14th International Particle Accelerator Conference,Venice, Italy JACoW Publishing
ISBN: 978-3-95450-231-8 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2023-MOPL163

THERMODYNAMIC CHARACTERISTICS OF HYDROGEN IN AN IONIZATION COOLING CHANNEL FOR MUON COLLIDERS *

B. Stechauner†[1] , J.A. Ferreira Somoza, E. Fol, D. Schulte, CERN, 1211 Geneva 23, Switzerland
C. Rogers, Rutherford Appleton Laboratory, Didcot, United Kingdom
J.Schieck[1], Institute of High Energy Physics, 1050 Vienna, Austria
[1]also at TU Wien, 1040 Vienna, Austria

Abstract

Ionization cooling is the only suitable approach to reduce the phase space volume occupied by a muon beam on a timescale compatible with the muon lifetime. Small normalized transverse emittances can be achieved by using hydrogen (H) as an absorber and high solenoid fields at low beam energy. The strong focusing suppresses emittance growth due to scattering occurring from muon beam interaction with nuclei in the absorber's atoms. This leads to very small beam sizes and therefore the deposition of energy in small volumes causing a high peak energy density. Temperature changes in H can cause pressure rises that may damage the absorber's H containment windows. This work presents the acceptable temperature ranges in liquid H and discusses an alternative method with low density H-gases.

INTRODUCTION

A muon beam, generated from pion decays, occupies a large phase space. The emittance characterizes the phase space volume and has to be reduced to get high luminosities for future $\mu^+ \mu^-$ collisons. A recent muon collider design study is based on a reduction (cooling) of the emittance in the different cooling sections [1].

Final Cooling Design Overview

The International Muon Collider Collaboration (IMCC) provides a target normalized transverse emittance of 25 µm. The target emittance is achieved in the final cooling section. Each cell of this section consists of one ultra high field solenoid (30-60 T) filled with an energy absorbing material followed by an RF accelerating cavity after the solenoid. A 45 T solenoid has already been previously demonstrated [2]. While muons penetrate the absorber, they lose momentum in all three spatial directions. As Fig. 1 illustrates, muons gain longitudinal momenta through the re-acceleration in an RF cavity, while the transverse velocities remain unchanged. Coulomb scattering of muons with the absorber's nuclei reduces the cooling effect. Low atomic number materials have more energy loss per scatter and counteract the transverse momentum increases. The strong focusing forces of solenoids inside an absorber follow to an emittance reduction.

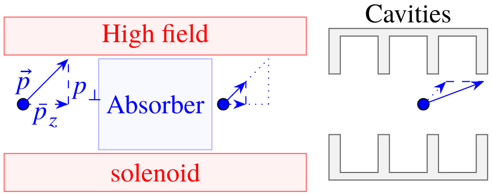

Figure 1: A final cooling cell initially reduces the momenta of muons. A longitudinal re-acceleration results to a decrease the divergence of the momenta.

Equilibrium Emittance

To achieve an efficient transverse phase space decrease in a final cooling cell, the normalized equilibrium emittance $\epsilon_\perp^{\mathrm{eq}}$ has to be as low as possible. An estimation of $\epsilon_\perp^{\mathrm{eq}}$ can be derived from the ionization cooling equation [3]

$$\epsilon_\perp^{\mathrm{eq}} = \frac{\beta_\perp \left(13.6\,[\mathrm{MeV}]\right)^2}{2\beta m c^2 L_\mathrm{R} \langle \partial E/\partial s\rangle}. \tag{1}$$

Equation (1) depends on the the betatron function $\beta_\perp\,[\mathrm{m}] \propto p\,[\mathrm{MeV}]/B\,[\mathrm{T}]$; the absorber's radiation length L_R [$\mathrm{g\,cm^{-2}}$]; its stopping force $\langle \partial E/\partial s\rangle$ [$\mathrm{MeV\,cm^2\,g^{-1}}$]; the Lorentz factor β; and the muon mass m [$\mathrm{MeV\,c^{-2}}$]. A comparison in Fig. 2 of this model (dashed lines) with $\epsilon_\perp^{\mathrm{eq}}$ evaluated by ICOOL [4] simulations (solid lines) shows discrepancies

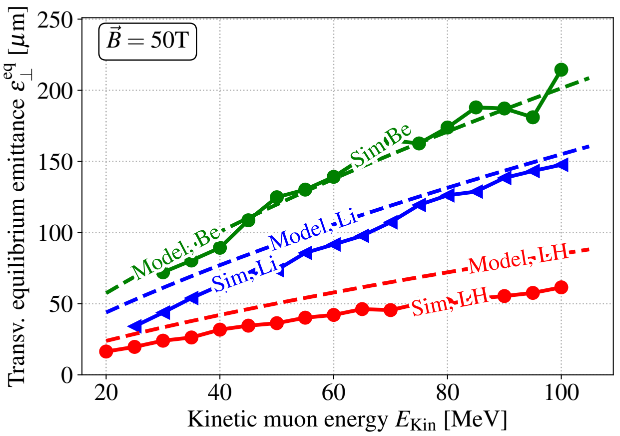

Figure 2: Equilibrium emittance discrepancy of ICOOL simulations compared with the theoretical model. The inconsistencies become stronger at lower E_{Kin}, which comes from approximations of the scattering angle in the model.

---

[*] TU Wien Funding for PhD students participating in conferences
[†] bernd.stechauner@cern.ch

with the model. ICOOL is a tracking program specialized in ionization cooling. The discrepancies are due to an approximated scattering model in the ionization cooling description. Nevertheless, the model as well the simulation clarifies that the lowest equilibrium emittance is achieved in a high magnetic field environment, when:

- the initial kinetic energy of the muons is low, and
- the absorber material has a large radiation length.

LIQUID HYDROGEN ABSORBER

The muon collider requires an emittance of 25 µm, which means $\epsilon_\perp^{\mathrm{eq}}$ has to be lower than that value. Figure 2 shows that such low equilibrium is possible for Liquid Hydrogen (LH) as an absorber when the beam energy is below 20 MeV and the field is at 50 T. The mean energy deposition $\langle \partial E/\partial s \rangle$ of muons follows the Bethe-Bloch equation [5, 6] and the deposition increases with decreasing beam energy.

The Last Cooling Cell

The target emittance is reached at the last cell of the final cooling section. This is observed, for instance, when a beam enters the last cell with a transverse emittance of 30 µm and a kinetic energy of 10 MeV. Figure 3 shows a simulation with ICOOL where 25 µm is reached when the absorber length is less than 3 cm. It is important to note that the longitudinal beam sizes are not discussed in this paper and the previous example is intended for illustrative purposes only.

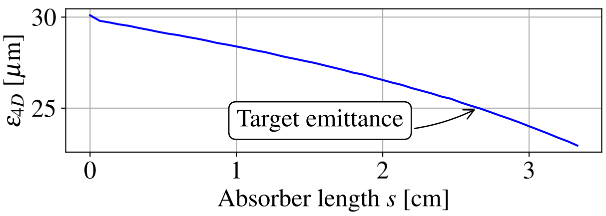

Figure 3: ICOOL simulation of a 3 cm LH absorber in a 50 T solenoid. The target emittance is achieved with an initial 10 MeV muon beam and 30 µm transverse emittance.

Energy Storage in LH

With ionization cooling, the beam diameter and the kinetic energy reduce, and coupled with the increasing energy loss at lower energy, a higher energy density is deposited in the absorber.

The volume V of this path corresponds to the beam area multiplied by the length of the absorber L. The molar number n of the absorber atoms/molecules inside the beam trace is $\rho V/A$, with the density ρ and the atomic mass A of the absorber. When LH looses its fluid properties at a certain temperature, the gas pressure inside the beam path can be expressed as

$$p = \frac{\rho R T}{A}, \tag{2}$$

with the gas constant $R = 8.314\,\mathrm{J\,K^{-1}\,mol^{-1}}$. The heat map in Fig. 4 illustrates the beam trace resulting from an ICOOL simulation of $4 \cdot 10^{12}$ muons[1] penetrating the last cooling cell. This can lead to temperatures over 100 K as shown in Fig. 4. According to Eq. (2), this corresponds several hundreds of bar inside the absorber.

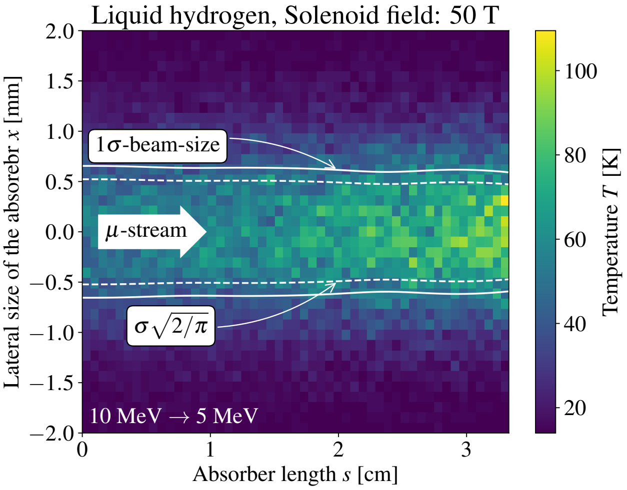

Figure 4: Example of the temperature distribution in the last cooling cell simulated with ICOOL. The beam size is indicated.

WINDOWS

Both liquid and gaseous H_2 have to be confined by a solid material. The area of the confinement where muons enter into or leave the absorber is known as a beam window. A window must have a small thickness and a low atomic number in order to mitigate Coulomb scattering. Some good candidates for these windows are Be, Si_3N_4 and Al.

Window Thermodynamics

Similar to the previous example with LH, the beam also deposits its energy into the window. The muon beam reaches the target parameters after passing the absorber of the last cell in the final cooling section. Every 0.2 s a new bunch passes through the cooling channel. The heat transportation per time can be expressed as $\dot{Q} = -\lambda\, A\, \nabla T$, which depends on the temperature gradient ∇T of the material; the solid's thermal conductivity λ; and the surface A of the beam window. The window is assumed to be cylindrical with radius R, a width d and the heat gets only transported through those surfaces which are not the absorber and the vacuum facing areas. For these conditions the duration for the cool down of the window to an equilibrium temperature T_{out} is calculated from

$$\frac{\partial T}{\partial r} = -\frac{1}{2\pi\lambda r d}\frac{\partial Q}{\partial t}. \tag{3}$$

For example, in the case where a beam passes through a 200 µm thick Beryllium (Be) window, the final beam has an emittance of 25 µm and its energy is 5 MeV. Inside a 50 T solenoid field the diameter of the muon beam is about

---

[1] Target muon number per bunch in the parameter list of the IMCC study [1]

1 mm. For these conditions the temperature increase would be approximately 175 K for 4×10^{12} muons per bunch. The hot beam trace inside the window cools down exponentially for various window diameters as shown in Fig. 5, when the window is cooled in a fixed temperature T_{out} at R. It can be concluded that 50 ms is sufficient time for the window to cool to its original temperature.

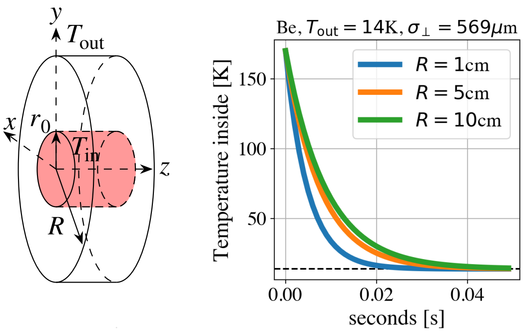

Figure 5: The left side visualize a cylindrical beam window with its geometrical parameters used in Eq. (3). The right plot shows the cooling rate for various window radii.

Window stress

For a given absorber pressure the minimum window thickness d [m] can be estimated by

$$d = \sqrt{\frac{3}{8} p (1 + \nu) \frac{R^2}{\sigma_{\text{max}}}}, \tag{4}$$

where the Poisson ratio ν and the maximum yield strength σ_{max} are properties of the material [7]; the variable p is the gas pressure acting on the window; and R is the radius of the beam window. Equation (4) shows that the minimum thickness of a round beam window scales with its radius and the pressure acting on it.

GASEOUS HYDROGEN ABSORBERS

As mentioned above, the evaporation of LH creates large pressure rises and can cause the window to burst. The thin windows are designed to prevent an increase of $\epsilon_{\perp,\text{N}}$, but they can withstand less pressure as shown in Eq. (4). An alternative to LH would be Gaseous Hydrogen (GH), since, as shown in Eq. (5), the pressure increase

$$\Delta p = \rho g_{\text{gas}} \frac{0.27 \times N \beta\gamma e}{2\, c_m \beta_\perp \epsilon_{\perp,\text{N}}} \left\langle \frac{\partial E}{\partial s} \right\rangle, \tag{5}$$

can be controlled with the density ρ of GH. In Eq. (5) c_m is the specific heat capacity; the number of muons N; and the electrical charge e. The gauge factor g_{gas} consists of the gas density at $T_0 = 273$ K and atmospheric pressure p_{atm}, which is for GH $g_{\text{gas}} = p_{\text{atm}}/(\rho_{\text{GH}} T_0) \approx 0.041\,\text{bar}\,\text{m}^3\,\text{K}^{-1}\,\text{kg}^{-1}$. Low GH densities would reduce pressure rises. The cooling of the emittance depends on the deposited energy in the gas, which leads for low gas densities to an extension of the absorber length. Subsequently, the absorber size would be limited to the length of the solenoid. One possibility would be to divide the absorber into distinct chambers separated by thin windows, each filled with GH of different ρ. Figure. 6 shows the case with five different densities in the absorber, where the beam reaches the target emittance. The pressure was kept in the range of 10.3 bar, which corresponds to the yield strength of a 100 μm thick Be window.

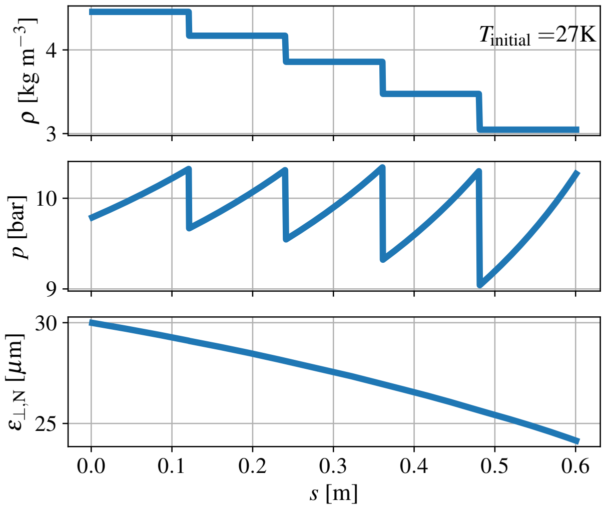

Figure 6: An absorber with chambers filled with different GH densities (top plot) prevents a destructive rise in pressure (middle plot) after the beam passage. The lowering down of the GH densities causes a variation of the gas pressure, when the beam goes through the absorber. The target emittance is still achieved in a reasonable small absorber length (bottom plot). The beam windows are not included in this estimation.

CONCLUSION

If the emittance of a muon beam is cooled to very low values, LH is no longer liquid, which could lead to a pressure rise sufficient to burst beam windows. The high pressures can be suppressed with matched densities of GH chambers in the absorber, while the absorber length remains short enough to be contained within a high field solenoid. The external cooling on the radial surfaces of the thin windows is sufficient to avoid any overheating caused by dense muon pulses.

ACKNOWLEDGEMENT

Funded by the European Union (EU). Views and opinions expressed are however those of the authors only and do not necessarily reflect those of the EU or European Research Executive Agency (REA). Neither the EU nor the REA can be held responsible for them.

REFERENCES

[1] C. Accettura et al., "Towards a Muon Collider", March 2023, `arXiv:2303.08533v1`

[2] S. Hahn et al., "45.5-tesla direct-current magnetic field generated with a high-temperature superconducting magnet", *Nature 570, 496–499*, 2019, `doi:10.1038/s41586-019-1293-1`

[3] D. Neuffer, "Muon cooling and applications", *Proc. Workshop on beam cooling and related topics*, Montreux, 1993, `doi:10.5170/CERN-1994-003.49`

[4] R. Fernow, "COOL: a simulation code for ionization cooling of muon beams", *Proc. of the 1999 Particle Accelerator Conference*, New York, NY, USA, 1999, pp. 3020-3022 vol.5, `doi:10.1109/PAC.1999.792132`

[5] H. Bethe, "Zur Theorie des Durchgangs schneller korpuskularstrahlen durch Materie", *Annalen der Physik, vol. 397*, 1930.

[6] F. Bloch, "Zur Bremsung rasch bewegter Teilchen beim Durchgang durch Materie", *Annalen der Physik, vol. 408*, 1933.

[7] C. Garion, "Discussion on beam window stresses", *Private communication*, CERN, 2022.