ISBN: 978-3-95450-231-8 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2023-WEPL096

ANALYSIS OF A DOUBLE-RESONANCE CROSSING FOR BEAM SPLITTING

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Abstract

Beam splitting can be performed by adiabatic crossing of a given one-dimensional non-linear resonance. This process is routinely used at the CERN Proton Synchrotron (PS) to deliver the proton beam to the Super Proton Synchrotron (SPS) fixed-target physics. To improve the efficiency of the intensity sharing between the various beamlets, a dipole kicker can be used to excite the beam during the resonance crossing process. This entails a double-resonance crossing phenomenon that will be described and discussed in detail in this paper.

INTRODUCTION

Phenomena induced by non-linear effects open the possibility of manipulating the transverse beam distribution. For instance, a beam can be split transversally by means of adiabatic crossing of a stable non-linear resonance. This is the core of the CERN PS Multiturn Extraction (MTE) method [1], which has been successfully implemented [2] and is used in routine operation to deliver proton beams to the SPS [3, 4] . That technique allows the generation of multiple beamlets from the initial single-Gaussian beam, and has been studied within the framework of the adiabatic theory of separatrix crossing [5].

A recent study [6], based on the same theoretical basis [7], opened the way to another possibility of beam splitting: a modulated exciter with a frequency close to a multiple of the tune that creates resonant islands and, therefore, enables splitting and transport phenomena.

Experimental evidence gathered from MTE operation has shown that the presence of a modulated AC dipole improves the performance of the process, increasing the trapped fraction of the beam [4, 8–10]. This method, which combines two phenomena in a double-resonance, two-frequency system, has not yet been investigated from a theoretical point of view on the basis of the separatrix-crossing theory. In this paper, we consider some basic models of particle dynamics in double-resonance condition and investigate the generated phase-space structure to understand how the trapping process is enhanced. We then performed numerical simulations of two possible operational protocols that have been tested during MTE studies and commissioning activities to observe their advantages and limitations.

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THE MODEL

The starting point of our analyses is a 2D Hénon map to which a modulated exciter is added. This accounts for an accelerator in which a non-linear amplitude detuning and a transverse AC dipole are present. Introducing the Courant-Snyder normalised variables in (x, p), the map reads [6]:

$$\begin{pmatrix} x_{n+1} \\ p_{n+1} \end{pmatrix} = R(\omega_{0,n}) \begin{pmatrix} x_n \\ p_n + x_n^2 + \varepsilon \cos(\omega_n n + \psi_0) \end{pmatrix}, \qquad (1)$$

where, being $\omega_\mathrm{r} = 2\pi/q$ the resonant frequency, we set $\omega_{0,n} = \omega_\mathrm{r} + \delta_n$, $\omega_n = \omega_\mathrm{r} + \Delta_n$, therefore keeping the resonance condition $\omega_n \approx \omega_{0,n} \approx \omega_\mathrm{r}$, and ω_n is modulated as in MTE varying δ_n in some neighbourhood of zero. Therefore, two resonance conditions coexist: one is given by the main tune $\omega_{0,n}/2\pi$ of the map being close to the rational value $1/q$, and the other is between the exciter frequency and the tune.

If $\Delta_n = 0$, i.e. the exciter frequency is kept fixed to ω_r, it is possible to directly study the phase space of the qth iterate of the map (1) at frozen values of ω_n. Some phase-space portraits of this iterated map for $q = 4$, using different values of the exciter phase ψ_0 and the amplitude ε are shown in Fig. 1.

We observe that depending on ψ_0, the relative size of the islands is different: one island (or two) becomes larger than the others. Using cardinal points w.r.t. the origin to identify each island, if $\psi_0 = 0$, the west island is the largest. For $\psi_0 = \pi/4$ the effect is shared between the west and north islands, at $\psi_0 = \pi/2$ it is the northern island that becomes dominant. The effect is more evident if the amplitude of the AC dipole ε is large enough, as in this case it is possible to destroy the central region.

Trapping Properties

From the phase-space structure observed in Fig. 1, according to the separatrix crossing theory [11], we expect that the islands that grow faster will trap most of the particles, the dominant island being selected by the value of ψ_0.

The trapping process is simulated using the map (1). First, we set $\omega_0/(2\pi) = 0.249$, lower than the resonant value, when there are no islands, and then increase ε from zero to $\varepsilon = 1 \times 10^{-3}$ (in $N = 10^5$ turns). This is performed to avoid mismatching the initial distribution when the elliptic fixed point is shifted from the origin because of the exciter effect. Then we vary linearly $\omega_0/(2\pi)$ from 0.249 to 0.251 over N turns, while keeping constant $\omega_n = \pi/2$ and ε. The initial distribution is an ensemble of 10^3 particles normally distributed in x and p with zero average and $\sigma_x = \sigma_p = 0.1$.

14th International Particle Accelerator Conference,Venice, Italy JACoW Publishing
ISBN: 978-3-95450-231-8 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2023-WEPL096

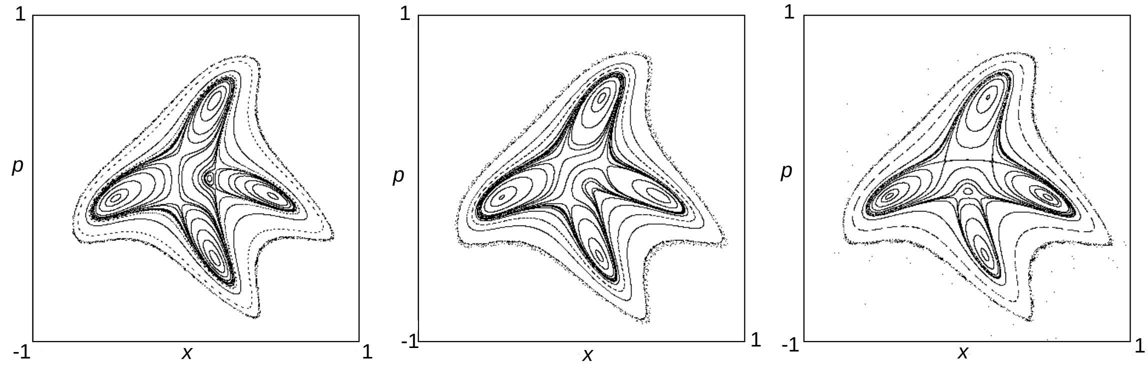

Figure 1: Phase-space portraits of the 4th iterate of the map (1) with $\omega_{0,n} = \pi/2$ at $\varepsilon = 0.001$ for three values of ψ_0: $\psi_0 = 0$ (left), $\psi_0 = \pi/4$ (centre), and $\psi_0 = \pi/2$ (right).

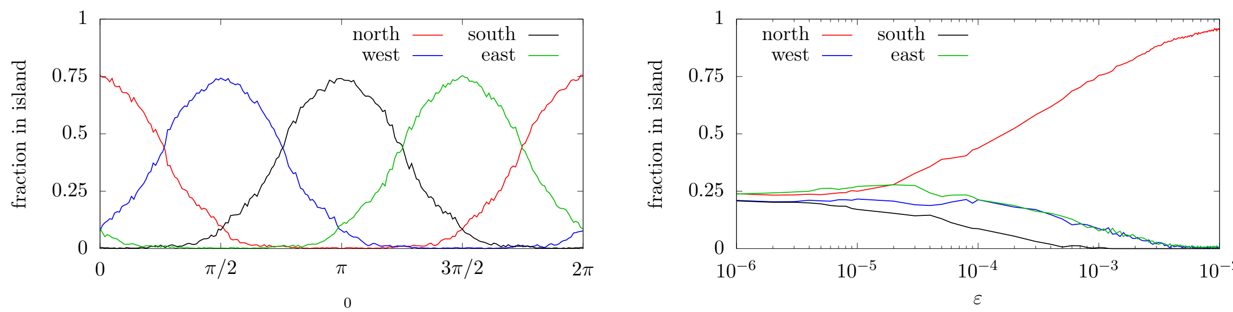

Figure 2: Trapping fraction in each island for a double-resonance simulation, as a function of the exciter phase ψ_0 (left) and of the exciter amplitude ε (log scale, with $\psi_0 = 0$) (right). The initial distribution is Gaussian in (x, p) with $\sigma = 0.1$. The map model (1) has been used, with $\omega_\mathrm{r} = \pi/2$, $\delta_n \in [-0.001, 0.001]$, $\varepsilon = 1 \times 10^{-3}$, $\Delta_n = 0$.

It is possible to count the number of particles trapped in each island by evaluating their tune, thanks to the tune-locking property of islands [12]. As expected, depending on ψ_0, we have a different dominant island that captures most of the beam. The fraction captured in each island depending on ψ_0 is shown in Fig. 2 (left), where this dependence is apparent. This effect, as seen in the right plot of Fig. 2 for $\psi_0 = 0$, increases as a function of the exciter amplitude ε. When the amplitude of the AC dipole is very small, the four islands trap a similar fraction of particles. As ε increases, not only one island (in our case, the northern one) captures most of the beam: the trapping fraction of the two adjacent islands (east and west) decreases at the same rate, while the opposite island (south) depletes faster.

Longitudinal Motion

If a bunch of particles undergoes the resonance-crossing process, and a non-zero chromaticity is assumed, two phenomena occur: the chromaticity, combined with the distribution of the longitudinal variables, causes a modulation of the transverse tune originating trapping and detrapping phenomena because of the periodic variation of the islands' size and position. Furthermore, the longitudinal dynamics and the distribution of the longitudinal variables imply that the effect of the AC dipole occurs with a slightly different phase, thus affecting the final trapping. To study this case, it is more appropriate to consider a model in which the transverse degree of freedom under consideration is coupled with the longitudinal one, adapting what was proposed in [13]:

$$\begin{pmatrix} x_{n+1} \\ p_{n+1} \end{pmatrix} = R(\omega_{0,n} + \alpha f(j_{n+1})) \begin{pmatrix} x_n \\ p_n + x_n^2 + \varepsilon \cos(\omega_n n + \psi_0) \end{pmatrix}$$

$$\begin{pmatrix} j_{n+1} \\ \xi_{n+1} \end{pmatrix} = \begin{pmatrix} j_n + \lambda \sin 2\pi h \xi_n \\ \xi_n + \eta j_{n+1} + \dfrac{\alpha}{2K_2\beta_x^3} f'(j_{n+1}) \left[x_n^2 + \left(p_n + x_n^2\right)^2 \right] \end{pmatrix}, \tag{2}$$

having defined $\lambda = eV/E_\mathrm{s}\beta^2$, $j = \delta p/p_\mathrm{s}$, $f(j) = (1/2\pi)\sin 2\pi j$; K_2 is the normalised strength of the sextupole, β_x the value of the beta function at the location of the sextupole; α represents the value of chromaticity, β the relativistic factor, η the slip factor and h the harmonic number.

The term α includes all sources of chromaticity (natural chromaticity and chromaticity generated by the sextupole included in our model). In our study, we are going to explore the small α regime that corresponds to what is done in the PS.

PROTOCOL SIMULATIONS

In MTE operation, two ways to employ the AC dipole have been studied [8]. In both cases, the exciter is switched

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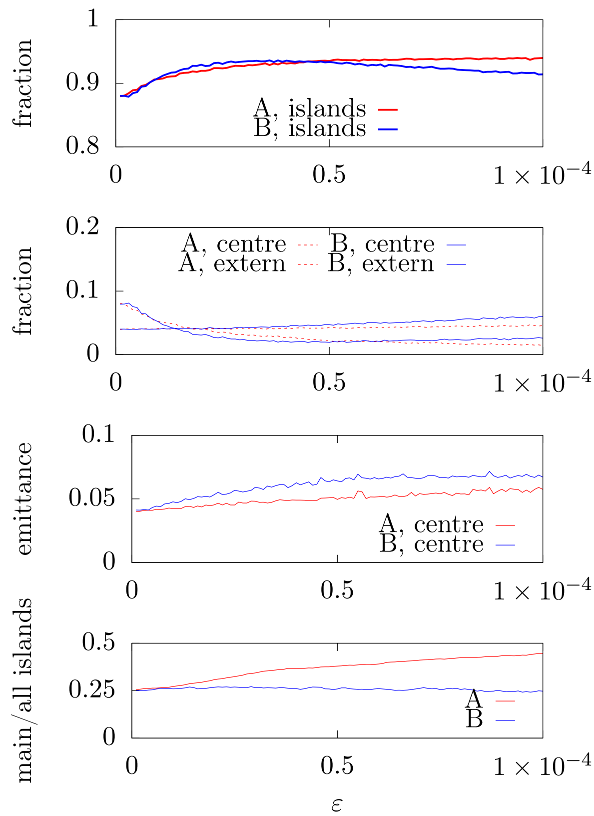

Figure 3: Top to bottom: Fraction of particles trapped in islands (first) and in the central and external regions (second) as a function of ε using Protocol A and B with $\alpha = 0$. Emittance of the central beamlet (third) and number of particles in the dominant island (north) divided by the population of all islands (fourth).

on and off in less than a turn. In Protocol A, $\omega_n = \pi/2$, i.e. it is time-independent, while in Protocol B, $\omega_n = \omega_{0,n}$, i.e. $\delta_n = \Delta_n$.

We show in Figs. 3 and 4 the main trapping properties of the two protocols for a beam of normally distributed particles ($\sigma_x = \sigma_p = 0.1$) depending on ε and chromaticity α. Another quantity of interest that we plot in Fig. 3 as a function of ε is the emittance of the core beamlet.

Unless otherwise stated, the simulation parameters are $\alpha = 0$, $\varepsilon = 1 \times 10^{-4}$ and the exciter is kept active for 50% of the simulation time. When $\alpha \neq 0$, we set $p_s = 14\,\mathrm{GeV/c}$, $\lambda = 859.1 \times 10^{-12}$, $\eta = 2.213 \times 10^{-2}$, $K_2 = \beta_x = 1$ and the initial distribution of longitudinal variables is normal, with $\sigma_j = 0.001$, $\sigma_\xi = 0.2$.

Note that we need to switch off the exciter before the end of the process to avoid that a single island captures all the beam. Of course, as the area of the dominant island reduces, some particles are lost to the external region of the phase space, thus causing the formation of a halo. However, the nonadiabaticity of exciter activation and deactivation does not cause harm, as the exciter strength used in PS is very small, with a maximum kick $\delta p/p_0 = 10\,\mu\mathrm{rad}$.

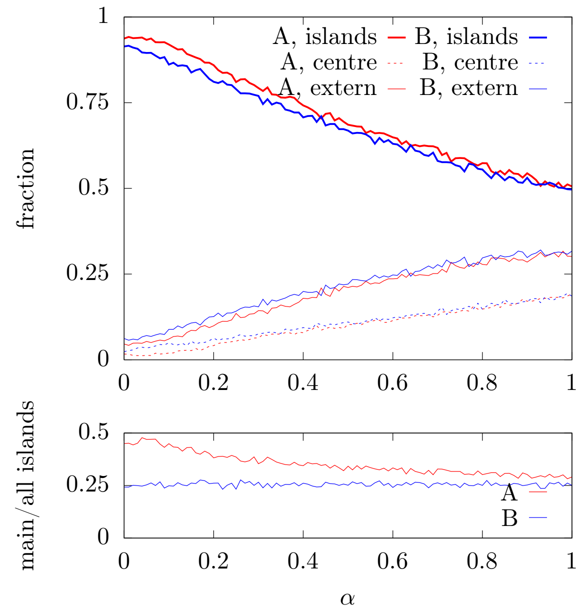

Figure 4: Top: Fraction of particles in each phase-space region after applying Protocol A and B as function of chromaticity α. Bottom: ratio between the population of the dominant island (north) and all islands.

We observe that both protocols are useful in increasing the trapping fraction of the MTE process; however, as ε increases, the number of particles released in the external region also increases, deteriorating the trapping process by forming a halo. Depending on the range of ε, one of the two protocols may be marginally better than the other. However, Protocol A better controls the emittance of the central beamlet, while Protocol B ensures that all islands receive the same share of particles, cancelling the dominant island effect. Furthermore, we see that chromaticity reduces the trapped fraction almost linearly, while, in Protocol A, it reduces the difference between islands.

CONCLUSIONS

Simulation results show that introducing a modulated AC dipole to cross a double resonance effectively helps trapping a larger fraction of particles in stable islands compared to a pure MTE approach to perform beam splitting. The two investigated protocols give comparable results. In general, the observed behaviour is explained by the peculiar phase-space structure that emerges under such conditions, where some islands grow larger and faster and, according to separatrix-crossing theory, are advantaged in trapping particle orbits. However, to fully understand and reproduce the experimental findings on double-resonance beam splitting, we plan to perform simulations using the realistic PS lattice.

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