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# IMPACT OF CRAB CAVITY RF NOISE ON THE TRANSVERSE BEAM PROFILES IN THE HL-LHC*

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*Abstract*

Crab Cavities (CCs) are a key component for the HL-LHC luminosity upgrade. To significantly reduce the Long-Range Beam-Beam (LRBB) effects a large crossing angle scheme is needed ($\frac{\theta_c}{2} = 250\,\mu$rad). The installation of 4 CCs per beam in each of the two main interaction points aims to restore the luminosity loss caused by the crossing angle. Noise injected through the Low-Level RF (LLRF) system in these cavities is known to be affecting the growth of the transverse bunch emittance. In this paper a new numerical study has been developed thanks to the new tracking tool Xsuite to study in depth this detrimental effect of both phase and amplitude LLRF noise. Both Long Range and Head On Beam Beam effects are included in the simulation together with the CC noise to evaluate the effects of the interplay between these strong non-linearities and the external noise. Furthermore, transverse bunch measurements show that the transverse distribution can be modeled as an heavy tailed q-Gaussian. To take this into account a particular focus is given to the linear matching and subsequent tracking of a multivariate q-Gaussian distribution in the lattice. The Emittance Growth Rate induced on both a Gaussian and a q-Gaussian bunch is computed. This study could serve as a basis to evaluate the cross-talk between the two beams introduced by their head-on interaction in this heavy tailed scenario.

## INTRODUCTION

The LHC [1] will undergo an upgrade in 2029 that aims to reach an integrated luminosity of 3000 1/fb, one order of magnitude higher than the initial design goal. This luminosity upgrade (HL-LHC, [2]) will be possible thanks in part to a doubling of the intensity per bunch ($2.2 \times 10^{11}$ protons) and a reduction of $\beta^*$ to 15 cm. This particular configuration would cause critical LRBB [3] effects that can be reduced thanks to a large crossing angle scheme ($\frac{\theta_c}{2} = 250\,\mu$rad). The installation of 4 CCs per beam in each of the two main interaction points was proposed to restore this geometrical luminosity loss. A schematic plot of the compensation achieved with CCs is visible in Fig. 1.

The E.M. field in the Crab Cavity acts as a dipolar kick that depends on the longitudinal position. Due to this the longitudinal motion is inevitably coupled with the transverse one, therefore noise injected through the LLRF system corresponds to an Emittance Growth Rate (E.G.R.) in the transverse planes, [4]. In this study, the impact of the CC noise on the transverse beam profiles is investigated using a series of numerical simulations. The lattice under study

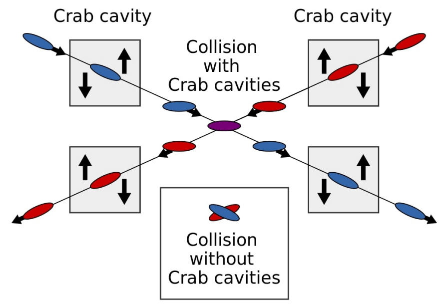


Figure 1: Crab crossing scheme layout.

is strongly non-linear due to the presence of sextupoles, octupoles and both LRBB and Head-On Beam-Beam (HOBB), hereby approximated in the weak-strong regime [5]. The weak-strong assumption is verified a posteriori evaluating the effects on the weak beam assuming that a similar effect is affecting the strong beam by symmetry: if such effect is not significant in a weak-strong approximation, then the method is valid.

## COMPUTATIONAL SETUP

All the simulations in this study have been performed using the newly developed tracking tool Xsuite [6]. Xsuite is a collection of Python packages for the simulation of the beam dynamics in particle accelerators supporting both CPU and GPU computing platforms. The choice of this type of tracking is useful to have a better picture of the incoherent effects that could arise due to the introduced lattice imperfections.

### *Lattice Specifications*

The main parametric aspects of the lattice under study are reported in Table 1. The optics is the top energy HLLHCV1.5 [7].

The noise injected in each cavity from the LLRF can impact the amplitude and the phase of the CC voltage. It is modeled as white noise, i.e. with a uniform frequency spectrum produced from a Gaussian distribution with $\mu = 0$ and $\sigma = \phi_{\text{noise,i}}$ or $\sigma = A_{\text{noise,i}}$, for the phase and amplitude noise, respectively. To define the total noise level of a single simulation we consider the noise of different CC as independent, therefore we sum in power the contributions coming from each cavity:

$$\phi_{\text{noise,tot}} = \sqrt{\sum_{i=1}^{8} \phi^2_{\text{noise,i}}} = \sqrt{8}\,\phi_{\text{noise}}, \quad (1)$$

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Table 1: Lattice Specifications

| **Parameter** | **Value** |
|---|---|
| $(\epsilon_{n,x}, \epsilon_{n,y})$ | (2.5 µm, 2.5 µm) |
| $\beta^*$ | 15 cm |
| $\sigma_z$ | 7.5 cm |
| $(\xi_x, \xi_y)$ | (15, 15) |
| Current in the arc octupoles | +100 A |
| $(Q_x, Q_y)$ | (62.315, 62.32) |
| $\frac{\theta_c}{2}$ | 250 µrad |
| Number of Crab Cavities | 4 per IP per beam |
| $V_{CC}$ | 3.4 MV |
| $\theta_{CC}$ | −190 µrad |
| LRBB per IP side | (25, 20, 25, 20) |
| Luminosity in IP8 (levelled) | $2.0 \times 10^{33}$ Hz/cm$^2$ |

$$A_{\text{noise,tot}} = \sqrt{\sum_{i=1}^{8} A^2_{\text{noise,i}}} = \sqrt{8}\, A_{\text{noise}}. \tag{2}$$

These two types of noise have different effects on the bunch:

- Phase noise shifts the bunch centroid (dipole motion);
- Amplitude noise kicks the head and the tail of the bunch in opposite directions, resulting in intra-bunch oscillations.

### *q-Gaussian and Matching*

It has been observed in Ref. [8] that the tails in the LHC bunches differ from those of a normal distribution. In particular, the transverse distribution appears to be heavy tailed and is modeled as a q-Gaussian [9]. q-Gaussian arises naturally being a stationary solution of a Fokker-Plank that can describe different statistical behaviors in dynamical systems, ranging from sub to super-diffusion processes [10].

The q-Gaussian distribution is a generalization of the Gaussian distribution and its PDF is defined as follows:

$$f(x) = \frac{\sqrt{\beta}}{C_q} e_q(-\beta x^2), \tag{3}$$

where:

$$e_q(x) = [1 + (1-q)x]_+^{\frac{1}{1-q}}, \tag{4}$$

with $C_q$ a normalization factor. The parameter q describes the weight of the tails. For a $q < 1$ the tails are lighter with respect to a Gaussian distribution, whereas the heavy tail domain is given by a $1 < q < 3$. The Gaussian distribution is recovered when $q \to 1$.

The strong non-linearities present in the lattice, together with the E.G.R. induced by noise, require these heavy-tailed distributions to be tracked to understand the diffusive behavior in the chaotic regions of the phase space and the detrimental effects on luminosity coming from the increased beam size [11].

The initial distribution considered for the tracking is a linear matched distribution i.e. independent on the azimuthal variable in the normalized phase space. A special case for this problem is given by the Gaussian distribution, where its factorizability in the phase space variables grants automatically matched beams in all the N-dimensions when extracting N independent Gaussian variables.

This is not true in general, and in particular it is not true for the q-Gaussian distribution: generating 4 q-Gaussian independent distributions (x, $p_x$, y, $p_y$) will inevitably lead to cross correlations between the different planes.

To solve this problem we begin by defining a multivariate 4D q-Gaussian [12], eliminating cross correlations between the 1D distributions by imposing the following: all 4 variables must be distributed with the same q-Gaussian PDF, an assumption well supported by the measurements where both x and y profiles follow a q-Gaussian with q≈1.1 .

To generate this N-variate q-Gaussian random vector a possible sampling algorithm is described in Ref. [13]. This method can be related to the following idea [14]: given a certain 1D radial distribution, the q-Gaussian in our case, one can define four angles of rotation and rotate it to obtain four independent distributions, each describing the required 1D profile. The role of the radial distribution in the algorithm is taken by the $\chi^2$ where the information about the q is contained.

The 2D projections shown in Fig.(2) show that this way of implementing the q-Gaussian in the tracking gives matched beams in (x, $p_x$),(y, $p_y$) planes while maintaining a physical circular distribution in the (x, y) plane.

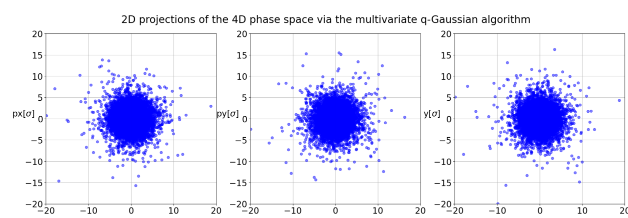


Figure 2: 2D projections of the 4D phase space for a q-Gaussian with q = 1.4. All the distributions are obtained via the multivariate q-Gaussian algorithm. Both x and y planes are matched, and no cross correlation appears in the x-y plane.

The crucial thing is, independently from the chosen algorithm, that for a non-Gaussian distribution the generation must always be performed with a top down approach: first one generates the multidimensional distribution to be tracked and then one can retrieve the 1D profiles.

## EMITTANCE BLOW-UP

E.G.R. due to LLRF noise is the first important effect that has been both theoretically predicted and measured in Ref. [15] and is therefore important to understand how this new way of tracking heavy tailed distributions let us make predictions on the real behavior of the bunches. However in both cases no BB effects have been included and a significant

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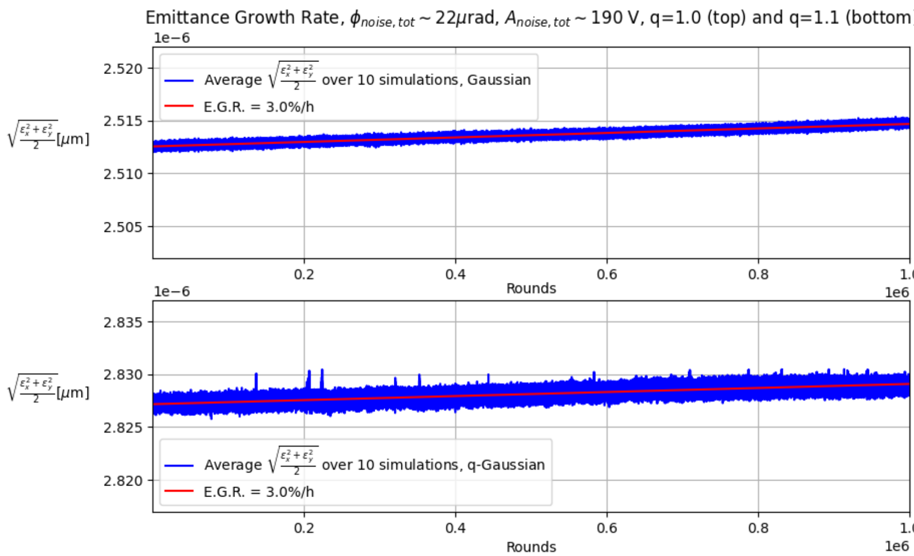


Figure 3: Emittance Growth Rate due to LLRF noise for a Gaussian and a q-Gaussian distribution. Each plot is obtained by averaging over multiple simulations (10). The simulation corresponds to ≈90 seconds of LHC.

contribution from the machine non-linearities is expected in HL-LHC.

The definition of emittance for a given transverse distribution $(\hat{x}, \hat{p}_x, \hat{y}, \hat{p}_y)$ in the physical space is:

$$\epsilon_{RMS,x} = \sqrt{\langle \hat{x}^2 \rangle \langle \hat{p}_x^2 \rangle - \langle \hat{x} \cdot \hat{p}_x \rangle^2}. \quad (5)$$

This definition is statistical, therefore it is subject to fluctuations due to the limited amount of particles that can be tracked within a reasonable amount of time. To reduce the statistical fluctuations of the observable under study the procedure adopted is to calculate a combination of both the x and y emittance:

$$\sqrt{\frac{\epsilon_{RMS,x}^2 + \epsilon_{RMS,y}^2}{2}}, \quad (6)$$

and then average over several different simulations.

The results of the tracking for both a Gaussian and a q-Gaussian with q = 1.1 are reported in Fig.(3), with a combination of both $\phi_{\text{noise,tot}} \approx 22$ μrad (≈ 0.05 ps) and $A_{\text{noise,tot}} \approx 190$ V and an expected E.G.R. of ≈ 3%/h, to be compared with the currently measured E.G.R. of ≈ 3%/h in LHC [16]. The study has also been performed for different levels of noise, including the case of no noise, with a comparable level of agreement between the two methods. The fact that both the simulations resulted in a very similar behavior is both a cross-check that the q-Gaussian tracking is working as expected but also that, behind a similar E.G.R., a very different beam profile could be present. The emittance is indeed dominated by the core of the bunch, so it carries partial to no information about the large-amplitude particles that, in the case of a q-Gaussian, represent a reasonable fraction of the whole distribution. This concept is at the core of the studies that are currently being performed to understand the problem of diffusion.

## CONCLUSIONS AND OUTLOOK

Crab Cavities are one of the greatest challenges of the HL-LHC upgrade. The study of the impact of the LLRF noise on the transverse beam profiles via simulations is therefore essential. In this study a strongly non-linear lattice with both LRBB and HOBB lenses with parameters close to the HL-LHC design optics has been used to perform multiparticle tracking and understand the effect of the noise on the transverse distributions. The beam profile data retrieved in the past has shown enhanced tails, which was never taken into account in these type of simulations. The problem of matching related to non Gaussian distributions has been solved in the case of q-Gaussian. Thanks to this it is now possible to perform new numerical studies with more realistic bunch shapes.

In this context the problem of E.G.R. due to LLRF noise in the Crab Cavities has been studied. This was the first time that a multivariate heavy tailed distribution was tracked in a strongly non linear lattice and the results show that, concerning E.G.R., no particular differences from the Gaussian case are found. This is both an indication that the model is working as intended but also that further studies on the beam profiles are needed to have a deeper comprehension of the bunch evolution under both an external source of noise and non-linearities when the fraction of high amplitude particles is increased. This study will serve as a basis for two different directions of research: the study of the cross-talk between the two beams introduced by their head-on interaction and the diffusion caused by it. The introduction of the cross-talk should first of all result in an increase by a factor ≈ 2 in the E.G.R. Thanks to the q-Gaussian it should be also possible in principle to directly measure the diffusion caused by this effect by analyzing the evolution of q with time.

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