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MEASUREMENT AND MODELLING OF DECAPOLE ERRORS IN THE LHC FROM BEAM-BASED STUDIES

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Abstract

Studies of third-order chromaticity in the LHC during its initial two runs have consistently demonstrated a substantial discrepancy between the expected Q''' at injection and that observed in beam-based measurements. In 2022 during Run 3, for the first time, studies of Q''' have been complemented by measurements of chromatic detuning, being the momentum-dependent amplitude detuning, and the decapole resonance driving term f_{1004}. In this paper, these beam-based measurements are presented and compared to the magnetic model, and the implications to the source of the previously identified Q''' discrepancy are discussed.

INTRODUCTION

The decapole fields in the LHC have been studied since Run 1 [1–3] via chromaticity measurements. The third order chromaticity, Q''', generated for the most part by decapoles, has shown a consistent discrepancy to the magnetic model [4]. Corrections for b_5 errors, based on the magnetic model, were applied during operation by decapole spool pieces, located next to every second main dipole. Those corrections are observed to over-correct, which led to a homogeneous reduction of the corrector strength [5] being computed for beam-based corrections during Run 3 commissioning. Figure 1 and Table 1 show a comparison of the chromaticity with these two correction schemes for Q''', with higher-order terms also visible [5]. The measured and simulated shift in Q''' is also present.

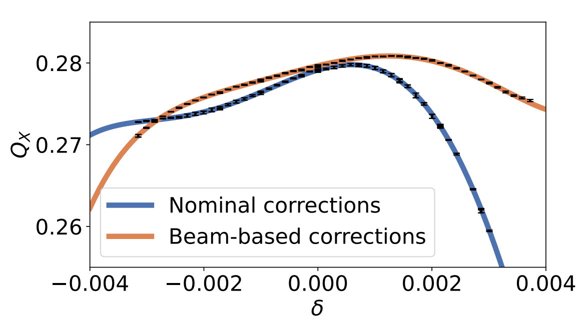

Figure 1: Chromaticity of the horizontal plane of beam 1 during Run 3's commissioning, with nominal corrections based on the magnetic model and beam-based corrections for Q'''.

During Run 3 commissioning and a dedicated machine development slot, measurements were conducted in order to better understand the decapole fields. The figures of merit of b_5, namely the third order chromaticity, the chromatic amplitude detuning and the f_{1004} resonance driving term (RDT) have been measured at injection energy. The chromaticity and chromatic amplitude detuning measurements were performed without any octupole and decapole correctors, to measure the bare machine and remove any possible corrector field error.

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Table 1: Third order chromaticity obtained during Run 3 commissioning, with nominal and beam-based corrections. The change in Q''', measured and expected via simulations, is also shown.

	Q''' [10^6]		$\Delta Q'''$ [10^6]	
B1	Nominal	Beam-based	Measured	Simulated
X	-3.36 ± 0.04	-1.02 ± 0.03	2.3 ± 0.1	2.5
Y	1.62 ± 0.05	0.12 ± 0.02	-1.5 ± 0.1	-1.4
B2				
X	-2.72 ± 0.08	-0.64 ± 0.03	2.1 ± 0.1	2.5
Y	1.54 ± 0.06	0.14 ± 0.03	-1.4 ± 0.1	-1.4

BARE CHROMATICITY

Previous studies [3] had demonstrated that octupole and decapole correctors were contributing to the b_4 discrepancy in the machine via hysteresis and feed-down. To evaluate the possible effect of decapole correctors on the third order chromaticity, a measurement was taken with these elements turned off.

The chromaticity measurement was performed using the standard method of varying the RF-frequency to induce a change of momentum-offset δ while measuring the tunes. The fit of the chromaticity function, given in Eq. 1 can be seen in Fig. 2 up to the third order.

$$Q(\delta) = Q_0 + Q'\delta + \frac{1}{2!}Q''\delta^2 + \frac{1}{3!}Q'''\delta^3 + \mathcal{O}(\delta^4). \quad (1)$$

A simulation has been run with MAD-X and the Polymorphic Tracking Code (PTC) with field errors from b_3 to b_8. The resulting values are shown in Table 2 along with the measured bare chromaticity and their ratio. It can be highlighted that a consistent discrepancy is observed between the model and measurements. The measured change in Q''', seen in Table1, also matches the model expectations, excluding the decapole correctors as the source of decapole discrepancy.

CHROMATIC AMPLITUDE DETUNING

The chromatic amplitude detuning is the tune shift dependant on both the action and the momentum offset, whose

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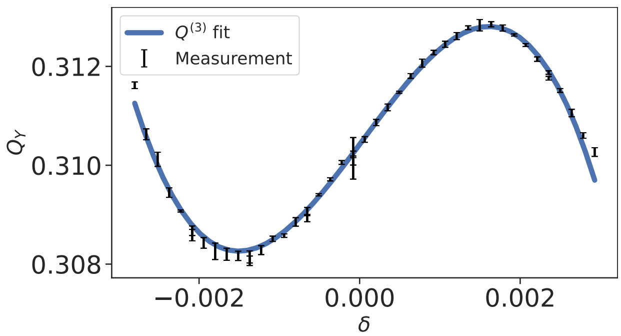

Figure 2: Vertical chromaticity of beam 1, measured during Run 3 with octupole and decapole correctors turned off. The fit is up to the third order.

Table 2: Measured and simulated third order chromaticity with octupole and decapole correctors turned off. The simulations include field errors from b_3 to b_8.

Plane	Meas. $Q'''[10^6]$	Sim. $Q'''[10^6]$	Ratio
Beam 1			
X	2.95 ± 0.04	6.94 ± 0.02	0.43 ± 0.01
Y	-1.82 ± 0.04	-4.29 ± 0.01	0.42 ± 0.01
Beam 2			
X	3.06 ± 0.07	7.03 ± 0.02	0.44 ± 0.01
Y	-1.72 ± 0.02	-4.27 ± 0.01	0.42 ± 0.01

decapole contributed terms are described via a Taylor expansion in Eq. 2. The last term is more commonly referred to as the third order chromaticity, Q'''.

$$\Delta Q(J_x, J_y, \delta) = \frac{\partial^2 Q}{\partial J_x \partial \delta} J_x \delta + \frac{\partial^2 Q}{\partial J_y \partial \delta} J_y \delta + \frac{1}{6}\frac{\partial^3 Q}{\partial \delta^3}\delta^3. \tag{2}$$

Each of those terms depend on the β-functions, the dispersion η and the normalized decapole field gradient K_5 [6], for a single source of length L:

$$\begin{aligned} \frac{\partial^2 Q_x}{\partial J_x \partial \delta} &= \frac{1}{16\pi} K_5 L \beta_x^2 \eta, & \frac{\partial^2 Q_x}{\partial J_y \partial \delta} &= -\frac{1}{8\pi} K_5 L \beta_x \beta_y \eta, \\ \frac{\partial^3 Q_x}{\partial \delta^3} &= \frac{1}{4\pi} K_5 L \beta_x \eta^3, & \frac{\partial^2 Q_y}{\partial J_x \partial \delta} &= -\frac{1}{8\pi} K_5 L \beta_x \beta_y \eta, \\ \frac{\partial^2 Q_y}{\partial J_y \partial \delta} &= \frac{1}{16\pi} K_5 L \beta_y^2 \eta, & \frac{\partial^3 Q_y}{\partial \delta^3} &= -\frac{1}{4\pi} K_5 L \beta_y \eta^3. \end{aligned} \tag{3}$$

The action dependant terms can be measured by exciting the beam with an AC-dipole [7] with increasing strengths at different momentum-offsets.

Such a measurement was taken with octupole and decapole correctors turned off to measure the bare machine. Some data could not be collected due to machine availability issues, restricting the measurement to low intensity kicks. Nevertheless, the terms $\frac{\partial^2 Q_x}{\partial J_y \partial \delta}$ and $\frac{\partial^2 Q_y}{\partial J_y \partial \delta}$ for beam 2 were measured for the first time in the LHC.

Figure 3 shows a fit of $\frac{\partial^2 Q_y}{\partial J_y \partial \delta}$ to measured Q_y vs J_y at two different momentum offsets. Expected shifts from MADX-PTC simulation, including field errors ranging from b_3 to b_8

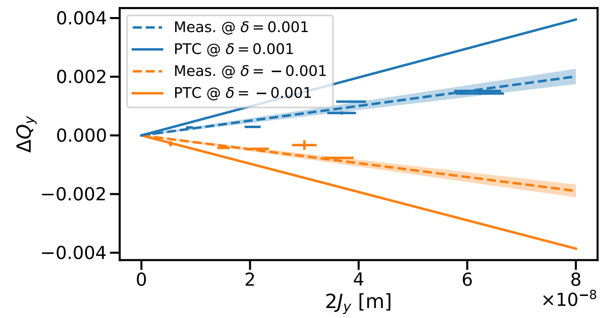

Figure 3: Measured and simulated tune shift due to a change of action at two different momentum-offsets. Fitted values correspond to chromatic amplitude detuning term $\frac{\partial^2 Q_y}{\partial J_y \partial \delta}$.

and a_4 to a_8 with unpowered correctors, are also shown. A consistent difference between simulation and measurement is observed. The values and ratios of measurement to model can be found in Table 3.

Table 3: Comparison of the measured and simulated terms $\frac{\partial^2 Q_x}{\partial J_y \partial \delta}$ and $\frac{\partial^2 Q_y}{\partial J_y \partial \delta}$. Simulations include errors from b_3 to b_8 an a_4 to a_8.

	$\frac{\partial^2 Q_x}{\partial J_y \partial \delta}\,[10^4\mathrm{m}^{-1}]$	$\frac{\partial^2 Q_y}{\partial J_y \partial \delta}\,[10^4\mathrm{m}^{-1}]$
$\delta = +0.001$		
Meas.	-1.16 ± 0.08	1.26 ± 0.15
PTC	-3.82 ± 0.01	2.47 ± 0.01
Ratio	0.30 ± 0.02	0.51 ± 0.06
$\delta = -0.001$		
Meas.	1.47 ± 0.12	-1.18 ± 0.13
PTC	3.92 ± 0.01	-2.41 ± 0.01
Ratio	0.38 ± 0.03	0.49 ± 0.05

The observed ratios of measurement to model, for the chromatic amplitude detuning, show small discrepancies to the bare chromaticity ones. Those could be attributed to the low intensity kicks not allowing a better fit. Nevertheless, the similarity of the ratios points to an issue with the b_5 error model of the main dipoles with measurements showing values about half that of the magnetic model.

RESONANCE DRIVING TERM f_{1004}

Normal Form techniques are routinely used to better understand the non-linear motion in accelerators [8–11]. Each multipole of order n will generate several Resonance Driving Terms (RDT) f_{jklm} with $j + k + l + m = n$ [12]. Each RDT will drive a resonance related to the tune:

$$(j-k)Q_x + (l-m)Q_y = p \quad ; \quad p \in \mathbb{N}. \tag{4}$$

RDTs can be measured via a frequency analysis of the turn by turn data, as they contribute to resonance lines that can be seen in one or both of the horizontal and vertical planes [9, 12]:

$$\begin{aligned} H_{f_{jklm}} \quad &\text{at} \quad (1-j+k)Q_x + (m-l)Q_y \quad ; \quad j \neq 0, \\ V_{f_{jklm}} \quad &\text{at} \quad (k-j)Q_x + (1-l+m)Q_y \quad ; \quad l \neq 0. \end{aligned} \tag{5}$$

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Their amplitudes are given by:

$$|H_{f_{jklm}}| = 2j(2I_x)^{\frac{j+k-1}{2}}(2I_y)^{\frac{l+m}{2}}|f_{jklm}|,$$
$$|V_{f_{jklm}}| = 2l(2I_x)^{\frac{j+k}{2}}(2I_y)^{\frac{l+m-1}{2}}|f_{jklm}|. \tag{6}$$

Of interest to LHC operation, is the RDT f_{1004}, driving the resonance $1Q_x - 4Q_y$. It can be seen as a line in the horizontal frequency spectrum at $-4Qy$ with an amplitude dependence on J_y^2. Figure 4 shows a frequency map [13] of a simulation including b_5 field errors, where its impact on the beam is easily noticeable.

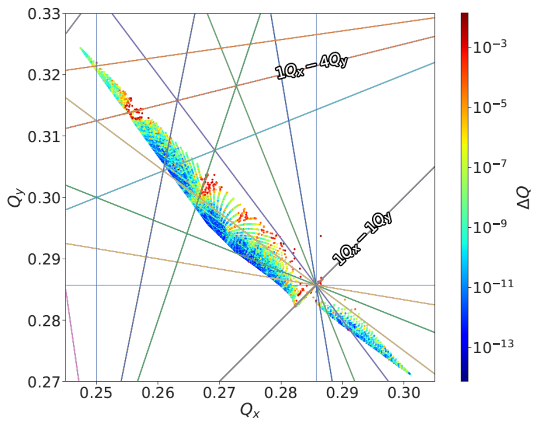

Figure 4: Frequency map at injection energy with b_5 field errors and nominal setting for landau octupoles. The highlighted resonance $(1, -4)$ excited by decapoles shows a degradation over 20,000 turns, with Q change between the start and the end of the simulation indicated in colour.

Forced f_{1004} RDT measurements, using driven oscillations from an AC-dipole [12], were taken for the first time at injection in the LHC. Measurements were performed before and after beam-based corrections for Q''', shown in Fig.1 and Table1. While reducing Q''', the beam-based corrections were observed to actually increase the RDT, as can be seen in the frequency spectrum in Fig. 5 and the amplitude of the RDT in Fig. 6.

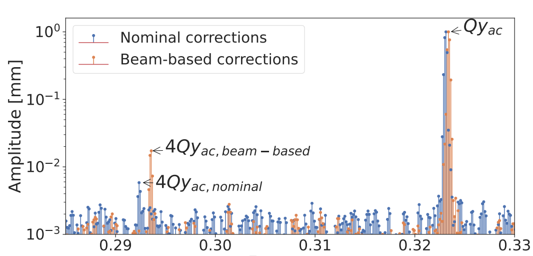

Figure 5: Horizontal frequency spectrum of turn-by-turn data, with nominal and beam-based corrections for the third order chromaticity Q'''. The $(-1, 4)$ resonance shows a different amplitude for each correction. The difference in frequency for both $4Q_y$ lines is explained by the slight AC-dipole tune difference between the measurements.

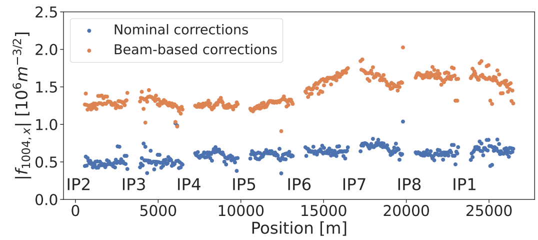

Figure 6: Amplitude of the RDT f_{1004} generated by normal decapoles, measured before and after having applied beam-based corrections on the third order chromaticity Q'''.

Simulations have been run with MADX-PTC to assess the response of the RDT to the K_5 strength applied on the correctors. Figure 7 shows the change to the real part of the RDT due to the application of the beam-based corrections for both the measurement and the simulation. Similar level of agreement is seen for the imaginary part and the rest of the ring. As the response of the model for this RDT agrees well with the measurement, corrections can be computed. Further studies are undertaken for the correction of this RDT while also keeping Q''' low.

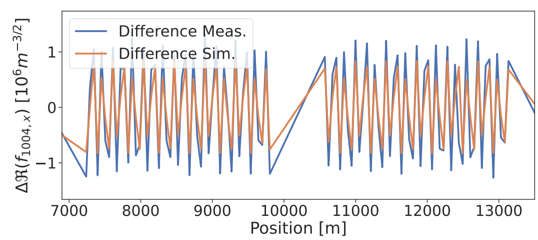

Figure 7: Change in real part of the RDT f_{1004}, due to application of beam-based corrections on top of the nominal corrections for Q''', for both measurement and MADX-PTC simulation.

CONCLUSIONS AND OUTLOOK

For the first time in the LHC, measurements of chromatic amplitude detuning and decapolar resonance driving terms have been performed. These open new avenues to understand the longstanding discrepancy between b_5 sources from magnetic and beam-based measurements. Decapole correctors are confirmed to respond as expected and not to be a source of b_5 errors. Similar discrepancies to the magnetic model are observed via both chromaticity and chromatic amplitude detuning, pointing to an error localized in the main dipoles. The deterioration of the f_{1004} RDT indicates the simple uniform correction over all arcs employed for Q''' did not achieve a local compensation of the decapole sources. This may point to some arc-by-arc b_5 variations which should be further studied.

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