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PS/DL/LEAR Note 84-2
5 March, 1984

MOULDING THE NOISE SPECTRUM FOR MUCH BETTER ULTRASLOW EXTRACTION

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1. Summary

Ultraslow extraction by swept filtered noise[1]) has made possible a spill duration of one hour in LEAR. But there were two major shortcomings: a pronounced spill modulation due to 50 Hz ripple, and a strong transient which made it difficult to obtain a constant average spill rate. In order to study the situation the requirements for the various regions of the noise spectrum are discussed.

An improvement is proposed which should be easy to attain since only inequalities have to be observed.

2. Introduction

Consider the particle density $\psi(x)$ and the diffusion constant

$$D(x) = \frac{1}{2(C\ B\rho)^2} \frac{d\langle V^2\rangle}{df} \quad (1)$$

where $x = \frac{\Delta p}{p}$; $x = 0$ chosen at resonance

C = circumference

$(B\rho)$= magnetic rigidity

$f = h.f_{rev}$= noise frequency

h = harmonic number of carrier frequency

f_{rev} = revolution frequency

$\langle V^2\rangle$ = time average of (gap voltage)^2 .

After shaping (with carrier frequency centered within "a", see fig. 1) the beam has assumed the constant density ψ_a over the initial width "a"; to this the constant spill rate ϕ and the spill duration T are related by

$$\psi_a\, a = \phi T \quad (2)$$

ψ obeys the Fokker-Planck equation :

$$\frac{\partial\psi}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial\psi}{\partial x} - v\,\psi\right) \qquad (3)$$

In the case of LEAR the streaming velocity v vanishes or reduces to an oscillating term $v = \hat{v}\, e^{i\omega t}$

ω = ripple frequency 2π
$\hat{v} = \omega r$
r = relative ripple amplitude.

3. The requirements

The average spill rate ϕ_0 is produced by sweeping the carrier frequency and thus the boundary (stack edge) so that for the quasi stationary solution of eq.(3) holds

$$\frac{\partial\psi_0}{\partial t} = 0; \quad D\,\frac{\partial\psi_0}{\partial x} = -\,\phi_0$$

Superimposed are oscillating parts $\phi_\sim$ which behave like diffusion waves. Their damping length is

$$ƛ = \sqrt{\frac{2D}{\omega}}$$

This yields already the first requirement: near the edge D should be small enough so that

$\left|\int \frac{dx}{ƛ(x)}\right| \gg 1$ provides sufficient damping or,

$$\left|\int \frac{dx}{\sqrt{D}}\right| \gg \sqrt{\frac{2}{\omega}} \qquad (4)$$

ψ_0 within the noisy region "b" is given by

$$\psi_b = \phi\int\frac{dx}{D}$$

and this should remain small compared to ψ_a. Thus we want (see equ.(2))

$$\int\frac{dx}{D} \ll \frac{T}{a} \qquad (5)$$

a condition which is the easier to meet the longer the spill duration T, even if D is small.

When particles travel from the stack edge to the resonance they have the average speed

$$\langle v \rangle = \phi \,/\, \psi_b(x)$$

(not to be confounded with v of equ. (3)).

Thus the "time constant" for region "b" is related to

$$t_b = \int \frac{dx}{\langle v \rangle} = \int \frac{\psi_o}{\phi_o}\, dx = \iint \frac{dx}{D}\, dx \qquad (6)$$

It seems reasonable to request $t_b < 1$ min.
At the resonance D should be large, namely

$$D = D_r \geq \text{a few } \omega r^2 \qquad (7)$$

in order to reduce the sensitivity of spill modulation versus ripple[1]).

4. The moulding of the Noise Spectrum

Conditions (4) to (7) cannot be met simultaneously with constant D for T $\simeq$ 1 h or more. Thus we should mould the noise spectrum suitably. This must be done in a sufficiently smooth way in order to avoid discontinuities which could be the source of new "diffusion" waves. We must also avoid intermodulation distortion beyond the edge in order not to degrade ψ_a. The noise could be composed of two parts (see fig. 1):

i) a swept relatively wide band, sharply cut and bell shaped,

ii) a narrow hump around the resonance, well rounded and with fixed carrier frequency.

part i) would be generated by switching a passive low-pass filter in between the noise generator and the present active low-pass cutting filter.

part ii) needs a second noise generator and else may consist essentially of a conventional passive low pass filter before mixing with the carrier frequency. (The hump may be cut sufficiently far in the tails in order to avoid discontinuities, so making sure that nothing is left beyond the edge.).

5. Comparisons and Remarks

At present the (constant) noise power for D is relatively low .($<10^{-7}$/s).

Thus eq.(4) is certainly satisfied, but eq.(5) merits only a $<$ symbol. This is not catastrophic as long as time constant t_b is small enough. But with the present long t_b (order 10 min) a transient problem arises.

With the moulded spectrum we shall bring t_b and ψ_b down by restricting the width of the low D region near the edge, but the width should remain larger than "r". We may choose D even as small as 10^{-8}/s, almost any finite D is sufficient to "scratch" enough particles from the stack.

The present $D = D_r$ seems insufficient to satisfy eq.(7). A remarkable improvement should already result with part i) of the moulded noise contribution, part ii) should lead to satisfaction. Both contributions can, of course, be limited by the intermodulation distortion still seen in region "a". As the extension of the intermodulation is proportional to the width of the spectrum and part ii) is narrow, D_r can hopefully be made large around the resonance even if LEAR is operated at lower energy (increased r) and at still longer T (for which a smaller $D = D_e$ near the edge would already satisfy eq.(5)) so increasing the ratio $D\ /D_e$.

It goes without saying that a reduction of the ripple amplitude r is beneficial to all these problems and thus efforts to improve the power supplies should continue.

Acknowledgements

The author would like to express his gratitude for suggestions and support to G. Molinari, D. Möhl, P. Lefèvre and in particular to R. Cappi.

Reference

1. R. Cappi, W.E.K. Hardt and Ch.P. Steinbach, Ultraslow extraction with good duty factor. XIth Int.Conf. on High Energy Accelerators, p. 335-340, Geneva, 1980.

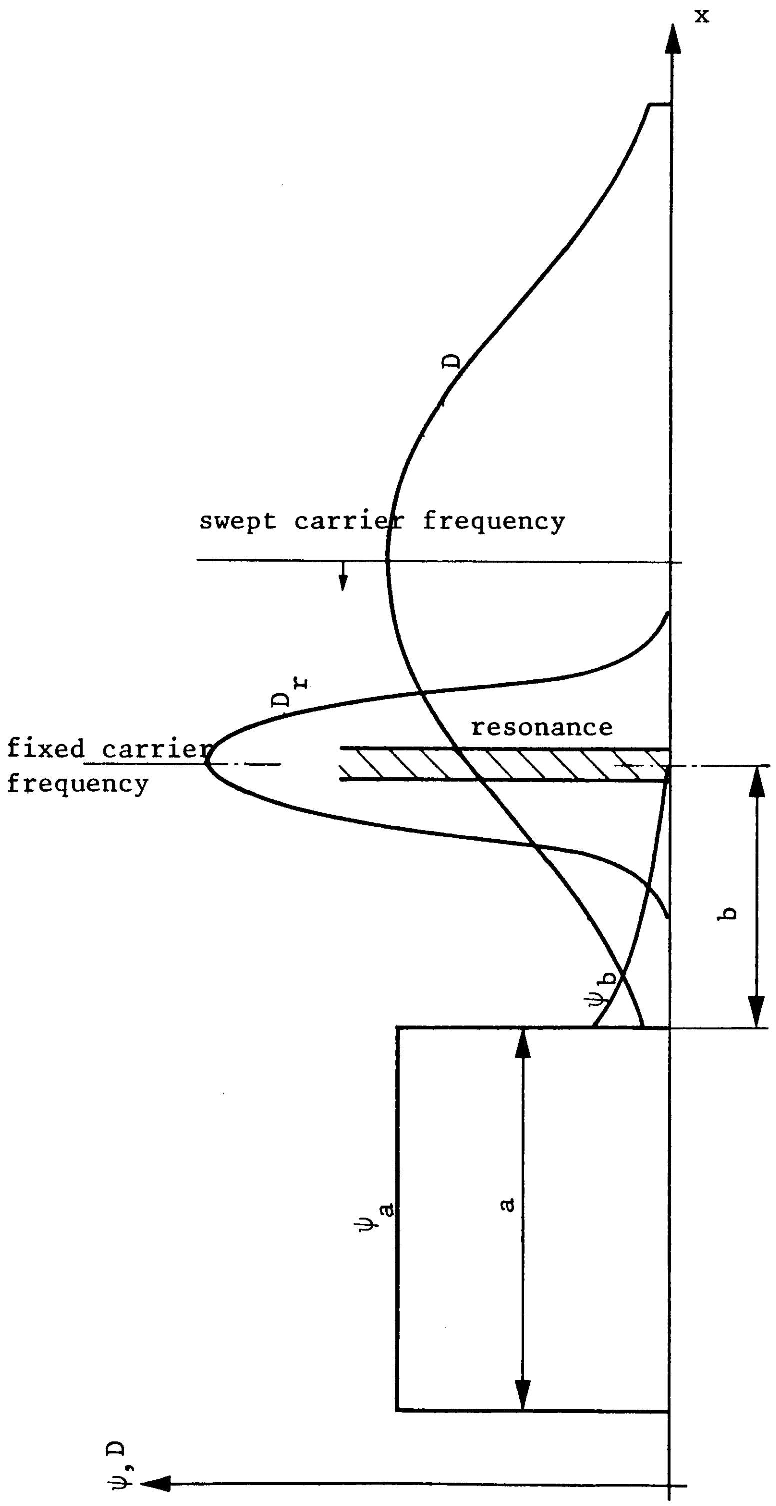
x
D
swept carrier frequency
D_r
resonance
fixed carrier
frequency
b
ψ_b
ψ_a
a
ψ, D

Fig. 1