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NOTE ON COULOMB SCATTERING IN THIN TARGETS

SUMMARY. It will be shown that Coulomb scattering is an effective deflecting device. Under certain conditions of the circulating beam the Coulomb scattering may be superior to the deflection method based on momentum loss through collisions and may be useful for a slow ejection system, or in emulsion techniques.

I. INTRODUCTION.

In the slow ejection system CERN 59-21 targets are used which shift the equilibrium orbit of a passing particle by an amount of 3 mm, due to collision loss. The relevant formula for the collision loss is

$$- \frac{dE}{dx} = 0,3 \frac{Z}{A} \left[\ln \frac{10^6}{(1-\beta^2)\, I} - 1 \right] \text{ MeV cm}^2 \text{ g}^{-1} \qquad (1)$$

where x is expressed in gr/cm^2 and I is the average ionisation potential (eV) of the target material.

The orbit shift is according to the momentum compaction formula

$$\Delta r/r = 0,027\ \Delta p/p \qquad (2)$$

The possible clearance with the circulating proton beam can be shown to be at the most twice the orbit shift, i.e. in the above example 6 mm.

Associated with collision loss is scattering. The relevant formula is the rms angle of scattering

$$\theta_{rms} = 0,6\ 10^{-3}\ (x/x_o)^{1/2} \qquad (3)$$

in which x_o is the radiation length expressed in gr/cm^2.
We note that for the tungsten target of CERN 59-21 this angle is about 10^{-3} radian.

For a scattering angle of this order we find that the possible clearance from the circulating beam is substantially larger than the 6 mm which could be obtained through collision loss. If the clearance has to be in the median plane (to accommodate a pulsed deflecting magnet or a stack of photographic emulsion), we put the target in the vertical plane of symmetry and just touching the circulating beam. For relative distances of k + 1/4 betatron wavelengths (k = integer) the transformation matrix is

$$M = \begin{bmatrix} 0 & \beta \\ -\frac{1}{\beta} & 0 \end{bmatrix} \tag{4}$$

and one appreciates that a scattering angle θ_s in the target gives a displacement equal to $\beta\theta_s$ superimposed on the displacement the particle in question nomally would have. Between F sections β is about 22 m, so that the displacement is about 22mm per m rad, and thus a factor of 3 to 4 larger than the collision loss displacement.

However, only a small fraction of the intercepted particles will get a clearance from the beam envelope larger than a given amount. Relevant for this is the distribution function for the scattering, which we assume to be Gaussian:

$$Q(\theta_s) = \frac{1}{\sqrt{2\pi}} \frac{1}{\theta_{rms}} \exp\left(- \frac{\theta_s^2}{2\theta_{rms}^2}\right) \tag{5}$$

In addition we have the distribution in angle of the incident particle, which can be anything depending on the history of the beam formation, but must be symmetric with respect to zero angle of incidence, and zero for angles of incidence larger than the maximum angle of incidence compatible with the beam radius. The calculation is feasible if we assume that

$$\begin{aligned} Q(\theta_i) &= \frac{1}{2\hat{\theta}_i} \qquad \text{for } |\theta_i| \leq \hat{\theta}_{max} \\ &= 0 \qquad \text{for } |\theta_i| > \hat{\theta}_{max} \end{aligned} \tag{6}$$

in which $\hat{\theta}_i$ is related to the beam radius by

$$R = \beta\,\hat{\theta}_i \tag{7}$$

II THE COMPOSITE DISTRIBUTION.

From the foregoing one sees that the distribution in angle (θ) of all particles leaving the target is given by:

$$Q(\theta) = \frac{1}{2\hat{\theta}_i} \frac{1}{\sqrt{2\pi}} \frac{1}{\theta_{rms}} \int_{-\hat{\theta}_i}^{+\hat{\theta}_i} \exp - \frac{(\theta - \theta_i)^2}{2\theta_{rms}^2} d\theta_i \qquad (8)$$

The fraction of the scattered particles found within a given interval is obtained with a further integration:

$$P = \int_{\theta_1}^{\theta_2} Q(\theta)\, d\theta \qquad (9)$$

and the corresponding interval of the possible displacement from the equilibrium orbit is found with

$$\Delta r = \beta(\theta_2 - \theta_1) \qquad (10)$$

Formula (8) is of the type $\int \exp(-t^2/2)\, dt$ and tabled in several hand-books. Having found $Q(\theta)$, one may evaluate formula (9) numerically. By trial and error one may determine a suitable set of parameters for a given problem.

III SLOW EJECTION SYSTEM.

The requirement is thus to catch as many protons in a pulsed deflecting magnet through the mechanism of scattering in a thin target. Target and deflecting magnet may be put conveniently in the same F section, because M is conform (4). We know that the deflecting magnet can be made practically without stray field, see PS/Int. EA 59-6, so that it can be brought very close to the circulating beam. However, we shall assume here that the narrow tolerances, required for setting up a slow ejection system with collision loss as mechanism (see CERN 59-21), can for some reason or another not be maintained. Accordingly we design the present system for larger clearances, and although the efficiently may drop, we provide for a system, which works under conditions, where the collision loss method would be very difficult to realise.

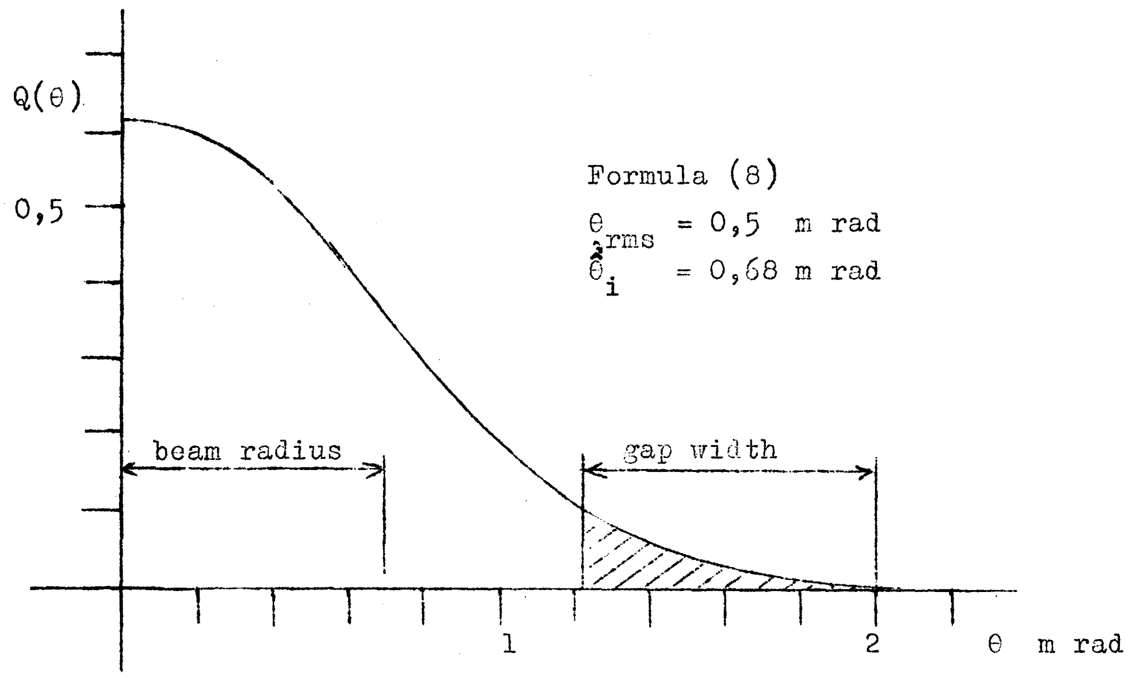
For the same reason we take the beam diameter somewhat larger than previously assumed.

Several parameters are now more or less fixed.

a) with a beam radius of 15 mm we find $\hat{\theta}_i$ = 15/22 m rad.
b) with a minimum clearance of 12 mm we find θ_1 = 27/22 m rad.
c) with a magnet gap width of 17 mm we find θ_2 = 44/22 m rad.
d) we select the trial value θ_{rms} = 0,5 m rad.

The latter choice is based on the following. The target should not be too thick and the collision loss should be negligible. Applying (3) and (1) respectively, one finds for a tungsten target a thickness (i.e. measured in the direction of the beam) of about 2 mm and a collision loss of about 7 MeV/c. A low value for θ_{rms} is suitable for reducing the height of the magnetic gap of the deflector. Relevant for this is the distribution function formula (5), but now considered in the vertical plane. We calculate thus the probability that the scattered particle is found within the limits given by this gap height. The value of β belonging to the vertical motion is about 15 m, so that with a gap height of 15 mm, the limits in the integral are ± 0,5 m rad, and so

$$P_{vert} = \frac{1}{\sqrt{2\pi}} \frac{1}{0,5} \int_{-0,5}^{+0,5} \exp\left(-\frac{\theta_s^2}{2 \cdot 0,5^2}\right) d\theta_s = 0,68 \qquad (11)$$



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In the figure is plotted $Q(\theta)$ of formula (8). The shaded area is the fraction which falls within the limits of the width of the magnet gap. One appreciates that on the negative θ axis an equal fraction is found, which is also admitted by the magnet, because after two more complete revolutions the matrix (4) has changed sign. The admitted fraction in the horizontal direction is then found to be 4,5%, and this multiplied with the admitted fraction in the vertical direction (11) gives an efficiency of 3%.

However, this efficiency is not the final figure, because the scattered particles, which are not taken away by the magnet (and this is a large fraction), may be scattered again in the target. Probably the vertical efficiency of the magnet drops for multiple scattered particles and the horizontal efficiency remains substantially the same. Detailed calculations are here required, but it seems very well possible to obtain in the end an overall efficiency of 10% under far less strin gent conditions as required for the collision loss method (Piccioni system). The key formula used in this report is the Gaussian distribution function. Perhaps a closer investigation is needed to make sure that also the tail of this curve is correct.

One will appreciate that the magnet can be put on either side of the circulating beam. There exists then the possibility that only one deflecting magnet (in section No. 1) will do to deflect the particles in the Experimental Hall. The target width (i.e. measured in the direction perpendicular to the circulating beam) can be as thin as technically possible. Probably alignment problems do not exist. The "feather weight" target moves with the magnet and may be monitored for constant interception. Possibly one could use the resistance effect in the target itself to obtain the monitoring signal. The moving mechanism could be a moving coil arrangement with a response of several hundred Hertz.

IV EMULSION WORK

The system consists of a stack of photographic plates located for instance in the median plane several cm from the circulating beam, and a target located in the vertical plane in the same section.

A very low track density is required which can be achieved through any of the following means:

a) decrease of θ_{rms}
b) increase of θ_1
c) short interception time
d) decrease of target width.

The technical limits of these quantities provide ample possibilities to reduce P, formula (9), to any desired low value. The reproducibility of the experiment is another question and probably considerable waste of material is involved. However, in each individual stack one finds probably several layers with just the appropriate amount of track density.

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