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NUMERICAL SIMULATIONS OF TRANSVERSE NONLINEAR BEAM MANIPULATIONS AT THE CERN PS

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Abstract

A new set of non-linear beam manipulations have recently been proposed, with the goal of extending the transverse beam splitting that is routinely used at the CERN PS to deliver beam to the SPS for fixed-target physics. Using a simple Hamiltonian model, it has been shown how the transverse emittances can be shared by crossing a two-dimensional non-linear resonance. Moreover, an AC dipole has been shown to be used to split the beam transversely. In this paper, numerical simulations of these manipulations performed using a realistic model of the PS ring, including longitudinal motion, will be presented and discussed in detail.

INTRODUCTION

The phenomena introduced by non-linear resonances in beam dynamics can be used to manipulate the transverse beam distribution for different purposes. This is, for example, the case of the beam splitting performed at the CERN Proton Synchrotron (PS) to perform Multiturn Extraction (MTE) [1, 2]. In recent years, a number of 2D and 4D manipulations have been proposed, studying symplectic map models of transverse dynamics (and their interpolating Hamiltonians) and analysing their properties according to the theory of adiabatic separatrix crossing [3, 4]. The proposed methods range from new beam splitting methods using resonant islands [5] to adiabatic transport of an annular beam distribution [6] to the sharing of transverse emittances between the horizontal and vertical degrees of freedom by crossing different non-linear resonances [7–10].

In this paper, we make one step further towards the experimental confirmation of the feasibility of such manipulations: we present the results of detailed, realistic simulations of the PS ring dynamics to identify machine configurations to be used for future experimental validations of the predicted phenomena. We focus on two kinds of manipulations: 1 : 2 emittance sharing using a third-order resonance driven by a normal sextupole, first proposed in Ref. [9] and studied in detail in Ref. [7], for which we provide full 6D simulation of the mechanism, and beam splitting in two resonant islands generated by a modulated AC dipole, as proposed in Ref. [5], using a 4D simulation of the transverse dynamics.

GENERAL CONSIDERATIONS

The CERN PS was put into operation in 1959 and is now part of the LHC injector complex. It is a 628 m long accelerator, composed of 10 superperiods of 10 magnet units of 4.4 m, 8 short straight sections of 1.6 m, and 2 long straight sections of 3 m each. We work with a proton beam at injection energy ($E_{\mathrm{kin}} = 2\,\mathrm{GeV}$) to maximise the strength of the non-linear elements. Furthermore, at injection, the pole-face winding magnets can be made inactive, thus simplifying the machine configuration and its modelling. For our manipulations, we will generate and control non-linearities using normal sextupole and octupole magnets installed at straight section (SS) 39 at large β_x (≈ 22 m) and small β_y (≈ 11 m) and an oscillating dipole that is installed in SS97. The MAD-X definition files for this ring configuration are available in the CERN Optics repository [11]. Particle tracking is performed using the MAD-X polymorphic tracking code (PTC) [12], simulating a beam made of 1000 normally-distributed particles with normalised emittances $\varepsilon_x = \varepsilon_y = 0.5\,\mu\mathrm{m}$.

EMITTANCE SHARING

The theory of emittance sharing predicts that, if the accelerator tune is adiabatically modulated close to a third-order resonance (for example, $Q_x - 2Q_y \approx 0$), the 4D phase space can be reduced to a 2D one (as a linear combination of the actions is conserved), and a separatrix appears in the phase space, forcing particles to cross it. Considerations on the area of the two phase-space regions at the crossing time can be made to deduce that, if before modulation the transverse emittance are $\varepsilon_{x,\mathrm{i}}$ and $\varepsilon_{y,\mathrm{i}}$, after resonance crossing they are exchanged and multiplied by a factor linked to the resonance, namely $\varepsilon_{x,\mathrm{f}} = \varepsilon_{y,\mathrm{i}}/2$ and $\varepsilon_{y,\mathrm{f}} = 2\varepsilon_{x,\mathrm{i}}$.

For our simulations, we set $Q_x = 0.414$ and keep it fixed and modulate Q_y from the initial value of $Q_{y,\mathrm{i}} = 0.190$ to $Q_{y,\mathrm{f}} = 0.224$ in $N = 10^4$ turns. This is achieved by finding the initial and final values of the strengths k_f and k_d of the two families of tuning quadrupoles, corresponding to the two extreme tune values, and changing the strengths turn-by-turn in a linear fashion.

To drive the third-order resonance, we use normal sextupole magnets located in SS39, with total magnetic length $\ell = 0.4$ m, setting $K_2 = 1\,\mathrm{m}^{-3}$.

We also include longitudinal dynamics, turning on an RF cavity in SS91, with $V_{\mathrm{rf}} = 12$ kV and $\omega_{\mathrm{rf}} = 3.62$ MHz, and we generate particles with an initial energy spread $\Delta E/E$ normally distributed with $\sigma_E = 4\times10^{-4}$. We ran 6D simulations using these as default parameter values and varying a single parameter at the time. As we set $\varepsilon_{x,\mathrm{i}} = \varepsilon_{y,\mathrm{i}} = \varepsilon_\mathrm{i} = 0.5\,\mu\mathrm{m}$, we measure the final values of $\varepsilon_{z,\mathrm{f}}$ and $\varepsilon_{z,\mathrm{i}}$ with $z = x, y$ and these ratios with their initial value of 0.5 μm. Note that, for the resonance considered, from the scaling properties of the Hamiltonian model [7], the emittance exchange performance

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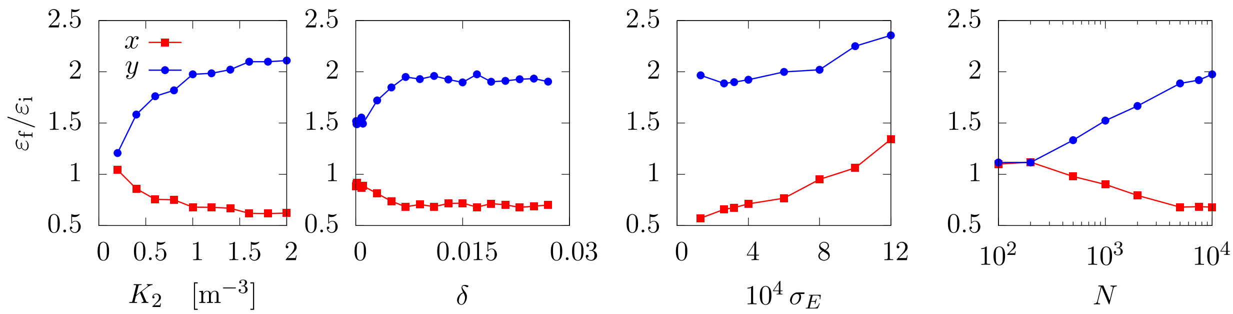

Figure 1: Ratio of the final to the initial normalised x and y emittances ($\varepsilon_z = \hat{z}^2 + \hat{p}_z^2 = [z^2 + (\alpha_z z + \beta_z p_z)^2]/\beta_z$) after crossing of the $Q_x - 2Q_y \approx 0$ resonance in the 6D simulations of the PS lattice, as a function of K_2 (first from left), frequency modulation width δ (second), σ_E (third) and the number of turns N (fourth). The default parameter values are $K_2 = 1\,\mathrm{m}^{-3}$, $\delta = 0.017$, $\sigma_E = 4 \times 10^{-4}$, $N = 10^4$.

should depend on the ratio $\kappa = \delta_{\max}/(K_2 \varepsilon_i^{1/2})$. Therefore, if we increase the initial emittance, we should reduce the sextupole strength or increase the frequency variation to keep κ constant.

Figure 1 shows $\varepsilon_{z,\mathrm{f}}/\varepsilon_{z,\mathrm{i}}, z = x, y$ as a function of K_2, the difference in tune 2δ ($Q_y \in [0.207 - \delta, 0.207 + \delta]$, the number of turns N, to evaluate the limits of adiabaticity of the process and the longitudinal emittance σ_E, to investigate the sensitivity of emittance sharing in the presence of longitudinal motion. As, for $K_2 = 1\,\mathrm{m}^{-3}$, the chromaticities are $\delta Q_x = -3.47$ and $\delta Q_y = -8.31$, this means that, for the range considered of σ_E, we work with tune variations up to 0.004 in Q_x and up to 0.01 in Q_y.

We observe that, as expected, the vertical emittance ratio increases toward the theoretical value of 2 while the horizontal one is reduced. However, unless the longitudinal emittance is small, reaching the expected final emittance value in the x plane is more difficult. In fact, as seen in the third plot of Fig. 1, both emittances increase with σ_E, but the effect on x is larger than the effect on y. The analysis of adiabaticity, shown in the fourth plot of Fig. 1, indicates that 10^4 turns ($\approx 2.1 \times 10^{-2}$ s) are sufficient to obtain the emittance sharing. Furthermore, we obtain a power law in the dependence of the final emittances values on N, as expected from the theoretical analysis [7].

BEAM SPLITTING

The proposal for beam splitting using a resonant condition generated by a transverse exciter relies on the fact that, due to the Poincaré-Birkhoff theorem [13], in presence of an oscillating perturbation whose frequency is close to a multiple m of the main frequency of the system (in this case, the accelerator tune), m stable islands will appear in the phase space. Modulating the perturbation frequency, one can control the size of the islands and perform adiabatic trapping of particles inside them, eventually splitting the beam between the core region and the m islands [14]. The effect of a modulated AC dipole such as the one available in the PS in SS97 can be implemented in the MAD-X simulation employing a horizontal dipole generating a sinusoidal kick whose amplitude a_n and frequency $Q_{\mathrm{AC},n}$ are varied turn by turn so that at turn n, the kick is given by $(\delta p/p_0)_n = a_n \cos(2\pi Q_{\mathrm{AC},n} n)$. Furthermore, we need sextupole magnets to generate and control the amplitude detuning: for this, we rely again upon the non-linear magnets in SS39. We choose the machine tunes as $Q_x = 0.1713$ and $Q_y = 0.618$. We perform 4D simulations of transverse motion of a beam of 1000 normally distributed particles following the following protocol, modulating the frequency f_n and the amplitude a_n of the AC dipole kick:

1. we ramp linearly a from 0 to its final value a_f in $N = 5000$ turns, keeping $Q_\mathrm{AC} = Q_{\mathrm{AC,i}} > mQ_x$ so that the islands cannot appear;

2. During the $2N$ turns, we keep $a = a_\mathrm{f}$ but we vary linearly Q_AC from $Q_{\mathrm{AC,i}}$ to $Q_{\mathrm{AC,f}}$. During this phase, islands are created and expanded, trapping particles;

3. Finally, we let the simulation run for 2048 turns with $a = a_\mathrm{f}$ and $Q_\mathrm{AC} = Q_{\mathrm{AC,f}}$ to evaluate the final tune of each particle using the algorithm of [15] to identify, due to the tune-locking phenomenon, whether the particles have or not been trapped in an island.

In Ref. [5], we analyse a third-order resonance to split the beam into four parts. However, we observed that the maximum kick available using the PS AC dipole (10 µrad) is insufficient to excite this resonance; nevertheless, to experimentally observe the beam splitting, the resonance order is not relevant, so we decided to use resonance 2 instead, which can be excited with a lower perturbation strength. For all our simulations, we set $Q_{\mathrm{AC,i}} = 0.351 = 2.05Q_y$. We then measured the trapped fraction of the beam as a function of the final exciter amplitude a_f, the final frequency $Q_{\mathrm{AC,f}}$ and the strength of the SS39 sextupole K_2. Unless otherwise stated, we set $a_\mathrm{f} = 10$ µrad (the maximum value), $Q_{\mathrm{AC,f}} = 0.345$ and $K_2 = 3\,\mathrm{m}^{-3}$. Figure 3 shows how the split beam would appear at the end of the manipulation in the horizontal phase space in two PS sections where the beam wire scanners are located and where the islands would be observed in an experiment (the particles in the islands are coloured). Note

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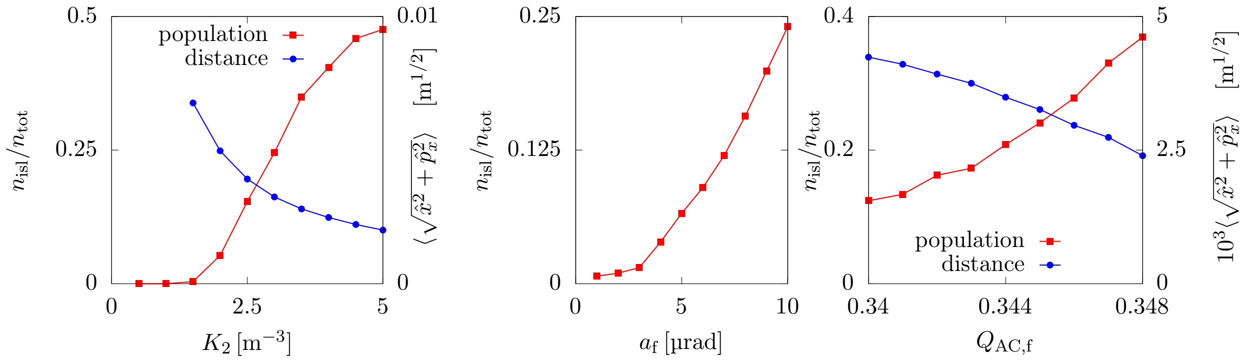

Figure 2: Trapping fraction (number of particles in islands $n_{\rm isl}$ divided by the total number of particles $n_{\rm tot}$ as a function of the sextupolar strength K_2 (left), the final amplitude of the AC dipole $a_{\rm f}$ (centre), and the final frequency $Q_{\rm AC,f}$ (right). For the left and right plots, the average distance (computed in the normalised phase space $(\hat{x}, \hat{p}_x)$) of the particles in the islands from the centre is also shown. The default values of the parameters are $K_2 = 3\,{\rm m}^{-3}$, $a_{\rm f} = 10\,\mu{\rm rad}$ and $Q_{\rm AC,f} = 0.345$.

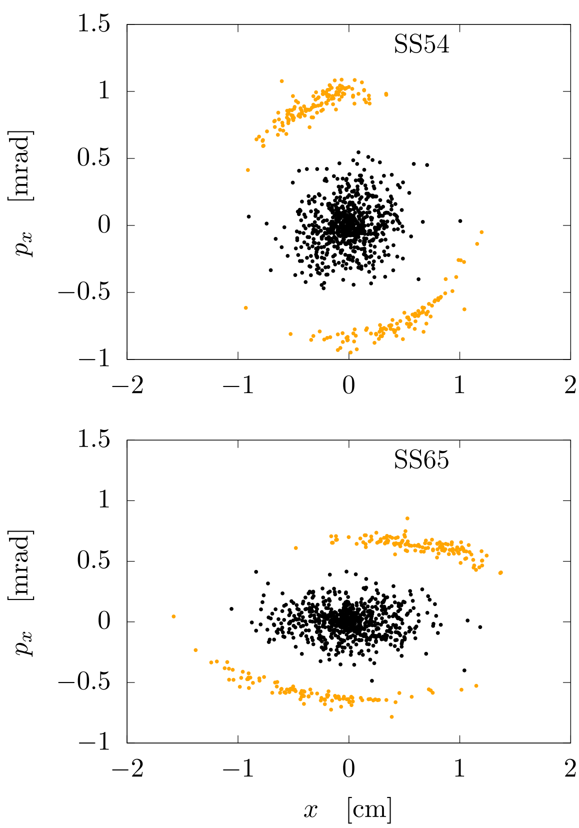

Figure 3: Final distribution in the physical (x, p_x) horizontal phase space at turn $n = 15000$, at the end of the beam splitting process at SS54 (top) and SS65 (bottom) where the PS horizontal beam wire scanners are located. The default parameters defined in the text have been used. The particles in the islands, identified by means of the final value of the horizontal tune, are coloured orange.

that the islands revolve around the centre, therefore, at each turn they would be found at different angles in the (x, p_x) plane.

In the left plot of Fig. 2, we show the fraction of trapped particles (red line) as a function of the strength of the sextupole in SS39, observing that increasing the detuning helps trapping, but the islands (blue line) stay closer to the centre. In the centre graph, we plot the trapped fraction as a function of the AC dipole kick amplitude, observing how it increases. In the right part of the figure, we plot the same quantity as a function of the final frequency $Q_{\rm AC,f}$. We see that if the frequency modulation is wider, we lose particles in the islands, trading that with more separation between the beamlets.

CONCLUSIONS

In this paper, we presented numerical simulations of the PS lattice in order to explore the possibility of experimental achievement of two non-linear transverse manipulations: the $(1, -2)$ emittance sharing crossing a third-order resonance and the beam splitting into the centre and two islands, thanks to a resonant transverse AC dipole. For the emittance sharing, 6D simulations of the particle dynamics confirm that this manipulation should be feasible in the Proton Synchrotron. The next step is an actual experimental test, but we also plan to analyse numerically on the PS simulations the emittance sharing using 4th order resonances following Ref. [7]. For what concerns beam splitting, the main limitation is given by the low strength of the PS AC dipole. However, stable resonant islands induced by the transverse exciter are still visible. Before the experiment, we will need to extend these simulations, improving statistics and taking into account longitudinal motion. We also need to further explore the parameter space to find an optimal trade-off between population of the islands and their separation from the centre of the beam.

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