the target produces a mean of n secondary particles which are isotropically emitted. (This is not strictly true. The secondaries are not emitted isotropically, nor is n independent of the angle to the beam axis at which the target is seen by the detectors. One can, however, define an effective solid angle Ω, and if Ω is small and the viewing angle does not change, n can be taken as constant.) Each of the particles detected in the first detector is then accompanied by n other particles, and the probability of detecting one of the particles in the second detector is $n\Omega_2$ where Ω_2 is the solid angle through which the second detector looks at the target. The number of true coincidences adding to the chance coincidences for delays $\tau < 10$ nsec will be, assuming $a_1 = a_2 = a =$ constant,

$$f_T = a\ T_s \cdot n\Omega_2 \qquad (61)$$

whereas the number of chance coincidences will be

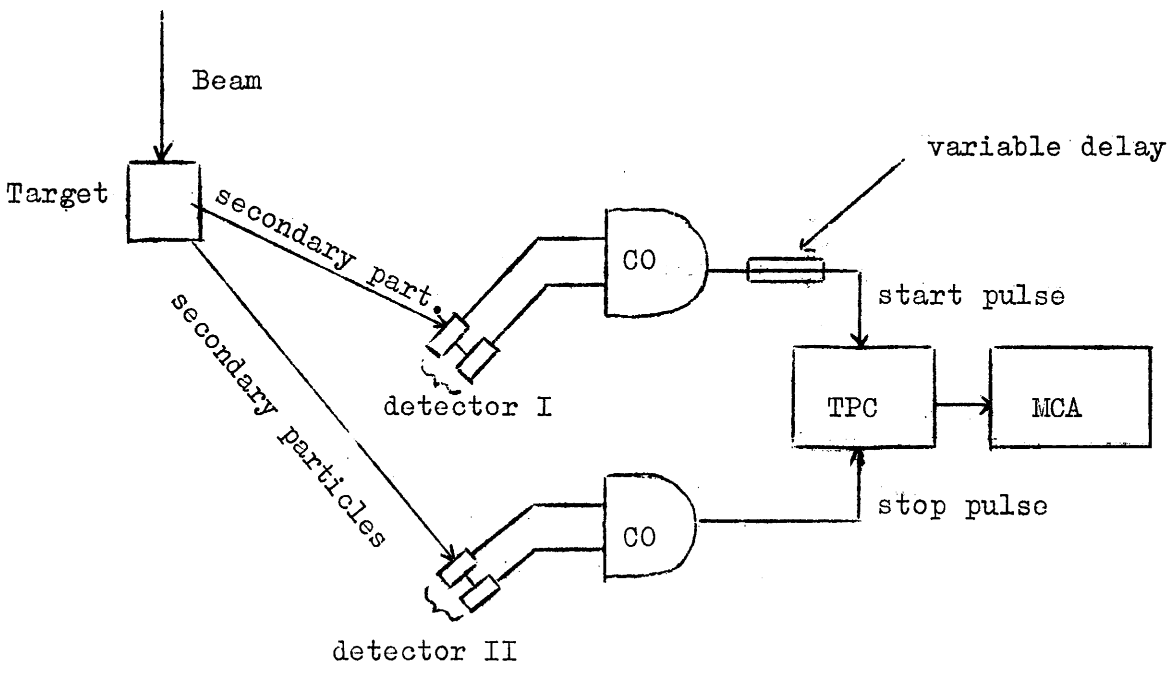
$$f_c = a^2 L T_s \tag{62}$$

so the percentage contribution of time coincidences is

$$\frac{f_T}{f_c} \times 100 = \frac{n\Omega_2}{a L} \times 100 \tag{63}$$

with a solid angle $\Omega_2 = 10^{-5}$, $n = 10$, $\Delta t = 20$ nsec and $a = 10^6$ counts per sec we would obtain a contribution of 0.5 %. For $\tau > 10$ nsec no more true coincidences would be observed.

The above suggests the following set-up for measuring the time-interval distribution:



8. The direct measure of the auto-correlation function

As mentioned above computer sampling techniques are used at low frequencies to obtain $A_1(\tau)$ and T_{elf}. This could also be done for the high frequencies if the true signal $m(t)$ is available. At low frequencies regular sampling is used but this is difficult at the higher frequencies. A procedure which could be used with two sampling heads is to take two samples of $m(t)$ separated by the time τ, read these into the computer, then form the product $m(t) \cdot m(t-\tau)$. This would be repeated for a range of delays $0 \leq \tau \leq 2.1$ µsec. The whole process could then be repeated Q times and the auto-correlation function formed as

$$A(\tau) = \frac{1}{Q} \sum m(t) \cdot m(t-\tau) \tag{64}$$

This assumes the sampling process to be asynchronous, i.e. the samples will be spread in a random fashion over the burst.

A signal $m(t)$ proportional to the flux of ejected protons could be obtained from a photo-multiplier looking at some light producing material in the beam. With the available proton flux ($\sim 10^4$ per nanosecond) an output signal of $\sim$ 10 mA should be obtainable from a low gain high current photo-multiplier. This would give a signal of $\sim$ 1 volt when matched directly into a cable so as to preserve the high frequencies. Such a signal could easily be sampled using sampling scope techniques, with a sampling aperture in the nanosecond region.

9. Comparison of the three methods

The above three methods of measuring the high frequency structure can be compared in terms of systematic accuracy, hardware requirements, and time for a given statistical accuracy.

The time interval method is the only one which shows a theoretical inaccuracy, this being due to the "wiggle factor" $e^{-\hat{a}\tau}$. This can always be diminished by reducing the average counting rate, with obvious repercussions on the measurement time.

Hardware requirements

The time interval method obtains its result from equipment which is standard for physics experiments and easy to use. It is rather expensive however. The delayed coincidence method again uses standard equipment. Two possibilities are available here: firstly one delay channel, coincidence unit, and counter can be used and the delay varied in order to plot the auto-correlation function; secondly a battery of delay channels, etc. can be used to build up all points on the auto-correlation function separately.

The first method is very economical in hardware but requires active operator intervention, is slower, and can give errors if the process is non-stationary. The second method requires a lot of equipment.

The direct method requires a special monitor and sampling circuits. The conversion and computer read-in equipment are standard. Side advantages of the special monitor are that it enables the instantaneous time structure to be viewed, allows analogue spectrum analysis, and provides an accurate low noise monitor for the observation of lower frequencies.

Time required for statistical accuracy

Assume a sinusoidal ripple on the burst of form

$$m(t) = 1 + H \cos \omega t$$

Section 3 gives the auto-correlation function as

$$A(\tau) = 1 + \frac{H^2}{2} \cos \omega\tau$$

and the duty factor as

$$D = 1 / \left(1 + \frac{H^2}{2}\right)$$

A 14 % ripple on the beam will therefore give a 1 % ripple on the auto-correlation function and a 1 % reduction in effective spill time. Now consider the measurement time required to resolve a 1 % ripple on the auto-correlation function.

a) Coincidence method

Equation (57) shows that the relative auto-correlation function is given by the number of coincidences. Thus a resolution of 1 % requires 100 coincidences, and an accuracy of 1 % requires approximately 10^4. Assuming the counting rate $a(t)$ approximately constant, equation (30) gives the time required for f coincidences as

$$T = \frac{f}{a^2 L} \tag{65}$$

Using a coincidence gate time L of 10 nanosec and the maximum practicable telescope counting rate of $a = 10^6$ per sec, the counting time T becomes 1 sec, i.e. 5 machine pulses at 200 msec per burst.

b) Time interval measurement

Equation (65) also gives the time required for the time interval measurement. In order to limit the effect of the wiggle factor $e^{-\hat{a}t}$ in equation (52), however, the counting rate must be limited. Allowing a droop of 10 % over the first 2.1 µsec gives a counting rate of 50 kHz. Again taking the resolution as 10 nsec (giving 210 ordinates over the first 2.1 µsec) the measuring time T is 20 secs or 100 machine pulses. This assumes that every time interval can be measured, as is the case in the equipment to be built at Nimrod by E. G. Sandels for CERN. A normal multichannel analyser has a dead time of 50 µsec which reduces its speed by a factor of 3 to 4 over the time mentioned above.

c) Direct method

Here each delayed product can easily be measured to better than 1 %. An even spread of delayed products across the 2.1 μsec period is required however, and if this is approximated by random sampling a statistical error is introduced. This is difficult to calculate and depends on the frequencies present. 200 samples may perhaps be adequate, this giving 10 samples on average over one period of the bunch frequency. A dead time of 50 μsec is reasonable for computer read in so that one average delayed product will require 10 msec. Again for 10 nsec resolution 210 points will be required, giving a total time of 2 secs or 10 machine pulses.

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