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CHROMATICITY CALCULATION AND CORRECTION FOR ACOL

Z.Y. Guo, C.D. Johnson and E.J.N. Wilson

1. Introduction

The aim of this note is to describe the methods which we have used to calculate the chromaticity and correct it in ACOL. The chromaticity is defined as the dependence of the betatron wave number on momentum:

$$Q' = \frac{\Delta Q}{\Delta p/p} \quad .$$

Unfortunately, there seems to be differences between the chromaticity formulae given in many published articles and computer programs. We had therefore to compare the several programs available at CERN and to develop an independent computer program, CHROMAT, which took into account the non-linear differential equations of motion in the end fields of magnets.

2. Comparison of the Calculations

There are several papers[1-2-3] which discuss the different methods of chromaticity calculation and several computer programs can be used for chromaticity calculation. As is well known, it is very important to consider the effects of magnetic edge field in chromaticity calculation for small machine such as AA, LEAR and ACOL. End field are treated differently in each program.

We compared the different results of chromaticity calculation from the programs PATRICIA[4], HARMON[5], MAD[6], CHROMAT for AA and ACOL (see Table 1). We believe the results from program MAD and CHROMAT because they are very close to the value of measurements in LEAR and AA at the same time. We also used the programs CHROMAT, PATRICIA and RACETRACK[7], matching the sextupole strength to cancel the chromaticity, and then using the program MAD to check the results. From Table 2 it will be found that the best results are from the program CHROMAT. So we can now use the program CHROMAT to calculate the chromaticity and to match the sextupole for ACOL with some confidence.

3. Formula for Chromaticity

The formula used in the program CHROMAT including the effects of edge magnetic field for chromaticity calculation and correction is as follows[3]:

$$Q'_H = -\frac{1}{4\pi}\int \kappa_s D\beta_H \, ds + \frac{1}{4\pi}\int \kappa\beta_H ds - \frac{1}{2\pi}\int \frac{1}{\rho^2}\beta_H \, ds$$
$$+ \frac{1}{2\pi}\int \frac{1}{\rho^3} D\beta_H \, ds + \frac{1}{4\pi}\int \frac{1}{\rho} D'\alpha_H \, ds - \frac{1}{4\pi}\sum\left[\frac{1+tg^2\theta}{\rho} D'\beta_H\right]_{ex}^{en}$$
$$+ \frac{1}{4\pi}\sum\left[\frac{1+2tg^2\theta}{\rho} D\alpha_H\right]_{ex}^{en} + \frac{1}{4\pi}\sum\left[\frac{tg\theta}{\rho}\beta_H\right]_{edge} + \frac{1}{2\pi}\sum\left[\frac{tg\theta}{\rho^2} D\beta_H\right]_{edge}$$

$$Q'_V = \frac{1}{4\pi}\int \kappa_s D\beta_V \, ds - \frac{1}{4\pi}\int \kappa\beta_V ds + \frac{1}{4\pi}\int \frac{1}{\rho} D'\alpha_V \, ds$$
$$+ \frac{1}{4\pi}\sum\left[\frac{1+tg^2\theta}{\rho} D'\beta_V\right]_{ex}^{en} + \frac{1}{4\pi}\sum\left[\frac{1-2tg^2\theta}{\rho} D\alpha_V\right]_{ex}^{en}$$
$$- \frac{1}{4\pi}\sum\left[\frac{tg\theta}{\rho}\beta_V\right]_{edge} + \frac{1}{4\pi}\sum\left[\frac{tg\theta}{\rho^2} D\beta_V\right]_{edge}$$

where

$$\kappa = -\frac{1}{B\rho}\frac{dB}{dx}$$

$$\kappa_s = -\frac{1}{B\rho}\frac{d^2B}{dx^2}$$

ρ, θ are the bending radius and bending angle, $\alpha_{H,V}$ and $\beta_{H,V}$ are the Twiss parameters of the lattice, D and D' are the dispersion and its derivative, and the symbols $[\,]_{edge}$, $[\,]_{en}$, $[\,]_{ex}$ indicate that the summation should be done at all edges, entrances and exits of the magnet, respectively.

4. Result

As an example, we obtained the natural chromaticity from the program CHROMAT for the lattice AC/83-13.

$$Q'_H = -5.362$$

$$Q'_V = -5.795$$

These values do not include the effects of the field between quadrupole and bending magnet. Table 3 gives many alternatives from which one can choose a pattern of sextupole for chromaticity correction.

References

1. J. Jäger, D. Möhl, PS/DL/LEAR/Note 81-7.
2. W. Hardt, J. Jäger, D. Möhl, PS/LEAR/Note 82-5.
3. S.Y. Chen, S.X. Fang, C. Zhang, Commun. in Theor. Phys. Vol. 1 No.2 (1982).
4. H. Wiedemann, PTM-230 (1980), SLAC.
5. M. Donald and D. Schofield, LEP Note 420.
6. F.C. Iselin, LEP Theory Note 12.
7. The program is developed at DESY.

APPENDIX A

The Program CHROMAT

The program CHROMAT is written for use on the IBM system computer at CERN. As the first step, one has to run the AGS program on the CDC system to get the lattice parameters, the sextupole pattern and then "CATALOG" TAPE3. After that, one can obtain the results of chromaticity and its correction from CHROMAT:

We give a listing of the program below together with the JCL instructions to run it. The program is stored in $ PZ.GUO.CHROMAT. It is not necessary to give any data input except for reading TAPE 3 by the program, but it is possible to change some names and strength of the sextupole after reading TAPE 3 for comparison of the different patterns (see the listing). The unit of the sextupole strength is m^{-2}.

Table 1. The different result of chromaticity from the different program based on the lattice 8308

		AA	ACOL
PATRICIA	Q'_H	-2.026	-6.644
	Q'_V	-3.737	-6.459
HARMON	Q'_H	-2.801	-7.072
	Q'_V	-1.251	-5.592
MAD	Q'_H	-1.480	-5.972
	Q'_V	-3.432	-5.856
CHROMAT	Q'_H	-1.390	-5.861
	Q'_V	-3.325	-5.719
Measurement*	Q'_H	-1.414	
	Q'_V	-3.562	

* from PS/DL/LEAR/Note 81-7

Table 2. The chromaticity correction from differen program

	MAD	CHROMAT	PATRICIA	RACETRACK
Q'_H *	−5.5890	−5.3623	−6.133	−6.122
Q'_V *	−6.0128	−5.7948	−6.430	−4.989
Q'_H **		−0.227	0.5435	0.4610
Q'_V **		−0.217	0.4171	−1.037

* without any sextupoles

** after correction ξ, from MAD

based on AC8313.

Table 3. The different pattern of sextupole based on 8313.

Job No	SDB ** / SFB	SN	SDQ (QD4)	SFQ (QF3)	SDP (QD6)	SFP (QF5)	SDS1	SFS1
181	0.08 / −0.08	0.05	0.2082	0.00	0.1834	−0.2300	0.3402	−0.2388
182	0.08 / −0.08	0.00	0.2082	0.00	0.1834	−0.2300	0.4302	−0.1532
201	0.08 / −0.05	0.05	0.2082	0.00	0.1834	−0.2300	0.3417	−0.2781
202	0.08 / −0.05	0.00	0.2082	0.00	0.1834	−0.2300	0.4317	−0.1925
203	0.10 / −0.05	0.05	0.2082	0.00	0.1834	−0.2300	0.3010	−0.2738
204	0.10 / −0.05	0.00	0.2082	0.00	0.1834	−0.2300	0.3910	−0.1882

** 8 bands without shims.

The list of the program with the JCL

```
XXXXX           FORTRAN 5
ACCOUNT(XXXXX)
FIND,CRNLIB5.
CRNLIB5,ALL.
NEWPROD.
FIND,TAPE3,XXXXX,ID=XXXXXXX.
FTN5,LO=O/S,DB=0/PMD,PL=2000.
LGO.
*EOR
      PROGRAM CHROMAT(INPUT,OUTPUT,TAPE3,TAPE1=INPUT,TAPE2=OUTPUT)
C     A PROGRAM FOR CHROMATICITY CALCULATION INCLUDING EDGE EFFECT
      DIMENSION NAME(300),EL(300),OK(300),SK(300),BETH(300),
     +BETV(300),ANG2(300),ALFH(300),RHO(300),ALINE(20),DUM(15)
      DIMENSION ALFV(300),DES(300),DESP(300),ANG(300),DS(300)
      DATA PI/3.14159/
C     READ TAPE3
      REWIND 3
   20 L=0
   30 READ(3,END=14) IEL,NUMB,NAMK,(ALINE(K),K=2,16)
   14 L=L+1
      NAME(L)=NAMK
      EL(L)=ALINE(2)
      OK(L)=ALINE(6)
      SK(L)=ALINE(7)
      BETH(L)=ALINE(10)
      BETV(L)=ALINE(9)
      ALFH(L)=ALINE(12)
      ALFV(L)=ALINE(11)
      DES(L)=ALINE(15)
      DESP(L)=ALINE(16)
      ANG2(L)=ALINE(5)
      DS(L)=EL(L)
      KEY=1
      IF(KEY.EQ.1) GOTO 300
      IF(NAME(L).EQ.2HSF) GOTO 301
      GOTO 302
  301 CONTINUE
      NAME(L)=3HSF9
      SK(L)=-6.0
      GOTO 300
  302 CONTINUE
      IF(NAME(L).EQ.2HSD) GOTO 303
      GOTO 304
  303 CONTINUE
      NAME(L)=3HSD9
      SK(L)=6.0
      GOTO 300
  304 CONTINUE
      IF(NAME(L).EQ.3HSF2) GOTO 305
      GOTO 306
  305 CONTINUE
      NAME(L)=2HSF
      GOTO 300
  306 CONTINUE
      IF(NAME(L).EQ.3HSD2) GOTO 307
      GOTO 300
  307 CONTINUE
      NAME(L)=2HSD
  300 CONTINUE
      IF(L.EQ.1) DS(L)=0.0
      IF(L.GT.1) DS(L)=DS(L-1)+DS(L)
      IF(NUMB.LT.IEL) GOTO 30
      BACKSPACE 3
      READ(3) IEL,NUMB,NAMK,(ALINE(K),K=2,16),
     +D1,D2,QH,D3,D4,D5,QV,DUM(15),C,D6
      PRINT 610
  610 FORMAT(1H1,'  NAME        L           K                  DK             BETH
     + BETV            ALFH         ALFV                DES          DESP              ANG')
      ILL=IEL+1
      DO 3 L=2,ILL
      PRINT 200,NAME(L),EL(L),OK(L),SK(L),BETH(L),BETV(L),
     +ALFH(L),ALFV(L),DES(L),DESP(L),ANG2(L)
    3 CONTINUE
  200 FORMAT (1X,A4,10E12.5)
      PRINT 201,QH,QV
  201 FORMAT (5X,2F15.5)
C     END OF TAPE3
      A1=0.0
      A2=0.0
      B1=0.0
      B2=0.0
      CHX1=0.0
      CHX2=0.0
      CHX3=0.0
      CHX4=0.0
      CHX5=0.0
      CHX6=0.0
      CHX7=0.0
      CHX8=0.0
      CHX9=0.0
      DO 50 I=1,IEL
      IF(SK(I).EQ.0.0) GOTO 10
      IF(NAME(I).EQ.2HSF) GOTO 410
      IF(NAME(I).EQ.2HSD) GOTO 420
      CHX1=CHX1-(SK(I)*DES(I)*BETH(I)*FL(I))/PI
   10 CONTINUE
      IF(OK(I).EQ.0.0) GOTO 40
      CHX2=CHX2+OK(I)*BETH(I)*FL(I)
   40 CONTINUE
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      IF(ANG2(I).EQ.0.0) GOTO 50
      ANG(I)=ANG2(I)/2.
      RHO(I)=EL(I)/ANG2(I)
      CHX3=CHX3-(BETH(I-1)+BETH(I))*EL(I)/RHO(I)**2.
      CHX4=CHX4+(DES(I-1)+DES(I))*(BETH(I-1)+BETH(I))
     +*EL(I)/(RHO(I)**3.*2.)
      CHX5=CHX5+(DESP(I-1)+DESP(I))*(ALFH(I-1)+ALFH(I))
     +*0.25*EL(I)/RHO(I)
      CHX6=CHX6-(DESP(I-1)*BETH(I-1)-DESP(I)*BETH(I))*
     +(1+(TAN(ANG(I)))**2.)/RHO(I)
      CHX7=CHX7+(DES(I-1)*ALFH(I-1)-DES(I)*ALFH(I))*
     +(1+2*(TAN(ANG(I)))**2.)/RHO(I)
      CHX8=CHX8+(BETH(I-1)+BETH(I))*TAN(ANG(I))/RHO(I)
      CHX9=CHX9-(DES(I-1)*BETH(I-1)+DES(I)*BETH(I))
     +*TAN(ANG(I))/(RHO(I)**2.*0.5)
      GOTO 50
  410 CONTINUE
      A1=A1-DES(I)*BETH(I)
      GOTO 50
  420 CONTINUE
      B1=B1-DES(I)*BETH(I)
   50 CONTINUE
      PRINT 100
  100 FORMAT (1H1,'      THE COMPONENTS OF HORIZONTAL CHROMATICITY')
      PRINT 110,CHX1,CHX2,CHX3,CHX4,CHX5,CHX6,CHX7,CHX8,CHX9
  110 FORMAT(5X,9(E12.5))
      CHY1=0.0
      CHY2=0.0
      CHY3=0.0
      CHY4=0.0
      CHY5=0.0
      CHY6=0.0
      CHY7=0.0
      DO 90 J=1,IEL
      IF(SK(J).EQ.0.0) GOTO 60
      IF(NAME(J).EQ.2HSF) GOTO 510
      IF(NAME(J).EQ.2HSD) GOTO 520
      CHY1=CHY1+(SK(J)*DES(J)*BETV(J)*EL(J))/PI
   60 CONTINUE
      IF(QK(J).EQ.0.0) GOTO 70
      CHY2=CHY2-QK(J)*BETV(J)*EL(J)
   70 CONTINUE
      IF(ANG2(J).EQ.0.0) GOTO 90
      ANG(J)=ANG2(J)/2.
      RHO(J)=EL(J)/ANG2(J)
      CHY3=CHY3+(DESP(J-1)+DESP(J))*(ALFV(J-1)+ALFV(J))
     +*0.25*EL(J)/RHO(J)
      CHY4=CHY4+(DESP(J-1)*BETV(J-1)-DESP(J)*BETV(J))*
     +(1+(TAN(ANG(J)))**2.)/RHO(J)
      CHY5=CHY5+(DES(J-1)*ALFV(J-1)-DES(J)*ALFV(J))*
     +(1-2*(TAN(ANG(J)))**2.)/RHO(J)
      CHY6=CHY6-(BETV(J-1)+BETV(J))*TAN(ANG(J))/RHO(J)
      CHY7=CHY7+(DES(J-1)*BETV(J-1)+DES(J)*BETV(J))*
     +TAN(ANG(J))/RHO(J)**2.
      GOTO 90
  510 CONTINUE
      A2=A2+DES(J)*BETV(J)
      GOTO 90
  520 CONTINUE
      B2=B2+DES(J)*BETV(J)
   90 CONTINUE
      CHX00=(CHX2+CHX3+CHX4+CHX5+CHX6+CHX7+CHX8+CHX9)/PI
      CHY00=(CHY2+CHY3+CHY4+CHY5+CHY6+CHY7)/PI
      PRINT 120
  120 FORMAT (1H1,'      THE COMPONENTS OF VERTICAL CHROMATICITY')
      PRINT 140,CHY1,CHY2,CHY3,CHY4,CHY5,CHY6,CHY7
  140 FORMAT(5X,7(E12.5))
      PRINT 150
  150 FORMAT (1H1,'       HORIZONTAL CHROMATICITY
     +        VERTICAL CHROMATICITY')
      PRINT 620
  620 FORMAT(1H1,'        WITHOUT ANY SEXTUPOLES')
      PRINT 160,CHX00,CHY00
  160 FORMAT(2(20X,E15.7))
      CHX01=CHX00+CHX1
      CHY01=CHY00+CHY1
      PRINT 530
  530 FORMAT(1H1,'        WITH SPECIAL SEXTUPOLES')
      PRINT 170,CHX01,CHY01
  170 FORMAT (2(20X,E15.7))
      C1=-CHX01*PI
      C2=-CHY01*PI
      DELT=A1*B2-B1*A2
      DELTX=C1*B2-C2*B1
      DELTY=A1*C2-A2*C1
      STSF=DELTX/DELT
      STSD=DELTY/DELT
      PRINT 190
  190 FORMAT(1H1,'        THE SEXTUPOLE        SF
     +        SD')
      PRINT 195,STSF,STSD
  195 FORMAT(2(20X,E15.7))
      STOP
      END
*EOR
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