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$\bar{p}p$ LEAR Note 98
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ULTRASLOW EXTRACTION OUT OF LEAR

(Transverse aspects)

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1. INTRODUCTION

Whereas the ultraslow nature of the LEAR extraction is achieved by manipulations in the longitudinal phase plane[1], the requirements in transverse phase plane are basically the same as for conventional resonance extraction.

A particularity is the necessity for economy in horizontal aperture, since an ample momentum bite must be provided for the various procedures to be carried out with the stack. In order to achieve this economy it was beneficial to determine the ejection parameters by means of analytical expressions which describe fairly well the general behaviour. The simplifications allowed in order to arrive at an analytical form could be shown to be insignificant when using a rigorous tracking program[3].

2. THE ESSENTIALS FOR EXTRACTION (Horizontal phase plane) (Fig. 1)

By means of the 16 quadrupoles, Q_H will be tuned to the resonance value 7/3 (coming from below). As this resonance value need not be met in general at the reference orbit, it will be described by a momentum fraction $(\frac{\Delta p}{p})_{res}$ (counted from the reference orbit).

Sextupoles are needed for two reasons (see also reference 5):

i) they tune the chromaticities ξ_V and ξ_H to the desired values, ξ_V near zero and ξ_H to a particular positive value as we will see;

ii) they excite the resonance harmonic m with a particular amplitude and phase, m being 7 for $Q_H = 7/3$. This excitation generates unstable fixed points and outgoing separatrices which enable the particles to jump over (within three revolutions) the critical element, which is :

The electrostatic septum (SE) whose location is assumed at the outside and at a positive value of the dispersion function $D_{He} > 0$ (e = suffix for a value at SE). Particles passing behind the SE are kicked to the outside so that they can overcome the thickness of the (stronger) magnetic septum (SM). The above mentioned value of ξ_H is important in order to make coincide trajectories of particles belonging to different emittances at SM, else the SM would have a larger effective thickness and eventually provoke losses. The horizontal position of SM can be chosen to arrive at a reasonable compromise between not to small an internal aperture (acceptance) and too large a local aperture needed at QF11 upstream of SM for the extracted beam. This is done with a local bump generated by four kicks around the long straight section 1 (SL1) housing SE and SM.

3. FIXED POINTS, TRAJECTORIES

It is convenient to ignore the slight dependence of Twiss parameters, lens strength's, etc, on momentum and ΔQ which allows to introduce the normalized co-ordinates

$$\begin{pmatrix} \overline{X} \\ \overline{X}' \end{pmatrix} = \sqrt{\frac{\beta_n}{\beta}} \begin{pmatrix} 1 & 0 \\ \alpha & \beta \end{pmatrix} \begin{pmatrix} X \\ X' \end{pmatrix} \qquad (3.1)$$

We may trace a particle near resonance ($Q = Q_{res} + \Delta Q$) at the azimuth of a sextupole, whose strength is indicated by $\Delta\overline{X}' = S\,\overline{X}^2$, over three revolutions but ignore terms with $(\Delta Q)^2$ and higher as was described by Barton[2]. His result can be written in condensed form using the complex notation $Z_\beta = \overline{X} + i\overline{X}'$, $Z_\beta^* = \overline{X} - i\overline{X}'$

$$\Delta Z_\beta = 3i\left(-2\pi\,\Delta Q\, Z_\beta + \frac{S}{4} Z_\beta^{*2}\right) \qquad (3.2)$$

$\Delta Z_\beta = 0$ yields the modulus of the fixed points, the stable fixed point has the trivial solution $Z_\beta = 0$, the three unstable fixed points have the modulus

$$|Z_{\beta f}| = 8\pi\left|\frac{\Delta Q}{S}\right| \qquad (3.3)$$

It can be shown that in this approximation the trajectories going through the fixed points are straight lines. They can be written (see also Fig. 2) for a phase ψ_s (see below) at an azimuth with phase ψ.

$$Z_\beta = \exp i\left[\frac{\pi}{6}(1 + 4N) + \psi_s - \psi\right]\cdot\left(r + i\,\frac{a\ \mathrm{sign}\ \xi}{\sqrt{3}}\right);\quad N = 0,1,2 \qquad (3.4)$$

The unstable fixed points are obtained by putting $r = a$ where $2a > 0$, say, is their mutual distance and a is related to the enclosed emittance area $\pi A_n = \pi \beta_n A$ by (A_n = normalized emittance)

$$a = \frac{\sqrt{3}}{2} |z_{\beta f}| = \sqrt{\frac{\pi A_n}{\sqrt{3}}} \tag{3.5}$$

Using the chromaticity definition $\frac{\Delta Q}{Q} = \xi \frac{\Delta p}{p}$ the momentum difference with respect to the zero emittance particle is given by

$$\left(\frac{\Delta p}{p}\right)_a = - \frac{a}{\sqrt{3}} \frac{S \text{ sign } \xi}{4 \pi Q \xi} = - \frac{a}{\sqrt{3}} \frac{S}{4 \pi Q |\xi|} \tag{3.6}$$

$S > 0$ as assumed in eq. (3.6) can always be arranged by imposing the proper phase.

M sextupoles of strength S_m located at phase ψ_m add up to an amplitude S and a phase ψ_s according to

$$\sum_{m=1}^{M} S_m \exp(3\, i\, \psi_m) = S \exp(3\, i\, \psi_s) \tag{3.7}$$

Eq.(3.2) holds at all phases $\psi = \psi_s \pmod{2\pi/3}$

4. THE JUMP OVER THREE REVOLUTIONS

Applying (3.2) to (3.4) yields for Δr the increment over three revolutions (jump)

$$r_+ = r_- + \Delta r = r_- + \frac{3}{4} S (r_-^2 - a^2) \tag{4.1}$$

This expression suffers from being not reversible. Replacing r_-^2 by $r_-.r_+$ yields the (better) symmetric expression

$$r_+ = \left(\frac{r_- - \frac{3}{4} S a^2}{1 - \frac{3}{4} S r_-}\right) = r_- + \left(\frac{r_-^2 - a^2}{\frac{4}{3S} - r_-}\right) \tag{4.2}$$

The difference between (4.1) and (4.2) should not be dramatized since higher order terms were ignored anyway in (3.2) and there is no easy trick to recollect them but (4.2) matches indeed better the results of a rigorous tracking program[3]. There is another feature of such a program which does not show up in the description by Eq. (3.4): the separatrices are curved when only a few sextupoles are taken. The sign of the curvature can change from revolution to revolution. But when many sextupoles (i.e. eight in LEAR) are considered, the curvature becomes negligibly small.

5. OTHER CONTRIBUTION TO Z

We have put the suffix β (for the betatron oscillations part) since there are other contributions to Z :

i) Z_b denoting a bump which steers the beam towards the septa;

ii) $Z_p = (\overline{D}_H + i\overline{D}'_H) . (\frac{\Delta p}{p})$ denoting the part coming from the momentum displacement and involving the (normalized) dispersion function $\overline{D}_H$.

It is convenient to compose $\frac{\Delta p}{p}$ of two parts $(\frac{\Delta p}{p}) = (\frac{\Delta p}{p})_a + (\frac{\Delta p}{p})_{res}$. $(\frac{\Delta p}{p})_a < 0$ was already introduced in Eq.(3.5) and is related to the emittance. $(\frac{\Delta p}{p})_{res}$ was mentioned in the beginning of chapter 2 and is common to all particles. $(\frac{\Delta p}{p})_{res}$ is needed for economic exploitation of the available aperture at critical places.

6. THE CONFIGURATION AT THE ELECTROSTATIC SEPTUM

Denoting the phase of SE by ψ_e we can rewrite Eq.(3.4) as

$$Z_{\beta e} = e^{i\phi_e} (r_e + i \frac{a \operatorname{sign} \xi}{\sqrt{3}}) \; ; \; \phi_e = \frac{\pi}{6} + \psi_s - \psi_e \bmod \frac{2\pi}{3}$$

the modulus being determined such that we obtain the main outgoing separatrix which belongs to the smallest $|\phi_e|$. See Fig. 2.
In general, this outgoing separatrix depends on "a", i.e. is different for different emittances.

As a consequence, particles belonging to different emittances experience their kick from different points in phase space (different values of $\overline{X}'_e$). At the SM where the kick $\Delta\overline{X}'_{eK}$ given to the particles by SE is translated into $\Delta\overline{X}_{mK} = \Delta\overline{X}'_{eK} \cdot \sin(\psi_m - \psi_o)$ no full use can be made of this separation unless the horizontal chromaticity ξ_H is tuned to a particular value. This value follows from the requirement that the main trajectories for all emittances should be superimposed

$$\frac{d}{da} \text{Im} \left[(Z_{\beta e} + Z_{pe}) e^{-i\phi_e} \right] = 0 \tag{6.2}$$

and implies

$$\xi_H = \frac{S}{4\pi Q_H} \left(\overline{D}'_{He} \cos \phi_e - \overline{D}_{He} \sin \phi_e \right) = \frac{S}{4\pi Q_H} |\overrightarrow{\overline{D}_{He}}| \sin (\delta_e - \phi_e) \tag{6.3}$$

(using $\overline{D}_{He} + i\overline{D}'_{He} = \left|\overrightarrow{\overline{D}_{He}}\right| \exp(i\delta_e)$).

As at SE in LEAR $\delta_e > \phi_e$, $\xi_H > 0$ is required.

The phase of the sextupolar excitation ϕ_e is not only involved in this proportionality factor between chromaticity and strength S but also when arranging for optimum economy of horizontal aperture. For a fixed distance of orbit for zero emittance to SE X_{eo} there is a maximum emittance (described by $\hat{a}$) whose one fixed point touches the SE.

Inspection of Fig. 3 shows that for $\hat{a}$ holds

$$\begin{aligned} X_{eo} &= \hat{a} \left(\cos \phi_e - \frac{1}{\sqrt{3}} \text{sign } \xi \sin \phi_e \right) + \left(\frac{\Delta p}{p}\right) \hat{a} \, \overline{D}_e \\ &= \hat{a} \left(\cos \phi_e - \frac{1}{\sqrt{3}} \text{sign } \xi \left(\sin \phi_e + \frac{\cos \delta_e}{\sin(\delta_e - \phi_e)} \right) \right) \end{aligned} \tag{6.4}$$

using (3.6) and (6.3).

7. APERTURE ECONOMY

The aperture should be matched at the F lenses where the β_H is maximum. (In LEAR there are also critical points in the centre of the bending magnets, but there the dispersion function is small). Thus by the aperture restriction at the F lenses, allowing for the orientation which is most space consuming, $\hat{a}$ is obtained and in turn X_{eo} using ϕ_e as a parameter in (6.4) .

By letting $(\frac{\Delta p}{p})_{res} = -(\frac{\Delta p}{p})_{\hat{a}}$ particles belonging to this maximum aperture fill the aperture symmetrically around the reference orbit. It is now a function of ϕ_e where the extreme particles revolve which belong to zero emittance, but undergo the jump belonging to (4.2).

The analytical program determines ϕ_e such that these particles have the same aperture requirement to the outside as the maximum emittance particle. The interpolation of the aperture needed to the outside and inside for intermediate emittance are about straight lines with "a", not exactly straight because the jump is involved, which is a more complicated than linear expression. See Fig. 4.

We can also link the space needed for the unstable fixed points obtaining straight lines versus "a". The space below the bottom line of this connection can be used for stack manipulations; specifying this space, then in turn limits the emittance which can be ejected in ultraslow fashion. The so-called "maximum emittance" is only of descriptive use, particles with such a big emittance cannot be really extracted since their jump vanishes, but their space requirement is easily described (Eq.(6.4)).
The underlined condition ensures optimum exploitation of the aperture for obvious reasons.

8. COMMENTS ON THE NOTATION OF THE COMPUTER PRINTOUT

Fig. 5 shows a printout with present default parameters. We give the following comments:

Notation	Meaning	Comments
XmØ/mm	Horizontal position of upstream inner edge of SM	Input parameter, determines then the bump
Xfnn/mm	Half-aperture of F quadrupoles	Input parameter
Kick/mrad	Kick given to the particles by SE	Input parameter. At high energy, this kick is as small as 3.5 mrad, and separation at SM becomes poor. A larger kick can be limited by the aperture requirement in the quadrupole F11.

Notation	Meaning	Comment
Dymb/mrad	$\Delta X'_{mb}$ = change of slope of bump at upstream edge of SM	Ignore.
JumpØ/mm	$\Delta r.\cos\,\phi_e$ = increase of X_e over three turns for zero emittance	Input parameter
Delps/E-3	$(\frac{\Delta p}{p})/10^{-3}$ provision for stack manipulation inside the peak emittance	Input parameter
Chrom'ty	ξ_H will (not) be found according to Eq.(6.3) if below is printed: (not) aligned	Input parameter, only for "not" aligned.
Phie	ϕ_e the excitation angle at SE	Value found for the underlined condition in paragraph 7
S	Value in normalized co-ordinates defined in Eq.(3.7)	
ΔK'11... ΔK'42	Required ΔK' values in real co-ordinates for the sextupolar excitation	
K'11..K'42	Required K' values for sextupolar excitation and chromaticity tuning	Holds for the two assumed values for the natural chromaticities
Chrm	ξ_H = horizontal chromaticity	
XeØ/mm	Distance from orbit for zero emittance to centre of SE	These two values differ by the bump at SE
Xe/mm	Horizontal position of SE	
Ye/mrad	Slope of trajectory at SE	
Xmb/mm	Bump at SM	
Delpr	$(\frac{\Delta p}{p})_{res}$ in 10^{-3}	
Remark on DymØ		Ignore
Emit	Emittance, in μrad.m various lines are arranged for equal spacing of "a" of Eq.(3.5)	The last line gives the maximum emittance, the last but one gives the peak emittance, which provides the specified Delps (see third column Dplft)

Notation	Meaning	Comment
Delpa	$(\frac{\Delta p}{p})_a$ of Eq.(3.6) in 10^{-3}	Evaluated for intermediate emittances
Deplft	$(\frac{\Delta p}{p})$ available for longitudinal manipulations (stack noise), in 10^{-3}	"
Jump	$\Delta r \cos \phi_e$ increase over three turns in mm	"
XmØ	This was specified for zero emittance (XmØ/mm). It is recalculated as a verification that the condition (6.3) is effective.	"
YmØ	The angle in mrad at which particles pass the leading edge of SM	"
X_{mk}	Distance at which particles pass behind the SM which have experienced the kick at SE	
X_{mkj}	as X_{mk} but the jump included	
X_{F11}	as K_{mkj} but (not at SM) at the F quadrupole 11 where β_H is maximum	Nearly the peak aperture requirement
$X_{f\ ext}$, $X_{f\ int}$	Extreme values to the outside (ext) quadrupole and to the inside (int) of F quadrupoles, outside the bump region	See chapter 7
$X_{b\ ext}$, $X_{b\ int}$	as ($X_{f\ ext}$ and $X_{f\ int}$) but in the centre of the bending magnets	

9. ADDITIONAL INFORMATION

Not all parameters which are needed to compute values for ejection are printed. The missing parameters follow from an AGS output either directly or by interpolation. They are stored on a data file from where they can be retrieved. The most important data are stored in matrix A which refers to the centre of the elements as printed in the left column. The last four columns represent the effect of a unit bump (at SM) or a bump with unit angle at SM as indicated in the SM line. The bumps are created by four kicks of which we only need to know the phase position. The strength is uniquely determined by the conditions : $\overline{X} = 0$ and $\overline{X}' = 0$ outside the bump region and the unity at SM.

BUMPER-ELEMENT:	DWH41	DHN11	DHN12	DWH11
PHASE/2/PI	-.24880	-.15961	.10133	.24880

MATRIX A (FOR Eject)

	BETA	ALPHA	D	D'	PSIBAR	UNIT BUMP Xb	UNIT BUMP Yb	UNIT ANGLE AT SM Xb	UNIT ANGLE AT SM Yb
SEH	6.05320	.91164	1.03670	.84354	-.22300	.29998	1.83426	-.04709	-.28792
QF11	10.61600	0.00000	3.46484	.69304	-.17800	.79981	1.67774	-.12554	-.26335
SMH	4.53450	1.13174	3.51841	0.00000	-.13483	1.00000	0.00000	0.00000	1.00000
BHN	10.73135	0.00000	-.86785	0.00000	-.29125	0.00000	0.00000	0.00000	0.00000

The necessary information for the sextupoles is contained in a further matrix

	Betah	Betav	D	PHASE/2/PI	LENGTH
F-Sext	9.19220	7.89860	3.51841	.17032	.38000
D-Sext	7.66570	15.32710	2.59422	.18874	.38000

Qv= 2.75000 Chrmh=-1.31000 Chrmv=-1.98000

Finally, we need to know Q_V and the (assumed) natural chromaticities as indicated below the matrix.

References

1. R. Cappi, W.E.K. Hardt, Ch. Steinbach, Ultraslow extraction with good duty factor, Proc. of the XIth Int. Conf. on High Energy Accelerators, CERN, Geneva, 1980, p. 335.

2. M.Q. Barton, Beam extraction from synchrotrons, Proc. of the VIIIth Int. Conf. on High Energy Accelerators, CERN, Geneva, 1971, p. 85.

3. P. Strolin, Third-order resonance slow extraction from alternating gradient synchrotrons, ISR/TH/66-40, 20.12.1966.

LEAR Notes in preparation

4. R. Giannini, Beam envelope in LEAR at third order resonance extraction.

5. Together with J. Jäger, Systematic resonances in competition with the $3Q_H = 7$ extraction resonance.

Distribution

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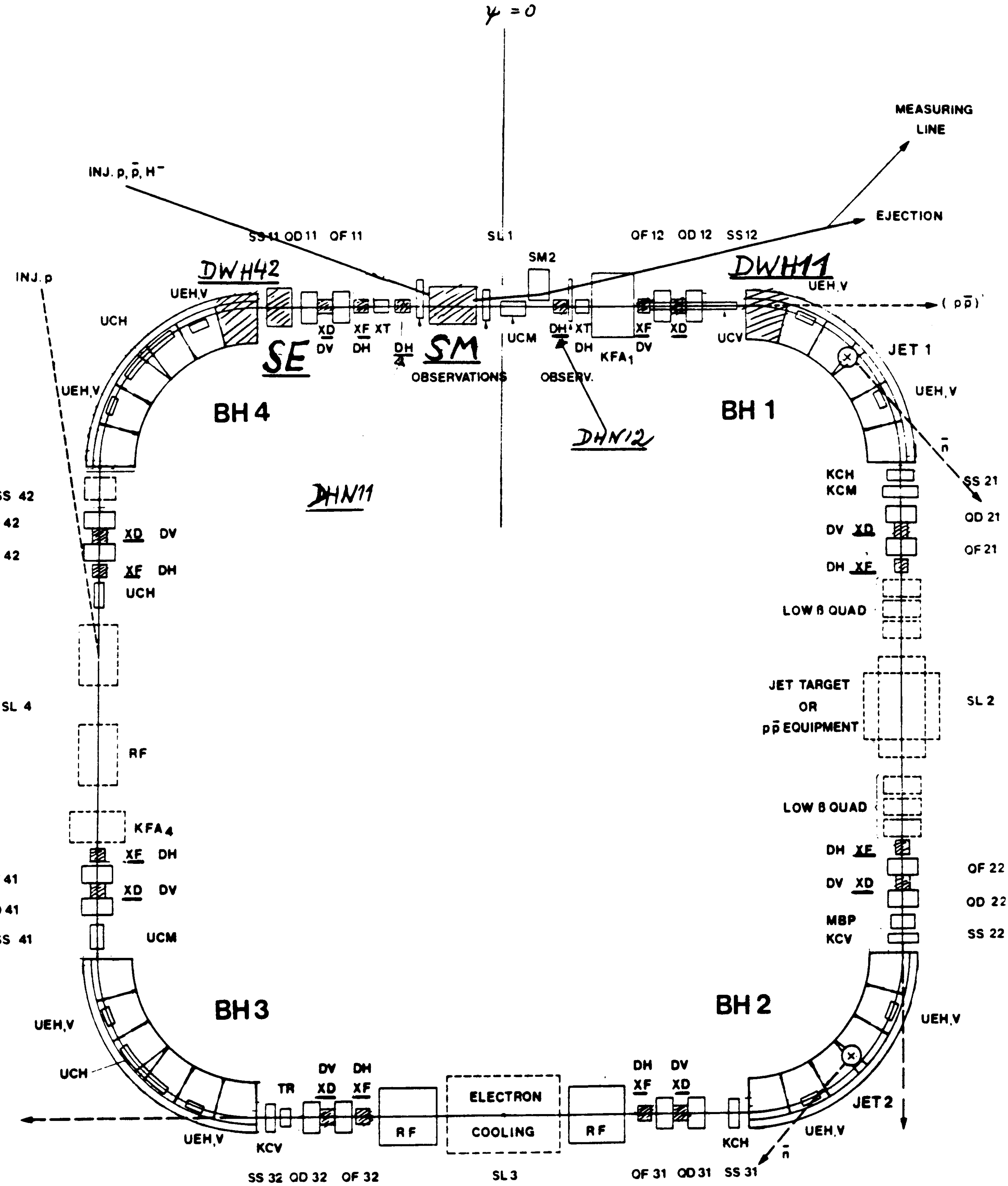


Fig. 1

LEAR General Lay out

EXTRACTION ELEMENTS UNDERLINED

SCALE 1 : 143

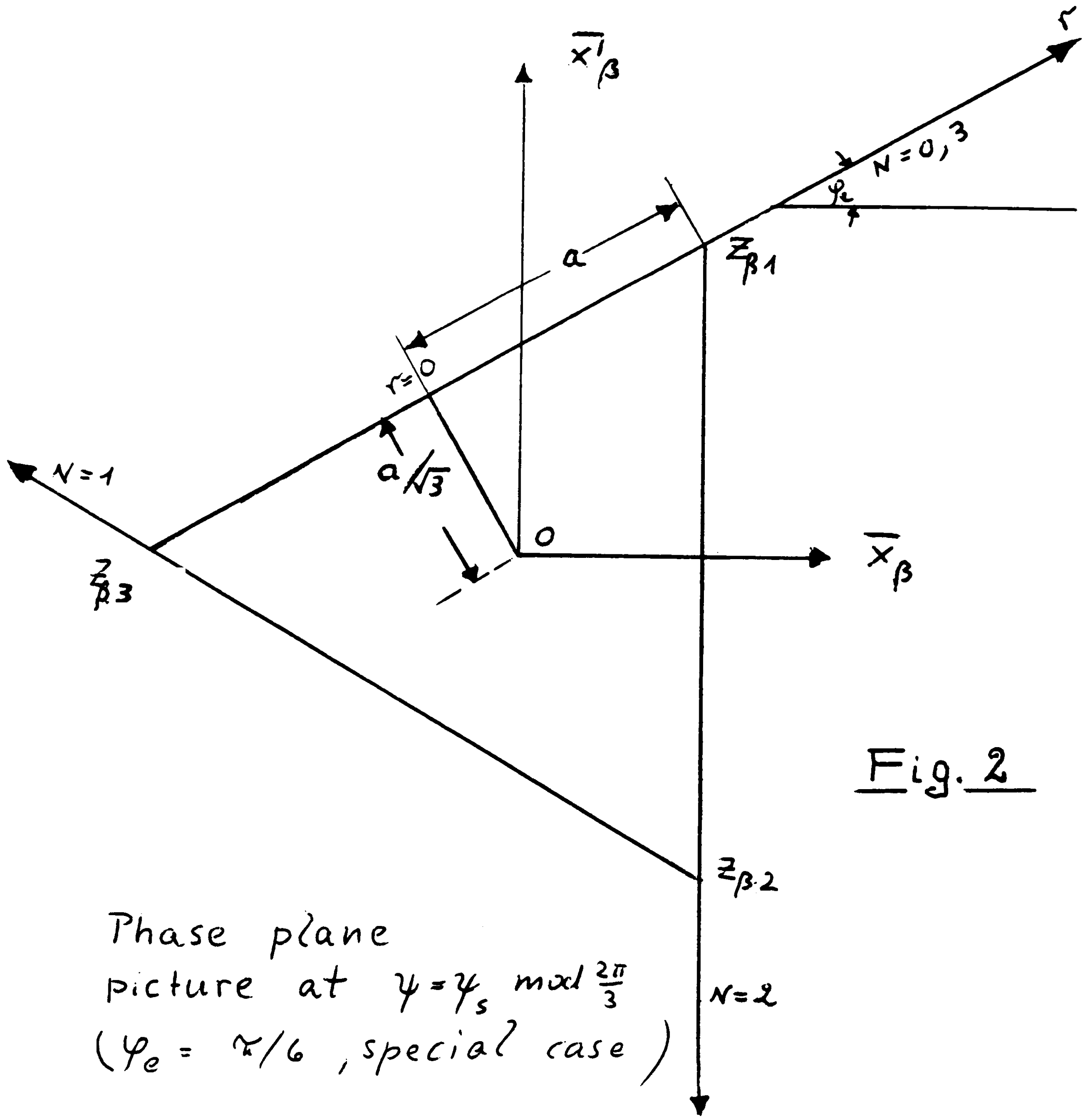
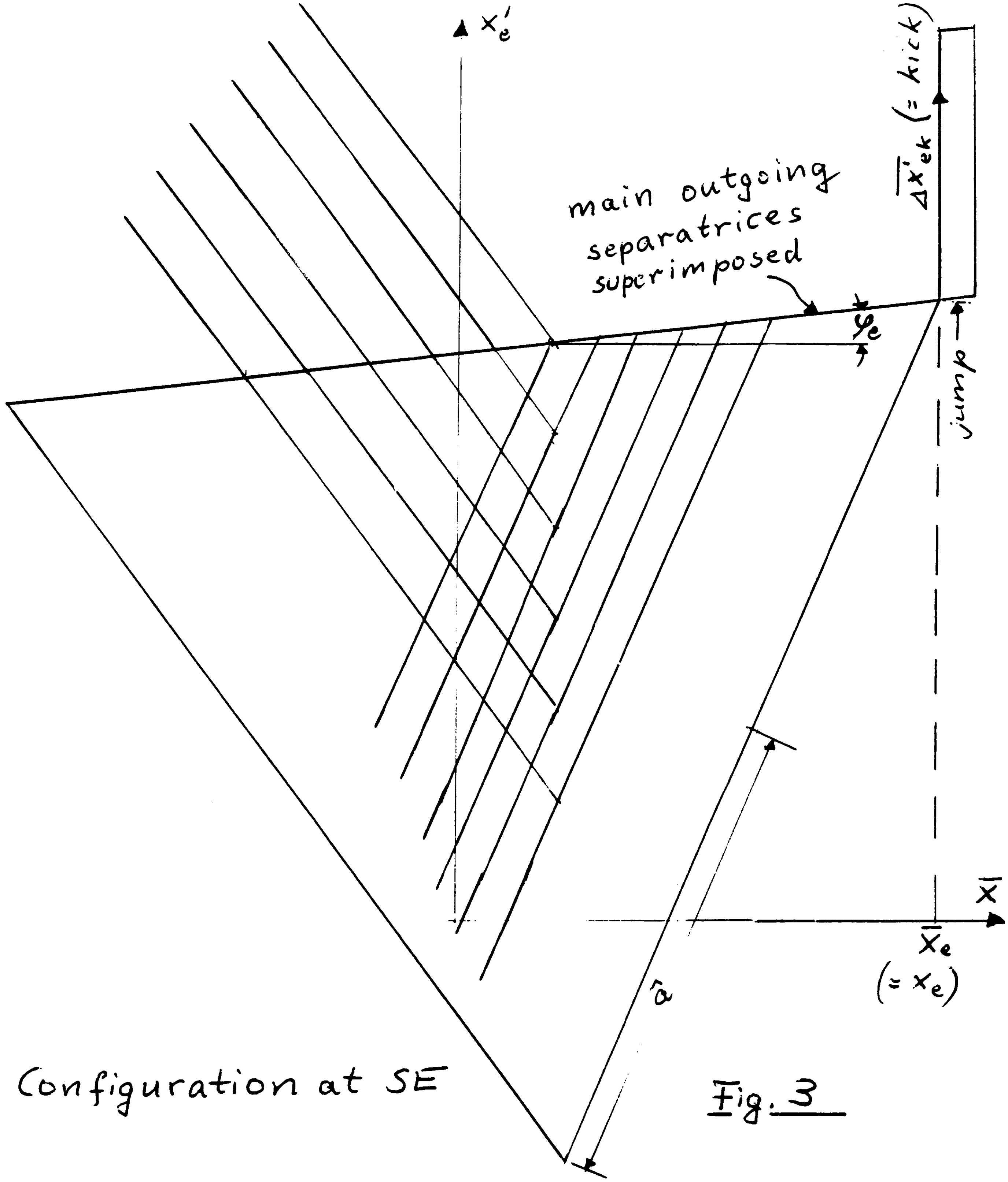


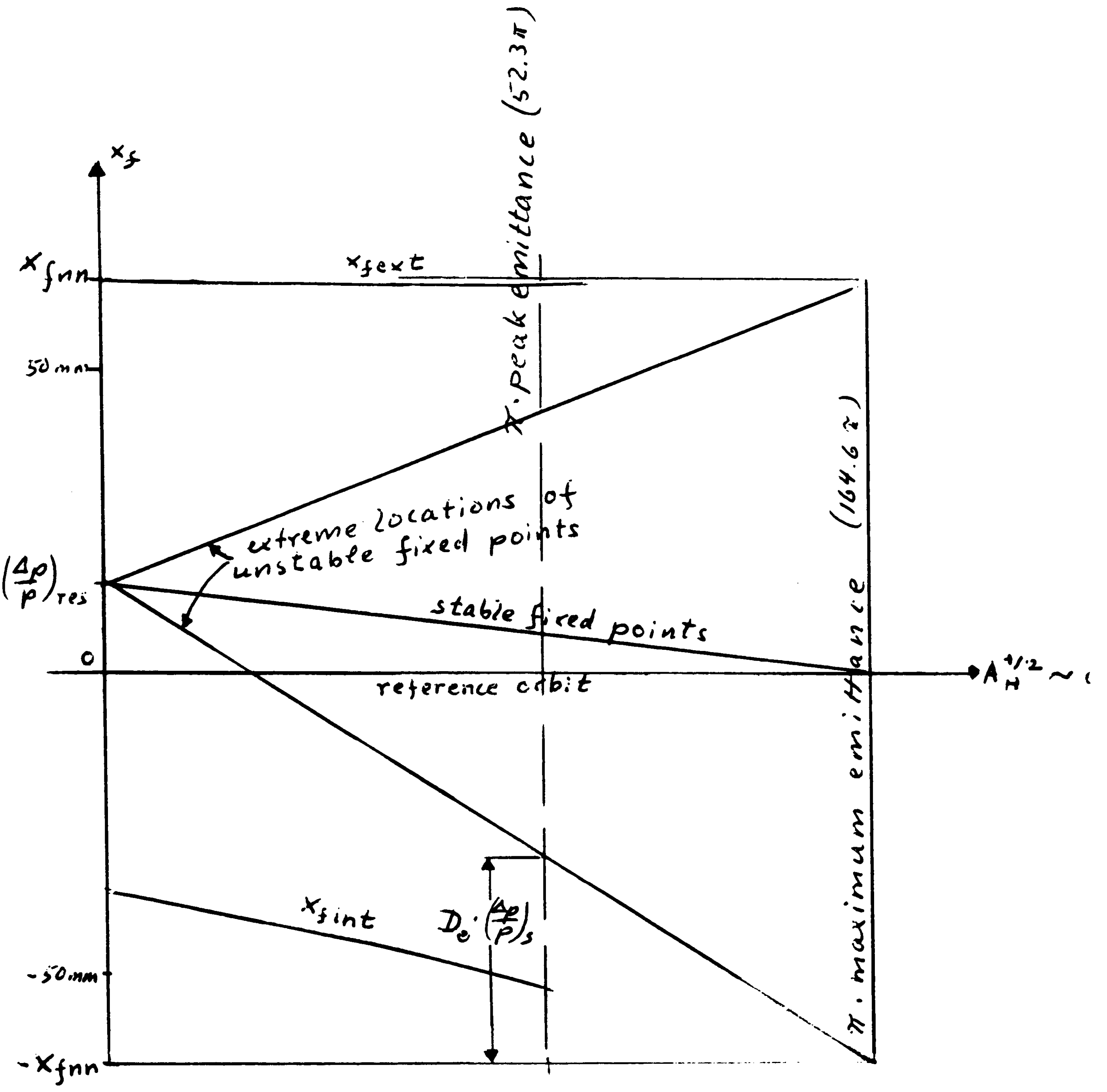
Fig. 2

Phase plane picture at $\psi = \psi_s \mod \frac{2\pi}{3}$ ($\varphi_e = \pi/6$, special case)



Configuration at SE

Fig. 3



Aperture economy

Fig 4.

DATA FILE:AGS81 AND THE Phie-INDEPENDENT VALUES:

Xm3/mm	Xfnn/mm	Kick/mrad	Dymb/mrad	Jump0/mm	Delps/E-3	Chrm'ty
55.000	65.000	5.000	0.000	3.000	10.000	aligned

THE ANGLE Phie= 6.985 DEG WITH S= 3.0050/m NEEDS THE K'-CHANGES IN XF(m-3):
ΔK'21=ΔK'22=ΔK'31= .2718;ΔK'32= .1770; ΔK'41=ΔK'42=ΔK'11 =-.2718; ΔK'12 =-.1770

(FOR TUNING THE CHROMAT'S FROM -1.31 TO Chrm HORIZ'LY AND FROM -1.98 TO ≈0 VERT'LY THE F SEXTUPOLES NEED THE K's:
K'21=K'22=K'31=-1.3379;K'32=-1.4327; K'41=K'42=K'11=-1.8814; K'12 =-1.7866
ALL 8 D-SEXTUP's NEED K'= 1.6911)

Chrm	Xe0/mm	Xe/mm	Ye/mrad	Xmb/mm	Delpr	Bump so that DYm0=0 in
3.603	34.880	43.607	2.247	12.696	4.174	normalized coordinates

Emit	Delpa	Dplft	Jump	Xm0	Ym0	Xmk	Xmkj	Xf11	Xfext	Xfint	Xbext	Xbint
3.00	0.00	22.93	3.000	55.00	-12.87	68.78	71.16	91.07	65.00	-36.08	47.19	-54.44
2.09	-.47	20.35	3.094	55.00	-12.87	68.78	71.23	91.23	64.72	-39.07	48.96	-55.39
3.37	-.94	17.76	3.080	55.00	-12.87	68.78	71.22	91.26	64.55	-42.15	50.84	-56.45
13.84	-1.41	15.17	2.956	55.00	-12.87	68.78	71.12	91.14	64.46	-45.33	52.80	-57.59
33.50	-1.88	12.59	2.723	55.00	-12.87	68.78	70.94	90.87	64.44	-48.57	54.82	-58.80
52.34	-2.35	10.00	2.379	55.00	-12.87	68.78	70.67	90.46	64.48	-51.87	56.91	-60.07
164.57	-4.17	-.00	-.000	55.00	-12.87	68.78	68.78	87.49	65.00	-65.00	65.35	-65.35

THE FIXED POINTS IN NORMALIZED COORDINATES AT SE ARE (STABLE F.P.WIHOUT BUMP):

Emit	(Xstp	Ystp)	Xst	Yst	Xin1	Yin1	Xin2	Yin2	Xin3	Yin3
3.00	4.327	25.255	8.727	52.162	8.727	52.162	8.727	52.162	8.727	52.162
2.09	3.839	22.406	8.239	49.314	12.661	52.644	3.144	51.478	8.912	43.819
3.37	3.351	19.558	7.751	46.465	16.595	53.126	-2.440	50.794	9.098	35.475
13.84	2.863	16.709	7.263	43.617	20.529	53.608	-8.023	50.110	9.283	27.132
33.50	2.375	13.861	6.775	40.768	24.464	54.090	-13.607	49.426	9.468	18.788
52.34	1.887	11.012	6.287	37.920	28.398	54.572	-19.190	48.741	9.653	10.445
164.57	-.000	-.000	4.400	26.907	43.607	56.436	-40.775	46.097	10.370	-21.811

********** END OF PROGRAM Eject **********

Fig 5.