PS/DL/LEAR Note 81-3
$\bar{p}p$ LEAR Note 93
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CHOICE OF A REFERENCE ORBIT INSIDE THE LEAR-MAGNET

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1. INTRODUCTION

Consider a magnet half-quadrant. It is made up of three blocks (central, inner, outer) which are positioned so that the centre of the good field region is kept tangentiel to an orbit of $\rho_G = 4.17$ m, each block having thus a length along the arc of $\ell = \frac{\pi}{12}\rho_G = 1.0197$ m.

For two reasons this arc can not be an orbit :

i.) The outer block has an effective length larger than ℓ by $\Delta\ell_Q$ due to the fringing field.

ii.) At the junction of the two half-quadrants (i.e. in the centre of the quadrant), a gap (of length g) was introduced by shortening each central block. g produces an effective magnetic hole of $2\Delta g$, say, which is related to the total vertical gap height H by

$$\Delta g = \frac{H}{\pi}\left(\frac{g}{H}\,\mathrm{arc\,tg}\,\frac{g}{H} - \frac{1}{2}\,\ell n\left(1 + \left(\frac{g}{H}\right)^2\right)\right)$$

Without further modifications the orbit would have the excessive peak to peak excursion of

$$\hat{r} = (\sqrt{2} - 1)\quad(\Delta\ell_o + \Delta g) \simeq 55\ \mathrm{mm}\ ;\ (\Delta g \simeq \Delta\ell_o \simeq 66\ \mathrm{mm})$$

2. SOLUTIONS

Various proposals are made in the following, in order to reduce the peak to peak excursion $\hat{r}$ to smaller values (between 7 and 16 mm). This is achieved by applying one or more of the following methods :

a) The outer block is shortened such that the magnetic length differs by a new value $\Delta\ell_o$ from ℓ, in particular $\Delta\ell_o$ might be put zero.

b) The bending radius of the central block is reduced by $\Delta\rho_c$ causing a maximum of the orbit deviation which forms together with the minimum deviation at the centre of the quadrant, the peak to peak value $\hat{r}$. Cases where this value is exceeded elsewhere are not admitted.

c) If the maximum is located in the central block ($\hat{\phi} \leq 15^o$), a spacer of the same thickness is inserted in the median plane of the inner and outer block as a means to increase their respective bending radii ρ_i and ρ_c (cases C, D).

If the maximum falls into the inner block ($15^o < \hat{\phi} < 30^o$) a spacer is only foreseen for the outer block (cases A, B, $\tilde{B}$).

d) In all cases, the parameters $\Delta\rho_c$, $\Delta\rho_o$ and $\Delta\ell_o$ were determined such that the orbit position at the magnet end remains at the centre of the good field region (and is centred with respect to the extreme values mentioned under b).

Each of the cases presented in table I is characterised by a particularity mentioned under remarks. In addition, the following statements are worth mentioning :

1) The bending radius in the central block is reduced by $\Delta\rho_c < 0$. There is a loss of maximum momentum attainable given by $\frac{\Delta p}{p} = \frac{\Delta\rho_c}{\rho_G}$ (see fig. 1).

2) The product of the peak to peak orbit excursion $\hat{r}$ and the loss of momentum is constant and given by

$$\hat{r} . \frac{\Delta p}{p} = - \frac{(\Delta g)^2}{2\rho_G}$$

This, of course, is an important reason, not to prefer simply the case with the smallest $\hat{r}$.

3) The spacer's thickness Δh results from both ρ_c and ρ_o. Assuming that the flux density in the mid-plane of the yoke equals that of the magnet gap, the spacer's action is doubled so that its thickness is given by

$$\Delta h = \frac{H}{2} \left(\frac{\rho_o}{\rho_c} - 1\right)$$

4) Clearly as the length of the modified orbit differs from the length of the geometric (naive) arc, (to which all comparisons are referred), there arises a circumference difference ΔC which is also found in table I. In order to re-establish the circumference, each magnet quadrant must be shifted towards the centre of LEAR by $\Delta C/(4\sqrt{2})$. The elements of the straight sections move towards the centre by $\Delta C/8$.

3. CONCLUSION

As two examples, the orbit excursions for B and $\tilde{B}$ are shown on fig. 2 as function of the geometric bending angle. Case B seems a good compromise which does not need too much shortening of the outer block, nor too big a spacer. The orbit excursion and the loss in maximum momentum seem both tolerable.

For convenience in fig. 3 and fig. 4 the Twiss functions are shown for case B (two working points valid on February 1, 1981).

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Table I: $\varrho_G = 4170$ mm ; $\Delta_g = 66.1$ mm

case	peak to peak orbit excursion $\hat{r}$/mm	relative loss of max. momentum $\frac{\Delta p}{p} \cdot 10^3$	$\Delta\ell_o$/mm	$\hat{\varphi}$	ΔC/mm $\frac{\Delta C}{8}\sqrt{2}$/mm	changes of bending radius $\varrho = \varrho_G + \Delta\varrho$	remarks (spacer thickness)
A	15.51	−33.78	−47.38	26.41°	22.79 4.03	$\left.\begin{matrix}\Delta\varrho_o\\ \Delta\varrho_i\\ \Delta\varrho_c\end{matrix}\right\} = -140.86$ mm	no spacer
B	12.34	−42.47	0.0	21.14°	15.53 2.75	$\Delta\varrho_o = 117.07$ mm $\left.\begin{matrix}\Delta\varrho_i\\ \Delta\varrho_c\end{matrix}\right\} = -177.1$ mm	$\Delta\ell_o = 0$ mm ($\Delta h_o = 2.95$ mm)
B̃	9.92	−52.82	64.5	17.07°	3.04 0.54	$\Delta\varrho_o = 469.47$ mm $\left.\begin{matrix}\Delta\varrho_i\\ \Delta\varrho_c\end{matrix}\right\} = -220.25$ mm	$\Delta\ell_o = \Delta\ell_{eff.}$ ($\Delta h_o = 6.98$ mm)
C	8.7	−60.2	−16.24	15°	15.12 2.67	$\left.\begin{matrix}\Delta\varrho_o\\ \Delta\varrho_i\end{matrix}\right\} = -28.13$ mm $\Delta\varrho_c = -251.04$ mm	$\hat{\varphi} = 15°$ ($\Delta h_i = \Delta h_o = 2.28$ mm)
D	7.37	−71.06	0.0	12.73°	8.18 1.45	$\left.\begin{matrix}\Delta\varrho_o\\ \Delta\varrho_i\end{matrix}\right\} = 25.76$ mm $\Delta\varrho_c = -296.33$ mm	$\Delta\ell_o = 0$ mm ($\Delta h_i = \Delta h_o = 3.33$ mm)

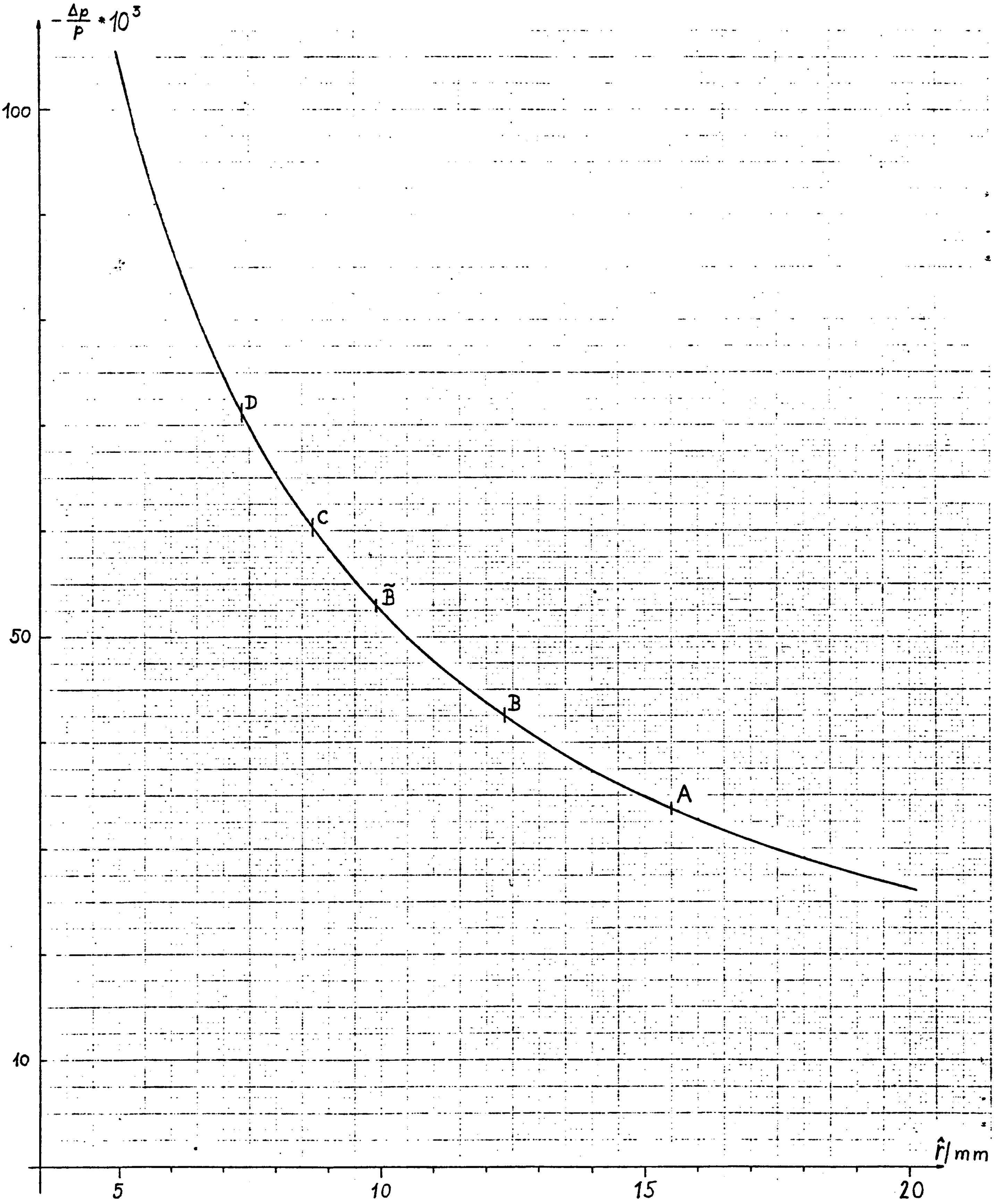

Fig. 1: Relative loss of maximum momentum versus peak to peak orbit excursion $\hat{r}$

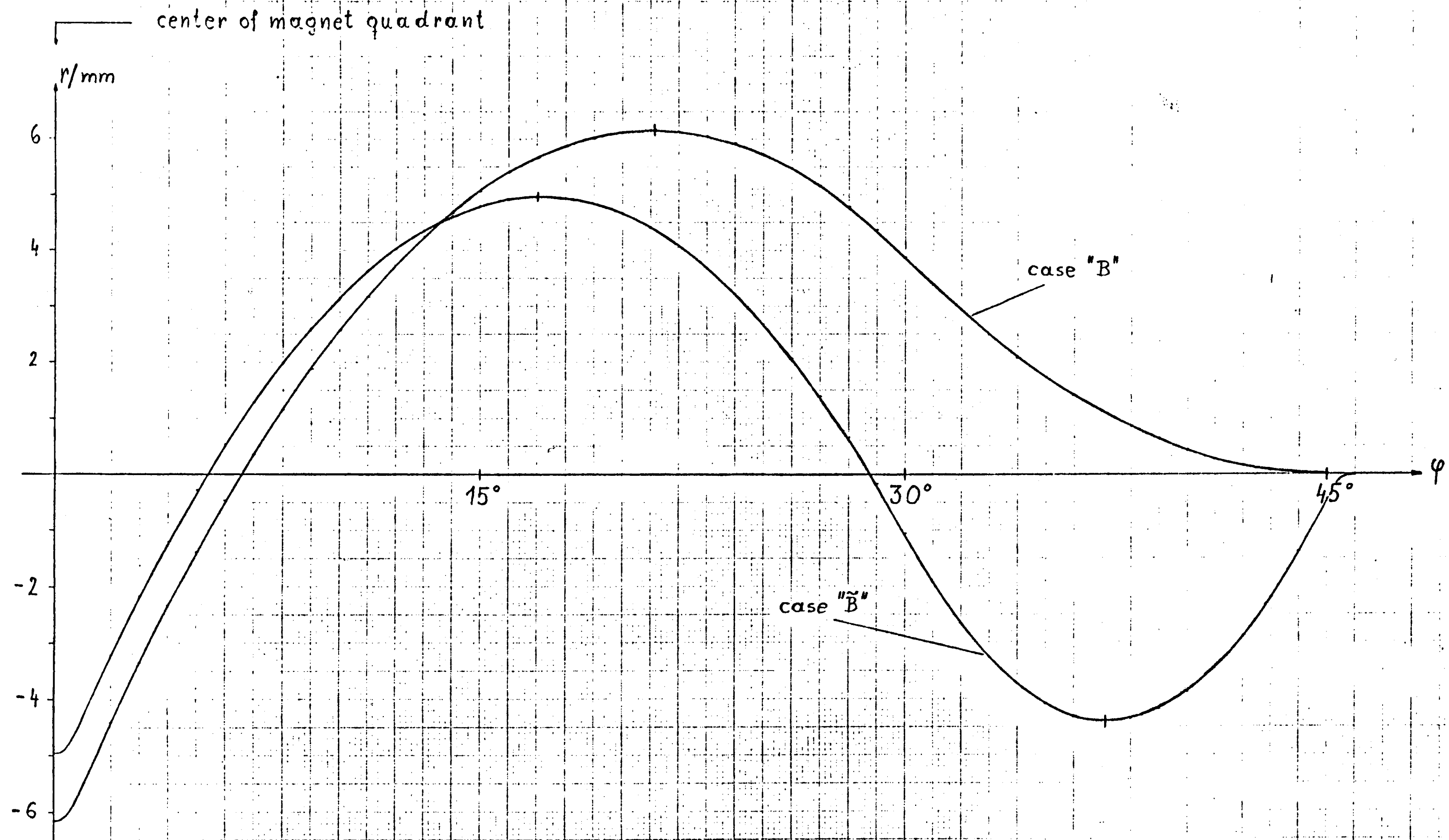

Fig. 2: Orbit excursion versus geometric angle

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 L E A R  REAL: *LWP* - CASE "B"                                                           AGS VERSION 75.03     10/02/81  10.58.43
CIRCUMFERENCE =   78.53982        4 HALF SUPERPERIODS                                 WEDGE MAGNETS    ALL VALUES AT EXIT OF ELEMENTS
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX


 NO  ELEM    L(M)      DIST(M)  ANG(MR)  K(1/M2)      BETAV(M)   BETAH(M)   ALPHAV     ALPHAH    MUV/2PI   MUH/2PI  ALPHAP(M)  ALPHAP'

 INITIAL                0.0000                         5.00743    1.96680   0.00000    0.00000   0.00000   0.00000   3.53655  -.000000
   1 SS       .5000      .5000  0.00000  0.0000000     5.05735    2.09391   -.09985    -.25422    .01584    .03962   3.53655  -.000000
   2 SS       .5000     1.0000  0.00000  0.0000000     5.20713    2.47524   -.19970    -.50844    .03137    .07486   3.53655  -.000000
   3 SS       .5000     1.5000  0.00000  0.0000000     5.45676    3.11079   -.29955    -.76266    .04632    .10370   3.53655  -.000000
   4 SS       .5000     2.0000  0.00000  0.0000000     5.80624    4.00056   -.39941   -1.01688    .06048    .12633   3.53655  -.000000
   5 SS       .5000     2.5000  0.00000  0.0000000     6.25557    5.14455   -.49926   -1.27110    .07370    .14391   3.53655  -.000000
   6 SS       .5000     3.0000  0.00000  0.0000000     6.80476    6.54277   -.59911   -1.52532    .08591    .15764   3.53655  -.000000
   7 SS       .5000     3.5000  0.00000  0.0000000     7.45379    8.19520   -.69896   -1.77954    .09709    .16852   3.53655  -.000000
   8 SS       .0455     3.5455  0.00000  0.0000000     7.51781    8.35819   -.70805   -1.80268    .09806    .16939   3.53655  -.000000
   9 CORRE    .1900     3.7355  0.00000  0.0000000     7.79408    9.06156   -.74599   -1.89928    .10201    .17287   3.53655  -.000000
  10 CORRE    .1900     3.9255  0.00000  0.0000000     8.08477    9.80165   -.78394   -1.99589    .10582    .17608   3.53655  -.000000
  11 GAP      .0990     4.0245  0.00000  0.0000000     8.24194   10.20181   -.80371   -2.04622    .10775    .17765   3.53655  -.000000
  12   QF     .2500     4.2745  0.00000 -1.2836607     9.35757   10.40519  -3.77751    1.25460    .11234    .18146   3.39563 -1.119818
  13   QF     .2500     4.5245  0.00000 -1.2836607    12.22477    9.01325  -7.99635    4.16345    .11611    .18552   2.98410 -2.150393
  14 CORRE    .1750     4.6995  0.00000  0.0000000    15.18618    7.61834  -8.92600    3.80748    .11815    .18888   2.60778 -2.150393
  15 CORRE    .1750     4.8745  0.00000  0.0000000    18.47296    6.34801  -9.85565    3.45150    .11982    .19288   2.23146 -2.150393
  16   QD     .2500     5.1245  0.00000  1.4337798    21.82732    5.23433  -3.15858    1.13551    .12177    .19989   1.78653 -1.435609
  17   QD     .2500     5.3745  0.00000  1.4337798    21.44620    5.14344   4.63725    -.76116    .12358    .20768   1.50289  -.850435
  18 GAP      .5830     5.9575  0.00000  0.0000000    16.39616    6.13524   4.02553    -.94016    .12853    .22423   1.00712  -.850435
  19 GAP      .5830     6.5404  0.00000  0.0000000    12.05932    7.33574   3.41382   -1.11916    .13513    .23807    .51135  -.850435
  20 FRING    .0645     6.6049 15.04510  0.0000000    11.62330    7.47971   3.34614   -1.11272    .13600    .23946    .45693  -.837089
  21   BOUT   .5138     7.1187119.84320  0.0000000     8.46193    8.58388   2.80701   -1.02609    .14425    .24965    .05535  -.724271
  22   BOUT   .5138     7.6325119.84320  0.0000000     5.85455    9.56831   2.26789    -.88079    .15587    .25865   -.28552  -.601063
  23   BMAG   .5459     8.1783136.70510  0.0000000     3.69134   10.39518   1.69512    -.62459    .17460    .26734   -.57268  -.449430
  24   BMAG   .5459     8.7242136.70510  0.0000000     2.15343   10.91512   1.12234    -.32200    .20561    .27547   -.77464  -.289412
  25   BMAG   .5459     9.2700136.70510  0.0000000     1.24082   11.08950    .54957     .00451    .25980    .28335   -.88764  -.123993
  26   BMAG   .4275     9.6975107.05250  0.0000000      .96272   10.97548    .10103     .26121    .32375    .28951   -.91261   .007318
  27 FRING    .0539     9.7514 13.49900  0.0000000      .95488   10.94561    .04447     .29297    .33270    .29029   -.91176   .023901
  28 HDEF     .0262     9.7776  0.00000  1.0000000      .95261   10.93783    .04194     .00365    .33708    .29067   -.91145   .000000
  29 GAP      .0399     9.8175  0.00000  0.0000000      .95094   10.93769   -.00000     .00000    .34375    .29125   -.91145   .000000
     DP/P   COSMU(H)        Q(H)  QPRIME(H)  BETAMAX(H)  COSMU(V)        Q(V)  QPRIME(V)  BETAMAX(V)     XMAX(H)   GAMMA TR.

   0.0000    -.86863      2.3300    -2.7183     11.0895   -.38268      2.7500    -5.7485     21.8273      3.5366     -5.2696

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TIME =     .037 SECONDS
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Fig. 3: AGS-output for case "B" ($Q_H = 2.33$, $Q_V = 2.75$)

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 L E A R  REAL; *UWP* - CASE "B"                                                                   AGS VERSION 75.03      10/02/81  11.03.12
CIRCUMFERENCE =    78.53982          4 HALF SUPERPERIODS                               WEDGE MAGNETS     ALL VALUES AT EXIT OF ELEMENTS
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX


 NO  ELEM     L(M)      DIST(M)  ANG(MR)  K(1/M2)      BETAV(M)    BETAH(M)    ALPHAV      ALPHAH     MUV/2PI     MUH/2PI    ALPHAP(M)   ALPHAP'

INITIAL                  0.0000                           5.24489     1.11291    0.00000     0.00000     0.00000     0.00000     2.47836    -.000000
  1  SS       .5000      .5000   0.00000  0.0000000       5.29256     1.33755    -.09533     -.44927      .01513      .06720     2.47836    -.000000
  2  SS       .5000     1.0000   0.00000  0.0000000       5.43555     2.01146    -.19066     -.89854      .02998      .11650     2.47836    -.000000
  3  SS       .5000     1.5000   0.00000  0.0000000       5.67388     3.13463    -.28599    -1.34781      .04433      .14841     2.47836    -.000000
  4  SS       .5000     2.0000   0.00000  0.0000000       6.00754     4.70708    -.38132    -1.79708      .05798      .16918     2.47836    -.000000
  5  SS       .5000     2.5000   0.00000  0.0000000       6.43653     6.72880    -.47665    -2.24635      .07079      .18334     2.47836    -.000000
  6  SS       .5000     3.0000   0.00000  0.0000000       6.96085     9.19979    -.57199    -2.69563      .08269      .19346     2.47836    -.000000
  7  SS       .5000     3.5000   0.00000  0.0000000       7.58050    12.12005    -.66732    -3.14490      .09366      .20100     2.47836    -.000000
  8  SS       .0455     3.5455   0.00000  0.0000000       7.64162    12.40810    -.67599    -3.18578      .09461      .20159     2.47836    -.000000
  9  CORRE    .1900     3.7355   0.00000  0.0000000       7.90538    13.65113    -.71222    -3.35650      .09850      .20392     2.47836    -.000000
 10  CORRE    .1900     3.9255   0.00000  0.0000000       8.18290    14.95904    -.74844    -3.52723      .10226      .20603     2.47836    -.000000
 11  GAP      .0990     4.0245   0.00000  0.0000000       8.33296    15.66624    -.76732    -3.61618      .10417      .20706     2.47836    -.000000
 12    QF     .2500     4.2745   0.00000  1.5828758       9.60667    15.91233   -4.49442     2.66449      .10869      .20954     2.35677    -.964641
 13    QF     .2500     4.5245   0.00000  1.5828758      13.12973    13.17404  -10.05948     7.92508      .11229      .21224     2.00395   -1.834635
 14  CORRE    .1750     4.6995   0.00000  0.0000000      16.88891    10.54859  -11.42156     7.07749      .11416      .21461     1.68289   -1.834635
 15  CORRE    .1750     4.8745   0.00000  0.0000000      21.12482     8.21979  -12.78365     6.22990      .11563      .21760     1.36183   -1.834635
 16    QD     .2500     5.1245   0.00000  1.5618576      25.58367     6.03966   -4.46760     2.77255      .11732      .22334      .96268   -1.384473
 17    QD     .2500     5.3745   0.00000  1.5618576      25.30731     5.26326    5.53686      .43347      .11886      .23051      .65827   -1.070561
 18  GAP      .5830     5.9575   0.00000  0.0000000      19.27689     4.83456    4.80764      .30190      .12306      .24895      .03418   -1.070561
 19  GAP      .5830     6.5404   0.00000  0.0000000      14.09668     4.55927    4.07842      .17033      .12869      .26876     -.58991   -1.070561
 20  FRING    .0645     6.6049  15.04510  0.0000000      13.57577     4.53721    3.99773      .17169      .12943      .27102     -.65841   -1.053325
 21    BOUT   .5138     7.1187119.84320  0.0000000       9.79806     4.35722    3.35505      .17695      .13652      .28949    -1.16282    -.907852
 22    BOUT   .5138     7.6325119.84320  0.0000000       6.68074     4.17703    2.71236      .17208      .14663      .30857    -1.58905    -.749355
 23    BMAG   .5459     8.1783136.70510  0.0000000       4.09236     3.98691    2.02956      .17405      .16328      .32986    -1.94474    -.551849
 24    BMAG   .5459     8.7242136.70510  0.0000000       2.24940     3.80173    1.34676      .16308      .19207      .35218    -2.18963    -.344046
 25    BMAG   .5459     9.2700136.70510  0.0000000       1.15185     3.63526     .66396      .14000      .24713      .37556    -2.31916    -.129824
 26    BMAG   .4275     9.6975107.05250  0.0000000        .81278     3.52603     .12926      .11455      .31996      .39457    -2.33842     .039827
 27  FRING    .0539     9.7514  13.49900  0.0000000        .80248     3.51388     .06184      .11094      .33059      .39700    -2.33569     .061228
 28  HDEF     .0262     9.7776   0.00000  1.0000000        .79955     3.51067     .05000      .01136      .33580      .39819    -2.33489     .000000
 29  GAP      .0399     9.8175   0.00000  0.0000000        .79756     3.51022    -.00000      .00000      .34375      .40000    -2.33489     .000000
     DP/P  COSMU(H)          Q(H)  QPRIME(H)  BETAMAX(H)  COSMU(V)         Q(V)  QPRIME(V)  BETAMAX(V)      XMAX(H)   GAMMA TR.

   0.0000    .30902        3.2000    -5.0760     15.9123   -.38268       2.7500    -7.3082     25.5837       2.4784     -2.6456

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TIME =     .038 SECONDS
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Fig. 4: AGS - output for case "B" ($Q_H = 3.2$, $Q_V = 2.75$)