

Physics with Kaons at NA62

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A broad physics program using K^+ mesons is pursued at CERN by the NA62 experiment. After having taken data in 2016,17 and 2018, the experiment resumed data taking in 2021 and is approved until CERN long shutdown 3. Recent results of NA62 on rare kaon decays are reviewed, concerning new physics searches, studies of low energy effective theories, searches for effects of lepton flavour, number and universality violation, and searches for production of feebly interacting particles predicted by dark scalar models. The future of kaon physics at CERN is also discussed.

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1. Introduction

NA62 is a fixed target experiment at CERN specifically designed to study the ultra-rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ [1]. The high performance required for this purpose makes the experiment perfectly suited to address a broad physics program with kaons. The physics program of NA62 can be grouped into four main areas: an indirect search for new physics (NP) through the study of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay; precision measurements of kaon decay observables to provide experimental inputs to low-energy effective field theories, such as Chiral Perturbation Theory (χ PT) [2, 3]; searches for lepton flavour/number violating (LFV/LNV) processes with kaons, and precision studies of lepton flavour universality violation (LFUV) in kaon decays; searches for production of feebly interacting particles (FIPs) in kaon decays, as predicted by proposed extensions of the Standard Model (SM) aiming at explaining the abundance of dark matter [4]. In addition, NA62 can study production and decay of FIPs with data taken in a special dump configuration.

The following sections briefly describe the latest results of NA62, with the exception of those related to the dump configuration, which have been presented in a specific contribution at this conference [5].

2. The NA62 experiment

The NA62 experiment is located in the North Area of CERN and exploits protons extracted from the SPS accelerator and hitting a Be target. A dedicated 100 m long beam line selects, focuses and transports a high-intense quasi-monochromatic 75 GeV/c secondary beam made of charged particles 6% of which are kaons. A differential Cherenkov counter and a silicon pixel tracker mounted on the beam line identify, timestamp and reconstruct the K^+ . The beam line is followed by a 80 m evacuated volume, called decay region. Detectors placed downstream of this region cover 10 mrad angular acceptance and reconstruct the products of the kaons decaying in the decay region. The main detectors are: a straw spectrometer in vacuum, a RICH detector, a quasi-homogeneous electromagnetic liquid Krypton calorimeters, a hadronic calorimeter and a plane of scintillator pads for muon detection. All the detectors have a cylindrical geometry with a hole in the center to accommodate the un-decayed particles of the beam. To this purpose an evacuated beam pipe passes through the RICH and the holes of the detectors downstream. Additional calorimeters surround the decay region and the beam pipe to detect particles down to 0 and up to 50 mrad.

NA62 took data in 2016, 17 and 2018, a period called RUN1. The data taking was resumed in 2021 after the CERN long shutdown 2 and the experiment is approved until long shutdown 3. This second period is called RUN2. In total, NA62 recorded about 3×10^{18} protons on target in RUN1 and at least twice as many are expected in RUN2.

3. $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

The $K \rightarrow \pi \nu \bar{\nu}$ decay occurs via the transition of a s to a d quark. The process is described by box and penguin diagrams dominated by the short distance exchange of a virtual t quark. This is one of the rarest meson decays in the SM, thanks to the quadratic GIM-mechanism and the $t \rightarrow d$ Cabibbo suppression. The corresponding decay amplitudes can be parametrized in terms of the

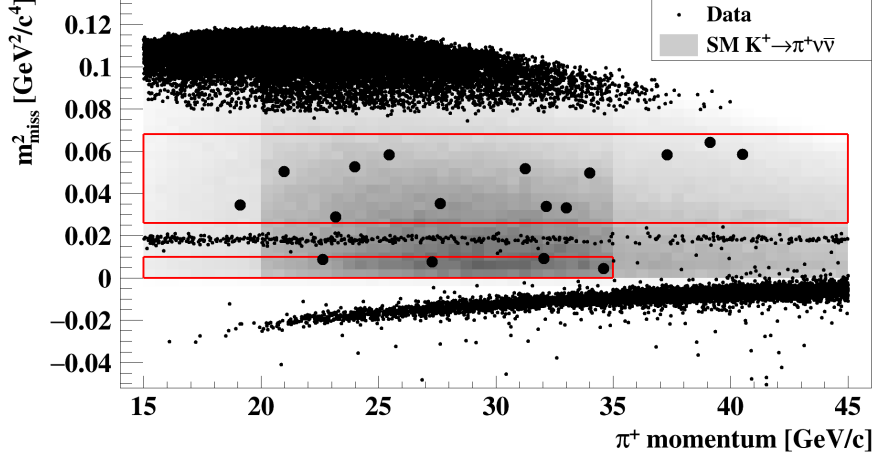


Figure 1: Reconstructed m_{miss}^2 as a function of π^+ momentum for events satisfying the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ selection criteria. The intensity of the grey shaded area reflects the variation of the SM signal acceptance in the plane. The two boxes represent the two signal regions and the events therein are the signal candidates. The events found outside the signal regions are also shown.

precisely measured $K^+ \rightarrow \pi^0 e^+ \nu$ decay, leaving the theoretical computation free from hadronic uncertainties. From a phenomenological point of view, the observable decays are $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$. The SM prediction of the branching ratio of the charged mode is about 8×10^{-11} [6–8]. The theory uncertainty in the prediction is less than 5%, while the overall uncertainty is about 10%, dominated by the precision in the knowledge of the CKM matrix elements.

The extreme SM suppression makes these decays particularly sensitive to NP. In model independent terms the $K^+ \rightarrow \pi^+ \nu \nu$ decays can probe NP beyond the 100 TeV mass scale [9]. Existing experimental constraints on NP affect the $K^+ \rightarrow \pi^+ \nu \nu$ weakly. Model dependent scenarios predict deviations from the SM predictions that can be probed with a $O(10\%)$ precision measurement of the branching ratio [10–17].

NA62 exploits the reconstruction of both K^+ and π^+ to select $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ events. This is essential to determine the $\nu \bar{\nu}$ invariant mass, m_{miss}^2 , and to kinematically separate the signal from the main background modes. Kaon decays with π^+ momentum lower than 45 GeV/c are selected, to guarantee at least additional 30 GeV energy produced together with the π^+ , given the 75 GeV momentum of the K^+ beam. Muon and π^0 rejection factors larger than 10^8 complement the kinematics to suppress tree level background processes like $K^+ \rightarrow \mu^+ \nu$ and $K^+ \rightarrow \pi^+ \pi^0$ decays. Precise timing of the charged particles with 100 ps resolution reduces backgrounds due to random coincidences between K^+ and accidental π^+ mesons. Nevertheless, events with accidental π^+ are the main source of background.

NA62 has analysed the data taken in RUN1, reporting the final observation of 20 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ candidates, with about 10 SM signal events and $7.03^{+1.05}_{-0.85}$ background events expected [18]. This is the first $> 3\sigma$ evidence for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay, and leads to a measured branching ratio $BR = (10.6^{+4.0}_{-3.4})_{stat} \pm 0.9_{syst} \times 10^{-11}$. Figure 1 shows the m_{miss}^2 versus the π^+ momentum of the observed $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay candidates selected in the data taken in 2018.

This measurement already constrains a significant fraction of the parameter spaces of NP models. In addition, this result can also be interpreted in terms of an upper bound on the production of an invisible scalar particle S of mass below $300 \text{ MeV}/c^2$ via the decay $K^+ \rightarrow \pi^+ S$.

4. χ PT studies

Unlike the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay, observables from kaon physics often requires χ PT or lattice-QCD for their interpretation, due to the sizable long distance (LD) contributions. The χ PT is the most common framework, but it requires experimental inputs to fix the many low energy parameters appearing at various orders of the momentum expansion. The study of specific kaon decay modes allows the extraction of these parameters.

4.1 $K^+ \rightarrow \pi^+ \gamma \gamma$

Long distance physics dominates the $K^+ \rightarrow \pi^+ \gamma \gamma$ decay [19]. The main kinematic variable is the di-photon invariant mass. The differential decay rate is parametrized in terms of functions of several low energy parameters known from other kaon decay modes, plus one, named \hat{c} , specific to this decay mode. This parameter is related to the pion and kaon loops modelling the dynamics of the $K^+ \rightarrow \pi^+ \gamma \gamma$ decay. The measurement of \hat{c} is a test ground for χ PT.

NA62 has performed the analysis of the $K^+ \rightarrow \pi^+ \gamma \gamma$ decay using the data collected in RUN1. About 4×10^3 $K^+ \rightarrow \pi^+ \gamma \gamma$ candidates have been selected, the largest world sample to date, with less than 10% background. The background is made of decays like $K^+ \rightarrow \pi^+ \pi^0 \gamma$ in which two electromagnetic clusters are merged, and $K^+ \rightarrow \pi^+ \pi^+ \pi^-$ with two tracks not reconstructed.

The di-photon spectrum is shown in Figure 2, left panel, where z is the square of the di-photon mass normalized to the kaon mass. The shape depends on \hat{c} , and exhibits a cusp structure due to the pion-loop. The non-zero value at low z comes from higher-order unitary corrections from $K^+ \rightarrow 3\pi$ decay. A fit to the spectrum using the χ PT description at order $O(p^6)$ allows the extraction of \hat{c} . The fit is limited to $z > 0.25$. The background is extracted from simulation and validated with data in suitable control regions orthogonal to the signal. A model independent branching ratio (BR) is also measured as the sum of the signal events in bins of z , normalized to the $K^+ \rightarrow \pi^+ \pi^0$ decay, in the same $z > 0.25$ region as for the fit.

The preliminary results are $\hat{c} = 1.713 \pm 0.075_{stat} \pm 0.037_{syst}$ and $BR = (9.73 \pm 0.17_{stat} \pm 0.08_{syst}) \times 10^{-7}$. These results are obtained with the external parameters fixed to reference [20, 21]. The result is consistent with, albeit significantly more precise than, previous measurements (Figure 2, right panel).

4.2 $K^+ \rightarrow \pi^0 e^+ \nu \gamma$

Chiral perturbation theory describes the dynamics of the decay $K^+ \rightarrow \pi^0 e^+ \nu \gamma$. The theory at $O(p^6)$ precisely predicts the ratio of the branching ratio of $K^+ \rightarrow \pi^0 e^+ \nu \gamma$ to $K^+ \rightarrow \pi^0 e^+ \nu$, depending on the kinematics of the radiated photon [22]. In addition, it is possible to define a T -violating asymmetry $A_\xi = (N_+ - N_-)/(N_+ + N_-)$ predicted by the SM to be below 10^{-4} , but sensitive to NP. Here N_{+-} is the number of $K^+ \rightarrow \pi^0 e^+ \nu \gamma$ candidates selected with $\xi = \vec{p}_\gamma \cdot (\vec{p}_e \times \vec{p}_\pi)/(M_K c)^3 > 0$ or < 0 , where $\vec{p}_{\gamma, e, \pi}$ are the 3-momenta of the radiated photon, positron and pion, respectively.

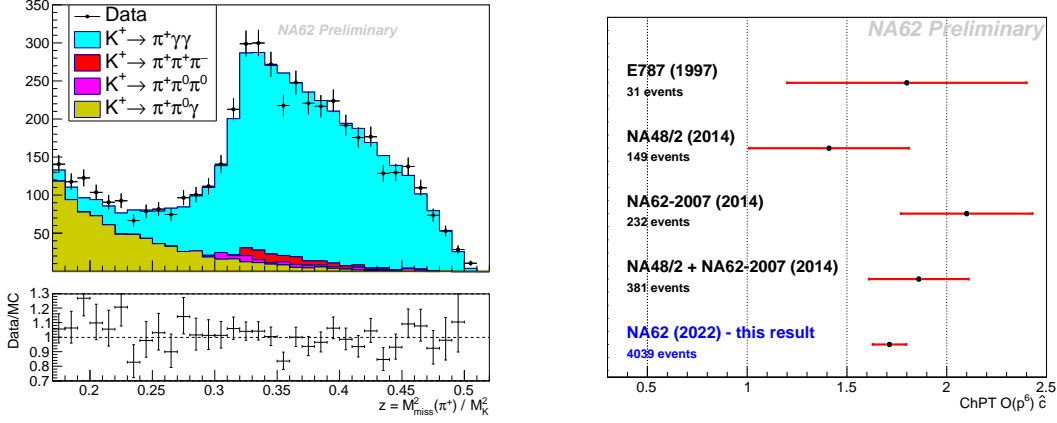


Figure 2: Fit procedure results within the χ PT $O(p^6)$ framework; the bottom insert shows the data-MC agreement (left). Comparison of measurements of \hat{c} (right).

NA62 has measured the ratios $R_{1,2,3}$ defined in table 1 and A_ξ on data collected in RUN1 [23]. Most of the selection criteria used in the analysis are common to $K^+ \rightarrow \pi^0 e^+ \nu \gamma$ (signal) and $K^+ \rightarrow \pi^0 e^+ \nu$ (normalization). The main differences are: an additional energy deposit in the electromagnetic calorimeter required to select signal events; different kinematic conditions applied to the total missing mass. The expected fraction of background events in both the signal and normalization sample is well below one percent.

The measurement of R_j relies on a counting approach. The acceptances are computed using the NA62 simulation tuned on data to reproduce at percent level the observed response of the electromagnetic calorimeter. The Monte Carlo generator of signal events makes use of the χ PT $O(p^6)$ description of the decay. The simulation of the normalization decay includes only the inner bremsstrahlung component [24]. The acceptance is also corrected for real multiple-photon emission using the PHOTOS package [25]. The asymmetry A_ξ is measured with respect to the experimental asymmetry evaluated with simulation.

The results are shown in table 1. The modelling of the calorimeter response and the corrections for multiple-photon emission are the main sources of the systematic uncertainty of $R_{1,2,3}$. The systematic uncertainty in the asymmetry is due to the limited statistics of the Monte Carlo sample.

This result from NA62 improves significantly on previous measurements especially in regions 2 and 3. The difference from the theory prediction is in the range of 3-5% depending on the region, and is significantly larger than the theory uncertainty. The precision of the asymmetry is at permille level, at least one order of magnitude above the prediction of both SM and NP models.

5. LFUV studies

The study of the $K^+ \rightarrow \pi^+ l^+ l^-$ ($l = \mu, e$) decay allows a precise test of LFUV. This process is theoretically described within the framework of χ PT [26, 27]. Dominant contributions are

Table 1: Definition, predictions and NA62 measurements of the observables related to the $K^+ \rightarrow \pi^0 e^+ \nu \gamma$ decay. E_γ and $\theta_{e\gamma}$ are the energy of the photon and the angle between the photon and the positron in K^+ rest frame.

Observable	Kinematic definition	χ PT prediction	NA62 measurement
$R_1 \times 10^2$	$E_\gamma > 10 \text{ MeV}, \theta_{e\gamma} > 10^\circ$	1.804 ± 0.021	$1.715 \pm 0.005 \pm 0.010$
$R_2 \times 10^2$	$E_\gamma > 10 \text{ MeV}, \theta_{e\gamma} > 20^\circ$	0.640 ± 0.008	$0.609 \pm 0.003 \pm 0.006$
$R_3 \times 10^2$	$E_\gamma > 10 \text{ MeV}, 0.6 < \cos \theta_{e\gamma} < 0.9$	0.559 ± 0.006	$0.533 \pm 0.003 \pm 0.004$
$A_\xi(S1) \times 10^3$	$E_\gamma > 10 \text{ MeV}, \theta_{e\gamma} > 10^\circ$		$-1.2 \pm 2.8 \pm 1.9$
$A_\xi(S2) \times 10^3$	$E_\gamma > 10 \text{ MeV}, \theta_{e\gamma} > 20^\circ$		$-3.4 \pm 4.3 \pm 3.0$
$A_\xi(S3) \times 10^3$	$E_\gamma > 10 \text{ MeV}, 0.6 < \cos \theta_{e\gamma} < 0.9$		$-9.1 \pm 5.1 \pm 3.5$

mediated by the $K^+ \rightarrow \pi^+ \gamma^*$ amplitude and involve LD effects described by a vector interaction form factor. The variable $z \equiv m(l\bar{l})^2/M_K^2$ describes the dynamics of the process, where $m(l\bar{l})$ is the dilepton invariant mass. The differential decay rate, $d\Gamma/dz$, includes a non-radiative and a radiative component. The non-radiative component is proportional to the square of a form factor $|W(z)|^2$, that depends on 2 parameters, a_+ , b_+ and an universal $K \rightarrow 3\pi$ loop term. Lepton universality requires that a_+ and b_+ are common to the electron and muon channel [28, 29]. In addition, NP can induce angular asymmetries.

NA62 has studied the muon decay using data collected in RUN1 [30]. The signal $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay is normalized to the $K^+ \rightarrow \pi^+ \pi^+ \pi^-$ channel. A 3-track vertex topology is required for both signal and normalization. Muon identification allows the selection of signal candidates, a full hadronic final state defines normalization events. Additional kinematic conditions contribute to the selection of signal and normalization samples with a background fraction below percent. In total about 28×10^3 $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ are selected over about 3.5×10^{12} K^+ decays.

The spectrum of the differential decay width is reconstructed and divided into 50 equi-populated bins. $|W(z)|^2$ is computed in each bin, assuming a linear dependence on z within the bin. A χ^2 fit to the reconstructed $|W(z)|^2$ provides the best values of a_+ and b_+ . Both positive and a negative values of a_+ and b_+ are statistically compatible with data. The positive solutions are discarded because they are theoretically disfavoured. The integral of the reconstructed differential decay width provides a measurement of the model-independent branching ratio of the decay.

Figure 3 shows the contour of the measured form factor parameters and the measurement of the model-independent branching ratio. Systematic studies have shown that the measurements are statistically limited. NA62 has improved significantly over previous measurements and a_+ and b_+ are compatible with those measured in $K^+ \rightarrow \pi^+ e^+ e^-$ decays.

An additional observable related to the $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay is $A_{FB} = (N_+ - N_-)/(N_+ + N_-)$; here N_{+-} are the number of signal events with positive or negative cosine of the angle between the kaon and the muon in the di-muon rest-frame. The measured A_{FB} is compatible with zero, setting an upper limit $A_{FB} < 0.9 \times 10^{-2}$ at 90% C.L.

NA62 has also performed direct searches for forbidden SM processes that can challenge charged lepton flavour and lepton number conservation [31–34]. Models such as leptoquarks can predict violation of the individual lepton number of charged leptons at levels incompatible with the SM. The conservation of total lepton number is an exact symmetry of the SM and can be violated only in the

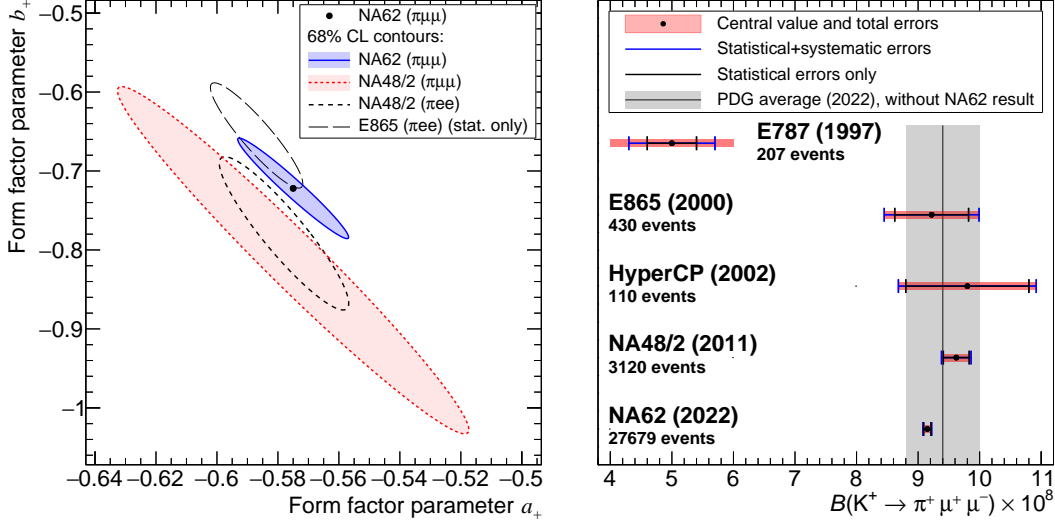


Figure 3: Combined statistical and systematic 68% CL contours in the (a_+, b_+) plane for the muon and electron modes compared to previous results (left). Measurements of the $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ branching ratio (right).

presence of Majorana neutrinos. These symmetries forbid several kaon decay channels involving muons and electrons in final state. The high resolution of the tracking system and the powerful particle identification makes NA62 ideal to search for these decay modes. The many searches performed so far with RUN1 data are statistically limited. No signals have been observed, but the sensitivity ranges between 10^{-10} and 10^{-11} and has generally improved by an order of magnitude on previous results.

6. FIPs searches

Kaons can decay to FIPs predicted by dark-sector models. In this context, NA62 has recently provided the preliminary result of a study of the $K^+ \rightarrow \pi^+ e^+ e^- e^+ e^-$ decay [35]. This decay is described within the SM in terms of a non-resonant and a resonant amplitude. The resonant amplitude corresponds to a branching ratio of $(6.9 \pm 0.3) \times 10^{-6}$ and proceeds via the $K^+ \rightarrow \pi^+ \pi^0$ channel with $\pi^0 \rightarrow e^+ e^- e^+ e^-$ [36]. The non-resonant part is suppressed in the SM, with the corresponding branching ratio predicted to be below 10^{-10} [37]. The presence of FIPs can induce sizable effects in the non-resonant branching ratio. The $K^+ \rightarrow \pi^+ e^+ e^- e^+ e^-$ could proceed through two short-lived QCD axions, a , decaying into $e^+ e^-$ pairs, through the sequence $K^+ \rightarrow \pi^+ a a$, $a \rightarrow e^+ e^-$. This mechanism could provide a possible explanation of the observed “17 MeV anomaly” in the mass spectra produced by the de-excitation of specific nuclei [38]. In particular, an axion mass of $17 \text{ MeV}/c^2$ would enhance the branching ratio by more than five orders of magnitude. Alternatively, the same final state could be reached by the cascade $K^+ \rightarrow \pi^+ S$, $S \rightarrow A' A'$, $A' \rightarrow e^+ e^-$, where S and A' are a dark scalar and vector, respectively [39].

NA62 has performed the first study of the $K^+ \rightarrow \pi^+ e^+ e^- e^+ e^-$ decay using data from RUN1. The signal exhibits a 5-track vertex topology. A combinatorial particle identification based on kinematics

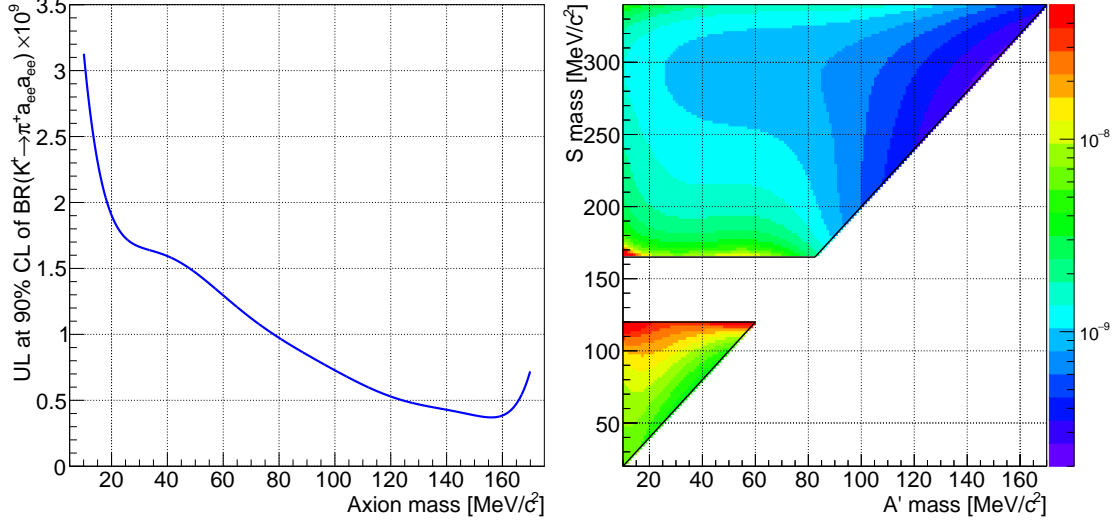


Figure 4: Left: upper limit at 90% CL of the branching ratio of the prompt decay chain $K^+ \rightarrow aa, a \rightarrow e^+e^-$ as a function of the assumed axion mass (left). Upper limit at 90% CL of the branching ratio of the prompt decay chain $K^+ \rightarrow \pi^+S, S \rightarrow A'A', A' \rightarrow e^+e^-$ as a function of the assumed dark-photon and dark-scalar masses.

is applied to the positive particles to assign the pion and electron masses to the reconstructed tracks. Events from the resonant component of the decay are used for normalization. The relevant kinematic variables to select the signal and the normalization are the total and the 4-electron invariant masses, $m_{\pi 4e}$ and m_{4e} , and the missing mass. In contrast to normalization events, signal candidates are selected outside the π^0 peak of m_{4e} . The signal region is defined around the kaon mass of $m_{\pi 4e}$. The background comes from 5-track SM kaon decays, like $K^+ \rightarrow \pi^+ \pi^0 \pi^0$ with both $\pi^0 \rightarrow e^+ e^- \gamma$, and 3-track SM kaon decays overlapping with accidental $K^+ \rightarrow \pi^+ \pi^+ \pi^-$ decays. About 0.18 events are expected in the signal region for the SM process, all due to background events. The sample used to search for the QCD axions or dark cascade is selected by grouping the electrons and positrons into two pairs and retaining those with the closest invariant mass. This condition reduces the background to 0.0004 ± 0.0004 events expected in the signal region.

The generator of the SM signal process includes only the non-resonant component. The signal acceptance is about 2×10^{-4} . No candidates are observed in the signal region leading to an upper limit on the non-resonant branching ratio of 1.4×10^{-8} at 90% C.L. To search for the QCD-axion mediated decay scan of the di-electron invariant mass is made in steps of $5 \text{ MeV}/c^2$. The $a \rightarrow e^+e^-$ decay is simulated as both prompt and isotropic. The acceptance for the $K^+ \rightarrow \pi^+ aa$ decay is in the range of $10^{-3} - 10^{-2}$ depending on the QCD axion mass. The mass scan is performed from 5 to $175 \text{ MeV}/c^2$ and the upper limit is shown in figure 4, left panel. A similar technique is used to look for the dark cascade process. In this case the search is performed in the di-vector and scalar mass spectrum. The result in the S versus A' mass plane is shown in figure 4, right panel. A smoothing technique has been applied to overcome the limited statistics of the simulation when computing the acceptance in this plane.

7. Future prospects

The NA62 experiment is approved to take data until long shutdown 3, with an intensity significantly larger than that of RUN1. The main goal is to improve on the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay. Several upgrades to the experimental apparatus have been implemented for this purpose. A first preliminary analysis of the data collected in 2022 exhibits a sensitivity similar to that of the whole RUN1. The yield of selected $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is such that precision of the order of 15% in the branching ratio can be attained by long shutdown 3, assuming a beam delivery similar to that of 2022 in the upcoming years. In addition, the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ study will profit from the hardware upgrades to further increase signal efficiency and reduce background. The program of precision measurements, and LNV, LFV and dark sector searches is continuing. A new trigger has been deployed to study the decays $K^+ \rightarrow \pi^+ e^+ e^-$ and $K^+ \rightarrow \pi^+ \mu^+ \mu^-$. The sensitivity of LFV and LNV searches is expected to improve by a factor of 2, thus approaching 10^{-11} .

The future of kaon physics at CERN relies on the HIKE project [40]. The idea is to use the kaon line of NA62 at CERN, suitably refurbished to deliver $\times 4$ more protons on target. The experimental area would become a kaon facility to study both K^+ and a K_L physics. The NA62 apparatus will be replaced by cutting-edge detectors able to cope with the higher intensity. Several detectors will serve both the K^+ and K_L phase, to guarantee a smooth experimental transition between the two phases. The physics program would include: the study of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay with precision approaching the theory error; the first measurement of the $K_L \rightarrow \pi^0 l^+ l^-$ decays, the short distant part of which is purely CP-violating and is essential to pin down NP effects at the highest mass scales; tests of LFUV at permille level through, for example, the precise measurement of the $K^+ \rightarrow \pi^+ l^+ l^-$; searches of LFV and LNV processes approaching a sensitivity of 10^{-12} ; a new measurement of the Cabibbo angle V_{us} , and of all the main kaon decay modes, an important step to profit of the full power of kaons as a probe of NP. Finally the dump physics program will be pursued and strengthened with dedicated runs in dump mode in synergy with a new off-axis experiment called Shadows [41].

8. Conclusions

A broad physics program using K^+ mesons is pursued at CERN by the NA62 experiment. The experiment has taken data in 2016,17 and 2018, the so called RUN1. In addition to the first $> 3\sigma$ evidence of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay, NA62 is performing studies in the framework of the χ PT theory, on lepton flavour, number and universality violation and is searching for the production of feebly interacting particles predicted by dark scalar models. The experiment is approved to take data until long shutdown 3, with the expectation to more than double the statistics collected in RUN1. The future of kaon physics at CERN relies on the proposal of a kaon facility able to study both charged and neutral kaons, name HIKE.

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